Effects of High-Power Amplification on Linear Multiuser Detectors Performance in DS-CDMA Frequency-Selective Fading Channels

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***Abstract*-The aim of this paper is to examine the effects of the nonlinear distortions introduced by high-power amplifiers (HPAs) in the downlink of direct-sequence code-division multiple-access (DS-CDMA) systems. By modelling the nonlinear distortion noise as coloured Gaussian noise, a semi-analytical expression for the symbol-error rate (SER) of linear multiuser detectors (LMDs) in frequency-selective fading channels has been derived. Simulation results confirm the effectiveness of the theoretical approach.**

I. INTRODUCTION

DS-CDMA is a widely employed technique in satellite and cellular mobile communication systems. This technique is characterised by an almost constant envelope for uplink transmissions, thereby reducing, with respect to other techniques like multicarrier CDMA, the system sensitivity to nonlinear amplifiers. Anyway, such a characteristic is lost in the downlink scenario, where the DS-CDMA signal transmitted by the base station is the superposition of many independent signals that belong to the different users. Thus the resulting signal has a non constant envelope as a consequence of the constructive and destructive superposition of each user contribution. Consequently, for downlink transmissions, the performance degradation introduced by the nonlinear HPA has to be carefully taken into account. Indeed, even if a predistortion technique [1] is applied to counteract the HPA nonlinearity, a residual clipping cannot be avoided because of the maximum amplifier output power.

The nonlinear distortion effects and the consequent SER performance degradation induced in the system link budget have been analysed in [2] for the matched filter (MF) detection of DS-CDMA signals in AWGN channels, and in [3] for the linear decorrelating detector (LDD) in AWGN and flat Rayleigh fading channels. Although the frequency-flat condition can be appropriate under some circumstances, in many cases DS-CDMA systems are subject to frequency-selective fading due to the multipath propagation. This effect is especially highlighted in CDMA cellular systems (e.g. IMT-2000) characterised by a wide frequency band. Since frequency-selective channels destroy the user orthogonality, high performance improvements can be obtained making use of LMD techniques [4] because of their capability in reducing the multiuser interference at the receiver side.

In this paper, we consider the SER evaluation of LMDs in presence of a HPA at the transmitter. Firstly, we extend to the minimum mean-squared error (MMSE) detector some of the results obtained in [2][3] for AWGN channels. Moreover, the degradation induced by the nonlinear amplifier is analysed in frequency-selective channels, obtaining the SER expression for quaternary phase-shift keying (QPSK) modulations. Simulation results, which validate the analytical approach, are presented for the MMSE, the LDD, and the RAKE receivers.

II. SYSTEM MODEL

The baseband signal transmitted by the base station to the kth user, in the downlink of a DS-CDMA system, is expressed by[1]

$$x_k(t) = A_k \sum_{i=-\infty}^{+\infty} b_k[i] s_k(t - iT), \tag{1}$$

where T is the symbol duration, A_k and $s_k(t)$ are the amplitude and the spreading waveform respectively, and $b_k[i]$ is the ith symbol of the user k. The spreading waveform is expressed by

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} c_k[j] p(t - jT_c), \tag{2}$$

where N is the processing gain, $T_c = T/N$ is the chip duration, $p(t)$ is the chip pulse shaping waveform and $c_k[j]$ is the jth value of the kth user spreading code, with $|c_k[j]| = 1$. The symbols $\{b_k[i]\}$ belong to a set of independent and equiprobable random variables, with $E\{|b_k[i]|^2\} = 1$.

The base station transmits synchronously the sum of the signals belonging to each user by a HPA that, if supposed to be instantaneous, can be modelled by its AM/AM and AM/PM distortion curves $G(\cdot)$ and $\Phi(\cdot)$ respectively [1], or, equivalently, by a complex nonlinear distortion $F(\cdot) = G(\cdot)\exp[j\Phi(\cdot)]$. Hence the sum $x(t)$ of the K users' signals

$$x(t) = \sum_{k=1}^{K} x_k(t) = |x(t)| e^{j \arg(x(t))} \tag{3}$$

is transformed by $F(\cdot)$ into

$$w(t) = F(|x(t)|) e^{j \arg(x(t))}, \tag{4}$$

which represents the baseband input-output relationship for the nonlinear amplifier. The output signal $w(t)$ can be expressed as the input signal multiplied by a complex coefficient α_0, which represents the average linear amplification gain, plus a nonlinear distortion complex noise $n_d(t)$, as expressed by

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[1] *Notations*: We use lower (upper) bold face letters to denote vectors (matrices), superscripts *, ^{T}, ^{H} and † to represent complex conjugate, transpose, Hermitian and Moore-Penrose pseudo-inverse operators, respectively, $E\{\cdot\}$ to represent the statistical expectation, $\lceil x \rceil$ and $\mathrm{csgn}(x)$ to denote the smallest integer greater than x and the complex signum of x, respectively. The Q-function is defined as $\mathrm{Q}(x) = (2\pi)^{-1/2}\int_x^{+\infty} \exp(-v^2/2)dv$. The symbols $*$, and $\otimes$ denote the convolution operator and the Kronecker matrix product, respectively, $\mathbf{0}_{M\times N}$ is the $M \times N$ all-zero matrix, and $\mathbf{I}_N$ is the $N \times N$ identity matrix. We define $(\mathbf{A})_{m,n}$ as the (m,n)th entry of the matrix $\mathbf{A}$, $(\mathbf{A})_{:,n}$ ($(\mathbf{A})_{m,:}$) as the nth column (mth row) vector of the matrix $\mathbf{A}$, $(\mathbf{a})_{m,1}$ ($(\mathbf{a})_{1,n}$) as the mth (nth) entry of the column (row) vector $\mathbf{a}$.

$$w(t) = \alpha_0 x(t) + n_d(t) . \quad (5)$$

This paper assumes that the signal $w(t)$, which is transmitted by the base station, passes through a slowly varying multipath channel, characterised by an impulse response

$$g(\tau) = \sum_{q=1}^{Q} \beta_q e^{j\theta_q} \delta(\tau - \tau_q) , \quad (6)$$

where Q is the number of paths, and β_q, θ_q and τ_q are the gain, the phase-shift, and the propagation delay of the qth path of the channel, respectively, and $\delta(\tau)$ is the delta Dirac function. By exploiting (5), the signal $r(t)$ at the input of the receiver can be expressed by

$$r(t) = \int_{-\infty}^{+\infty} g(\tau) w(t-\tau) d\tau + n(t) = r_{\text{SIG}}(t) + r_{\text{NL}}(t) + n(t) , \quad (7)$$

where $r_{\text{SIG}}(t) = \sum_{q=1}^{Q} \beta_q |\alpha_0| e^{j\theta_q + j\arg(\alpha_0)} x(t-\tau_q)$ is the useful received signal, $r_{\text{NL}}(t) = \sum_{q=1}^{Q} \beta_q e^{j\theta_q} n_d(t-\tau_q)$ is the nonlinear distortion noise, and $n(t)$ is the thermal noise.

At the receiver side, assuming perfect synchronisation and channel state information, $r(t)$ is filtered by a chip-matched filter, and successively sampled at the chip rate, obtaining

$$\begin{aligned} r_n[l] &= \int_{-\infty}^{+\infty} r(t) p^*(t - lT - nT_c) dt \\ &= r_{n,\text{SIG}}[l] + r_{n,\text{NL}}[l] + r_{n,\text{AWGN}}[l] \end{aligned} . \quad (8)$$

The received chip $r_n[l]$, expressed by (8), is characterised by three additive parts: $r_{n,\text{SIG}}[l] = \int_{-\infty}^{+\infty} r_{\text{SIG}}(t) p^*(t - lT - nT_c) dt$ is the useful component related to $x(t)$, $r_{n,\text{NL}}[l] = \int_{-\infty}^{+\infty} r_{\text{NL}}(t) p^*(t - lT - nT_c) dt$ is the in-band nonlinear distortion noise introduced by the quantity $n_d(t)$, and $r_{n,\text{AWGN}}[l] = \int_{-\infty}^{+\infty} n(t) p^*(t - lT - nT_c) dt$ is the in-band thermal noise with power $\sigma_{\text{AWGN}}^2 = E\{|r_{n,\text{AWGN}}[l]|^2\}$.

Assuming that the maximum delay spread of the channel is smaller than the symbol interval (i.e. $\max\{\tau_Q\} < T$), the channel spreads the information related to a particular symbol over two symbol intervals, and consequently $2N$ consecutive chips contain all the energy related to the symbol of interest. Hence, we will consider a receiving window of two symbol intervals.

Defining the channel order $L = \lceil \max\{\tau_Q\}/T_c \rceil$, and $\mathbf{r}[l] = [r_0[l] \cdots r_{N-1}[l]]^T$, $\mathbf{r}_{\text{NL}}[l] = [r_{0,\text{NL}}[l] \cdots r_{N-1,\text{NL}}[l]]^T$, $\mathbf{r}_{\text{AWGN}}[l] = [r_{0,\text{AWGN}}[l] \cdots r_{N-1,\text{AWGN}}[l]]^T$, $\mathbf{A} = \text{diag}(A_1, \cdots, A_K)$, $\mathbf{b}[l] = [b_1[l] \cdots b_K[l]]^T$, $g[l] = \left[\int_{-\infty}^{+\infty} g(\tau) R_{pp}(t-\tau) d\tau \right]_{t=lT_c}$, where $R_{pp}(\tau)$ is the autocorrelation function of the pulse waveform $p(t)$, $\underline{\mathbf{r}}[l] = [\mathbf{r}[l]^T \ \mathbf{r}[l+1]^T]^T$, $\underline{\mathbf{r}}_{\text{NL}}[l] = [\mathbf{r}_{\text{NL}}[l]^T \ \mathbf{r}_{\text{NL}}[l+1]^T]^T$, $\underline{\mathbf{r}}_{\text{AWGN}}[l] = [\mathbf{r}_{\text{AWGN}}[l]^T \ \mathbf{r}_{\text{AWGN}}[l+1]^T]^T$, $\underline{\mathbf{A}} = \mathbf{I}_3 \otimes \mathbf{A}$, $\underline{\mathbf{b}}[l] = [\mathbf{b}[l-1]^T \ \mathbf{b}[l]^T \ \mathbf{b}[l+1]^T]^T$,

$$\underline{\mathbf{G}} = \begin{bmatrix} g[L] & \cdots & g[0] & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & g[L] & \cdots & g[0] \end{bmatrix}_{2N \times 2N+L} , \quad (9)$$

$$\mathbf{C} = \frac{1}{\sqrt{N}} \begin{bmatrix} c_1[0] & \cdots & c_K[0] \\ \vdots & \cdots & \vdots \\ c_1[N-1] & \cdots & c_K[N-1] \end{bmatrix} \quad \underline{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{C}} & \mathbf{0}_{L\times K} & \mathbf{0}_{L\times K} \\ \mathbf{0}_{N\times K} & \mathbf{C} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{N\times K} & \mathbf{0}_{N\times K} & \mathbf{C} \end{bmatrix} , \quad (10)$$

where $\tilde{\mathbf{C}} = (\mathbf{C})_{N-L+1:N,:}$, we obtain

$$\underline{\mathbf{r}}[l] = |\alpha_0| e^{j\arg(\alpha_0)} \underline{\mathbf{G}} \underline{\mathbf{C}} \underline{\mathbf{A}} \underline{\mathbf{b}}[l] + \underline{\mathbf{r}}_{\text{NL}}[l] + \underline{\mathbf{r}}_{\text{AWGN}}[l] . \quad (11)$$

Focusing on the user k, the estimate $\hat{b}_k[l]$ of the QPSK symbol $b_k[l]$ is obtained by applying the $2N$-row vector $\underline{\mathbf{d}}_k$ to the received signal $\underline{\mathbf{r}}[l]$ in (11), as expressed by

$$\hat{b}_k[l] = 2^{-1/2} \text{csgn}(\underline{\mathbf{d}}_k \underline{\mathbf{r}}[l]) . \quad (12)$$

The extension to other constellation mapping, or longer delay spreads, is straightforward.

III. SER Performance in Nonlinear AWGN Channels

First of all, we survey some known theoretical results for AWGN channels, which turn out to be useful for frequency-selective channels also. The background hypotheses are herein summarised as:

i) High number K of active users, or equivalently high number K of physical channels, which belong to $\tilde{K}$ users with $\tilde{K} < K$ (multicode transmission). Hence, also the processing gain N must be sufficiently high.

ii) The amplitudes $\{A_k\}$ are almost equal.

Under these assumptions, the HPA input $x(t)$ in (3) can be approximated by a cyclostationary Gaussian random process, making possible the application of the Bussgang theorem as in [2]. As a consequence of this theorem, the linear component $\alpha_0 x(t)$ and the nonlinear one $n_d(t)$ in (5) are mutually uncorrelated, and the HPA output autocorrelation function can be evaluated [5] as $R_{ww}(\tau) = |\alpha_0|^2 R_{xx}(\tau) + R_{n_d n_d}(\tau)$, where all the quantities are averaged over the cyclostationarity period T [6], with

$$R_{n_d n_d}(\tau) = \sum_{i=1}^{+\infty} \gamma_i R_{xx}(\tau)^{2i+1} . \quad (13)$$

The coefficients $\{\gamma_i\}$ and α_0, which depend on the HPA function $F(\cdot)$ and on the HPA input power σ_x^2, can be calculated as in [5] or as in [7].

In AWGN channels, since $L = 0$, the channel matrix $\underline{\mathbf{G}}$ in (9) becomes $\underline{\mathbf{G}} = g[0]\mathbf{I}_{2N}$, and a receiving window of one symbol interval (N samples) is wide enough to recover the symbol of interest. In this situation, it holds true

$$\mathbf{r}[l] = |\alpha_0 g[0]| e^{j\varphi_0} \mathbf{C} \mathbf{A} \mathbf{b}[l] + \mathbf{r}_{\text{NL}}[l] + \mathbf{r}_{\text{AWGN}}[l] , \quad (14)$$

where $\varphi_0 = \arg(\alpha_0) + \arg(g[0])$ is the phase-shift due to the channel and the HPA. Therefore the receiver can be expressed by a N-row vector

$$\mathbf{d}_k = e^{-j\varphi_0} (\mathbf{D})_{k,:} , \quad (15)$$

where $\mathbf{D}$ is a $K \times N$ matrix that depends on the used detector. For the MF receiver, the detector acts like a simple despreader, therefore

$$\mathbf{D}_{\mathrm{MF}} = \mathbf{C}^H , \tag{16}$$

and the SER performance can be obtained in closed form by modelling the multiple-access interference (MAI) and the nonlinear distortion noise as Gaussian random variables. These approximations are reasonably justified by the high number K of active users and by the high processing gain N [2]. Hence, we obtain

$$P_{\mathrm{AWGN},k} = f\left(\sqrt{\sigma^2_{\mathrm{SIG},k} / (\sigma^2_{\mathrm{MAI},k} + \sigma^2_{\mathrm{NL},k} + \sigma^2_{\mathrm{AWGN},k})}\right), \tag{17}$$

where $f(x) = 2\mathrm{Q}(x) - \mathrm{Q}^2(x)$, $\sigma^2_{\mathrm{AWGN},k} = \sigma^2_{\mathrm{AWGN}}$, $\sigma^2_{\mathrm{SIG},k} = |\alpha_0 g[0]|^2 A_k^2$, $\sigma^2_{\mathrm{MAI},k} = |\alpha_0 g[0]|^2 \sum_{\substack{j=1\\ j\neq k}}^{K} (\mathbf{R})^2_{k,j} A_j^2$, and $\mathbf{R} = \mathbf{C}^H \mathbf{C}$ is the matrix containing the crosscorrelation coefficients of the users' spreading codes. In [2], the nonlinear distortion noise power $\sigma^2_{\mathrm{NL},k}$ has been evaluated supposing a rectangular chip pulse shaping waveform $p(t)$, leading to $\sigma^2_{\mathrm{NL},k} = |g[0]|^2 R_{ww}(0) - |\alpha_0 g[0]|^2 R_{xx}(0)$. Anyway, this assumption is not always realistic in bandlimited channels. By supposing a bandlimited pulse shaping waveform $p(t)$, the nonlinear distortion noise power can be evaluated as $\sigma^2_{\mathrm{NL},k} = |g[0]|^2 (\mathbf{C}^H \boldsymbol{\psi} \mathbf{C})_{k,k}$, where the $N \times N$ matrix $\boldsymbol{\psi}$ is expressed by $(\boldsymbol{\psi})_{m,n} = [R_{n_d n_d}(\tau) * R_{pp}(\tau)]\big|_{\tau=(n-m)T_c}$, and $R_{n_d n_d}(\tau)$ is computed using (13).

If we use the decorrelating detector [4], as expressed by

$$\mathbf{D}_{\mathrm{LDD}} = \mathbf{C}^{\dagger} , \tag{18}$$

and assuming that the code matrix $\mathbf{C}$ is full rank, the MAI is completely eliminated, and the SER performance can be expressed [3] as in (17), with $\sigma^2_{\mathrm{NL},k} = |g[0]|^2 (\mathbf{C}^{\dagger} \boldsymbol{\psi} \mathbf{C}^{\dagger H})_{k,k}$, $\sigma^2_{\mathrm{SIG},k} = |\alpha_0 g[0]|^2 A_k^2$, $\sigma^2_{\mathrm{AWGN},k} = (\mathbf{R}^{-1})_{k,k} \sigma^2_{\mathrm{AWGN}}$, where the quantity $(\mathbf{R}^{-1})_{k,k}$ is the thermal noise amplification factor due to the decorrelating operation, and $\sigma^2_{\mathrm{MAI},k} = 0$. Also in this case the nonlinear noise $(\mathbf{C}^{\dagger} \mathbf{r}_{\mathrm{NL}}[l])_{k,1}$ can be considered Gaussian because of the high processing gain N [3].

A scaled version of the MMSE receiver is expressed by [8]

$$\mathbf{D}_{\mathrm{MMSE}} = \mathbf{M} , \tag{19}$$

where $\mathbf{M} = (\mathbf{I}_K + |\alpha_0 g[0]|^2\, \mathbf{A}\mathbf{C}^H \mathbf{W}^{-1} \mathbf{C}\mathbf{A})^{-1} \mathbf{A}\mathbf{C}^H \mathbf{W}^{-1}$, and $\mathbf{W}^{-1}$ is the inverse of the nonlinear-plus-thermal noise covariance matrix $\mathbf{W} = |g[0]|^2\, \boldsymbol{\psi} + \sigma^2_{\mathrm{AWGN}} \mathbf{I}_N$. As far as the SER of the MMSE detector is concerned, we propose as in [3] to model the nonlinear distortion noise as Gaussian. Moreover, even the residual MAI at the MMSE output is well approximated by a Gaussian random variable, as shown in [9], leading to a SER that can be expressed by (17) as well, where $\sigma^2_{\mathrm{SIG},k} = |\alpha_0 g[0]|^2 A_k^2 (\mathbf{MC})^2_{k,k}$, $\sigma^2_{\mathrm{NL},k} = |g[0]|^2 (\mathbf{M}\boldsymbol{\psi}\mathbf{M}^H)_{k,k}$, $\sigma^2_{\mathrm{AWGN},k} = (\mathbf{M}\mathbf{M}^H)_{k,k} \sigma^2_{\mathrm{AWGN}}$, $\sigma^2_{\mathrm{MAI},k} = |\alpha_0 g[0]|^2 \sum_{\substack{j=1\\ j\neq k}}^{K} (\mathbf{MC})^2_{k,j} A_j^2$.

IV. SER in Nonlinear Frequency-Selective Channels

For frequency-selective channels, the detector

$$\underline{\mathbf{d}}_k = e^{-j\arg(\alpha_0)} (\underline{\mathbf{D}})_{K+k,:} \tag{20}$$

has to consider not only the MAI but it has also to combine the resolvable paths. The received vector in (11) suggests to regard the multipath situation, summarised by $\underline{\mathbf{G}}$, as the AWGN scenario in (14), with a modified code matrix $\underline{\mathbf{H}} = \underline{\mathbf{G}}\underline{\mathbf{C}}$. As a consequence, likewise (16), (18) and (19), for frequency-selective channels we can define the RAKE receiver, the LDD and the MMSE detector as

$$\underline{\mathbf{D}}_{\mathrm{RAKE}} = \underline{\mathbf{H}}^H , \qquad \underline{\mathbf{D}}_{\mathrm{LDD}} = \underline{\mathbf{H}}^{\dagger} , \qquad \underline{\mathbf{D}}_{\mathrm{MMSE}} = \underline{\mathbf{M}} , \tag{21}$$

where $\underline{\mathbf{H}} = \underline{\mathbf{G}}\underline{\mathbf{C}}$, $\underline{\mathbf{M}} = (\mathbf{I}_{3K} + |\alpha_0|^2\, \underline{\mathbf{A}}\underline{\mathbf{H}}^H \underline{\mathbf{W}}^{-1} \underline{\mathbf{H}}\underline{\mathbf{A}})^{-1} \underline{\mathbf{A}}\underline{\mathbf{H}}^H \underline{\mathbf{W}}^{-1}$, $\underline{\mathbf{W}} = \underline{\mathbf{G}}\underline{\boldsymbol{\Psi}}\underline{\mathbf{G}}^H + \sigma^2_{\mathrm{AWGN}} \mathbf{I}_{2N}$, and $\underline{\boldsymbol{\Psi}}$ is equivalent to the squared matrix $\boldsymbol{\Psi}$ with higher dimension $2N + L$.

By applying the detector $\underline{\mathbf{D}}$ in (21) to the received signal $\underline{\mathbf{r}}[l]$ in (11), we obtain

$$\underline{\mathbf{v}}[l] = e^{-j\arg(\alpha_0)} \underline{\mathbf{D}}\underline{\mathbf{r}}[l] = \underline{\mathbf{s}}_k[l] + \underline{\mathbf{m}}_k[l] + \underline{\mathbf{n}}[l] + \underline{\mathbf{a}}[l] \tag{22}$$

where $\underline{\mathbf{s}}_k[l] = |\alpha_0|\, \underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{SIG},k} \underline{\mathbf{A}}\underline{\mathbf{b}}[l]$ is the useful signal of the user k, $\underline{\mathbf{m}}_k[l] = |\alpha_0|\, \underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{MAI},k} \underline{\mathbf{A}}\underline{\mathbf{b}}[l]$ is the intersymbol interference (ISI) plus MAI term, $\underline{\mathbf{n}}[l] = e^{-j\arg(\alpha_0)} \underline{\mathbf{D}}\underline{\mathbf{r}}_{\mathrm{NL}}[l]$ is the nonlinear noise, $\underline{\mathbf{a}}[l] = e^{-j\arg(\alpha_0)} \underline{\mathbf{D}}\underline{\mathbf{r}}_{\mathrm{AWGN}}[l]$ is the thermal noise part, and $\underline{\mathbf{H}}_{\mathrm{SIG},k} = [\mathbf{0}_{2N\times K+k-1} (\underline{\mathbf{H}})_{:,K+k} \mathbf{0}_{2N\times 2K-k}]$ and $\underline{\mathbf{H}}_{\mathrm{MAI},k} = [(\underline{\mathbf{H}})_{:,1:K+k-1} \mathbf{0}_{2N\times 1} (\underline{\mathbf{H}})_{:,K+k+1:3K}]$ are obtained by partitioning the channel-code matrix $\underline{\mathbf{H}} = \underline{\mathbf{G}}\underline{\mathbf{C}}$ in (11) as $\underline{\mathbf{H}} = \underline{\mathbf{H}}_{\mathrm{SIG},k} + \underline{\mathbf{H}}_{\mathrm{MAI},k}$.

As in the AWGN scenario, we suppose that both the MAI and the nonlinear distortion noise can be approximated as Gaussian. Since these approximations work reasonably well in AWGN channels, a similar behaviour is expected in multipath channels, because the received signal can be thought as the superposition of many replicas of AWGN-like contributions, and the sum of Gaussian random variables is still Gaussian. By (20), the decision variable $\underline{\mathbf{d}}_k \underline{\mathbf{r}}[l]$ in (12) is equal to the (K+k)th element of the vector $\underline{\mathbf{v}}[l]$ in (22). Therefore, in order to evaluate the SER conditioned to a given channel realisation $g(\tau)$, we need to calculate the power of the (K+k)th element of the vectors in the right hand side of (22).

As far as the thermal noise is concerned, the elements of the vector $\underline{\mathbf{a}}[l]$ are jointly complex Gaussian random variables, because obtained as linear combination $\underline{\mathbf{D}}$ of jointly complex Gaussian random variables contained in $\underline{\mathbf{r}}_{\mathrm{AWGN}}[l]$. The covariance matrix $\underline{\boldsymbol{\Phi}}_{\mathrm{AWGN}}$ of $\underline{\mathbf{a}}[l]$ is expressed by

$$\underline{\boldsymbol{\Phi}}_{\mathrm{AWGN}} = E\{\underline{\mathbf{a}}[l]\underline{\mathbf{a}}[l]^H\} = \sigma^2_{\mathrm{AWGN}} \underline{\mathbf{D}}\underline{\mathbf{D}}^H , \tag{23}$$

and hence the (K+k)th element of the thermal noise vector $\underline{\mathbf{a}}[l]$ in (22) has power $\sigma^2_{\underline{\mathrm{A}},k}$ expressed by

$$\sigma^2_{\underline{\mathrm{A}},k} = (\underline{\mathbf{D}}\underline{\mathbf{D}}^H)_{K+k,K+k} \sigma^2_{\mathrm{AWGN}} . \tag{24}$$

For the nonlinear noise, we observe that the element $(\underline{\mathbf{n}}[l])_{K+k,1}$ of the vector $\underline{\mathbf{n}}[l]$ is the sum of the $2N$ elements of $\underline{\mathbf{r}}_{\mathrm{NL}}[l]$, weighted by the $2N$ elements of $e^{-j\arg(\alpha_0)} (\underline{\mathbf{D}})_{K+k,:}$, and consequently, if the processing gain N is high enough, the nonlinear distortion noise $(\underline{\mathbf{n}}[l])_{K+k,1}$ can be well approximated as a Gaussian random variable. The accuracy of this approximation depends not only on the processing gain N, but also on the channel realisation $g(\tau)$ contained in $\underline{\mathbf{H}} = \underline{\mathbf{G}}\mathbf{C}$, which affects the values of the weighting vector $\overline{e^{-j\arg(\alpha_0)}}(\underline{\mathbf{D}})_{K+k,:}$, and on the power input back-off (IBO) to the HPA. Indeed, if few elements of $(\underline{\mathbf{D}})_{K+k,:}$ are characterised by a higher modulus with respect to the others, the central limit theorem, and consequently the Gaussian approximation, will tend to fail. On the contrary, if many elements of $(\underline{\mathbf{D}})_{K+k,:}$ have significant modulus, the approximation accuracy is very good, because many elements of the vector $\underline{\mathbf{n}}[l]$ are weighted with coefficients having almost-equal values. Moreover, if the IBO is too high, most of the elements of $\underline{\mathbf{r}}_{\mathrm{NL}}[l]$ are close to zero (at least for class A or ideally predistorted amplifiers) and consequently $\underline{\mathbf{n}}[l]$ is practically obtained by the linear combination of few significant elements, thus violating the central limit theorem hypothesis as well. The covariance matrix $\underline{\mathbf{\Phi}}_{\mathrm{NL}}$ of the vector $\underline{\mathbf{n}}[l]$ is expressed by

$$\underline{\mathbf{\Phi}}_{\mathrm{NL}} = E\{\underline{\mathbf{n}}[l]\underline{\mathbf{n}}[l]^H\} = \underline{\mathbf{D}}\underline{\mathbf{G}}\mathbf{\Psi}\underline{\mathbf{G}}^H\underline{\mathbf{D}}^H , \qquad (25)$$

and therefore the (K+k)th element of the nonlinear distortion noise vector $\underline{\mathbf{n}}[l]$ in (22) has power $\sigma^2_{\underline{\mathrm{N}},k}$ expressed by

$$\sigma^2_{\underline{\mathrm{N}},k} = (\underline{\mathbf{D}}\underline{\mathbf{G}}\mathbf{\Psi}\underline{\mathbf{G}}^H\underline{\mathbf{D}}^H)_{K+k,K+k} . \qquad (26)$$

As far as the ISI plus MAI term $\underline{\mathbf{m}}_k[l]$ in (22) is concerned, the good accuracy of the Gaussian approximation has been already tested in [10] for the RAKE and MMSE receivers in linear scenarios. Moreover, in many cases (e.g. when $2K + \max\{K, L\} \le 2N$, [6]) the LDD is able to eliminate both ISI and MAI, i.e. $(\underline{\mathbf{m}}_k[l])_{K+1:2K,1} = \mathbf{0}_{K\times 1}$. The covariance matrix $\underline{\mathbf{\Phi}}_{\mathrm{MAI}}$ of the column vector $\underline{\mathbf{m}}_k[l]$ is expressed by

$$\underline{\mathbf{\Phi}}_{\mathrm{MAI}} = E\{\underline{\mathbf{m}}_k[l]\underline{\mathbf{m}}_k[l]^H\} = |\alpha_0|^2\ \underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{MAI},k}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H_{\mathrm{MAI},k}\underline{\mathbf{D}}^H , \quad (27)$$

and therefore the (K+k)th element of the vector $\underline{\mathbf{m}}_k[l]$ in (22) is characterised by a power $\sigma^2_{\underline{\mathrm{M}},k}$ expressed by

$$\sigma^2_{\underline{\mathrm{M}},k} = |\alpha_0|^2\ (\underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{MAI},k}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H_{\mathrm{MAI},k}\underline{\mathbf{D}}^H)_{K+k,K+k} . \qquad (28)$$

The power of the (K+k)th element of the signal vector $\underline{\mathbf{s}}_k[l]$ in (22) can be obtained as in the previous case, leading to

$$\sigma^2_{\underline{\mathrm{S}},k} = |\alpha_0|^2\ (\underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{SIG},k}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H_{\mathrm{SIG},k}\underline{\mathbf{D}}^H)_{K+k,K+k} , \qquad (29)$$

which is equal to $|\alpha_0|^2 A_k^2$ for the LDD [6].

Being the symbol-error probability conditioned to a given channel realisation $g(\tau)$ equal to

$$P_k(g) = f\left(\sqrt{\sigma^2_{\underline{\mathrm{S}},k} / (\sigma^2_{\underline{\mathrm{M}},k} + \sigma^2_{\underline{\mathrm{N}},k} + \sigma^2_{\underline{\mathrm{A}},k})}\right), \qquad (30)$$

where the quantities $\sigma^2_{\underline{\mathrm{S}},k}$, $\sigma^2_{\underline{\mathrm{M}},k}$, $\sigma^2_{\underline{\mathrm{N}},k}$ and $\sigma^2_{\underline{\mathrm{A}},k}$ depend on the detector that is chosen among the ones in (21), the average SER can be obtained by averaging (30) over the joint probability density function $p(g)$ of the channel parameters $\beta_1,...,\beta_Q,\theta_1,...,\theta_Q,\tau_1,...,\tau_Q$, as expressed by

$$P_{\mathrm{SEL},k} = \int P_k(g)p(g)dg . \qquad (31)$$

We want to point out that our approach used to obtain (31) is quite similar to the one of [10] for linear scenarios. The main difference is that, since we assume the spreading codes as fixed, the average over the spreading codes is not needed.

V. SIMULATION RESULTS

The situation in which the base station transmits data to $K = 40$ users is considered. The amplitudes $\{A_k\}$ are equal for all users. Gold sequences of length $N = 63$ have been chosen for the short spreading codes $\{c_k[j]\}$. For the chip pulse shaping waveform $p(t)$, a square-root raised cosine with roll-off factor equal to $\rho = 0.22$ has been chosen. The nonlinearities considered are the soft-limiter model (32),

$$G(|x|) = \begin{cases} |x| , & |x| \le A_{sat} \\ A_{sat} , & |x| > A_{sat} \end{cases} , \qquad \Phi(|x|) = 0 , \qquad (32)$$

which is the envelope input-output characteristic of an ideal predistorted HPA, and the Saleh model (33)

$$G(|x|) = \frac{2|x|}{1+|x|^2} , \qquad \Phi(|x|) = \frac{\pi}{3}\frac{|x|^2}{1+|x|^2} . \qquad (33)$$

The apparent signal-to-noise ratio ($\mathrm{SNR}_{\mathrm{app}}$) is defined at the input of the LMD as

$$\mathrm{SNR}_{\mathrm{app}} = (E\{|r_{n,\mathrm{SIG}}[l]|^2\} + E\{|r_{n,\mathrm{NL}}[l]|^2\}) / \sigma^2_{\mathrm{AWGN}} , \quad (34)$$

while the IBO and the output back-off (OBO) are defined as

$$\mathrm{IBO} = P_{z,\max} / P_z , \quad \mathrm{OBO} = P_{w,\max} / P_w , \qquad (35)$$

where $P_{z,\max}$ ($P_{w,\max}$) and P_z (P_w) represent the maximum and the average HPA input (output) power, respectively.

Fig. 1 shows the SER performance of the MMSE receiver in AWGN channels when the soft-limiter model (32) is used for the HPA. It is evident that there is a very good agreement between the analytical model and the simulation results for high OBO and for very low OBO values, while there is a little mismatch at high SNR for low OBO values (OBO = 1.30 dB). As explained in [3], this mismatch is due to the Gaussian approximation of the nonlinear distortion noise.

Fig. 2 shows the SER performance of the three linear receivers (RAKE, LDD, MMSE) in frequency-selective fading channels. The soft-limiter model (32) with OBO = 1.99 dB has been chosen for the HPA. The amplitudes $\{\beta_q \exp(j\theta_q)\}$ of the $Q = 15$ channel paths are modelled as independent zero-mean complex Gaussian random variables with variance $E\{|\beta_q|^2\} = 1/Q$, while the path delays $\{\tau_q\}$ are multiple of the chip duration T_c. The theoretical SER has been evaluated by averaging (30) over $N_{ch} = 400$ independent channel realisations. As expected, the MMSE receiver outperforms the other two detectors.

Figs. 3-4 show the SER performance of the MMSE detector with the frequency-selective channel model described above, using the HPA model (32) and (33), respectively. As in AWGN channels, there is a good agreement between analytical model and simulation results, especially if the OBO is very low or quite high. Moreover, little mismatch is present when the OBO is roughly 2 dB (Fig. 4).

VI. CONCLUSIONS

An analytical framework to evaluate the SER performance of LMDs for DS-CDMA systems subject to amplifier nonlinear distortions in frequency-selective fading channels has been introduced. Results for QPSK mapping with square-root raised cosine pulse shaping have been presented. Simulation results have shown how the analytical model is appropriate under the hypotheses of high number of users and high spreading gain.

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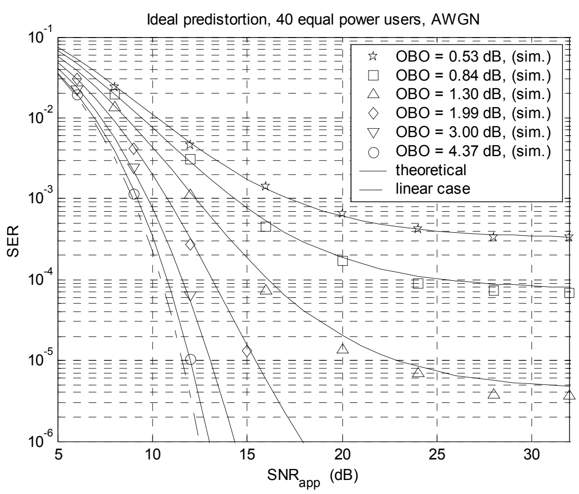

Fig. 1. SER of the MMSE detector in AWGN channels.

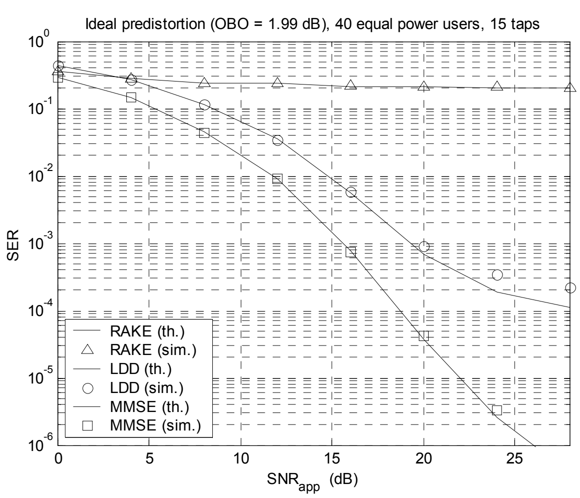

Fig. 2. SER comparison in frequency-selective channels.

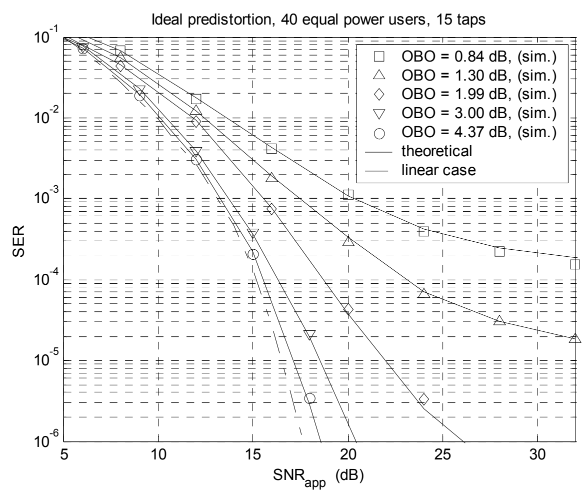

Fig. 3. SER of the MMSE in frequency-selective channels.

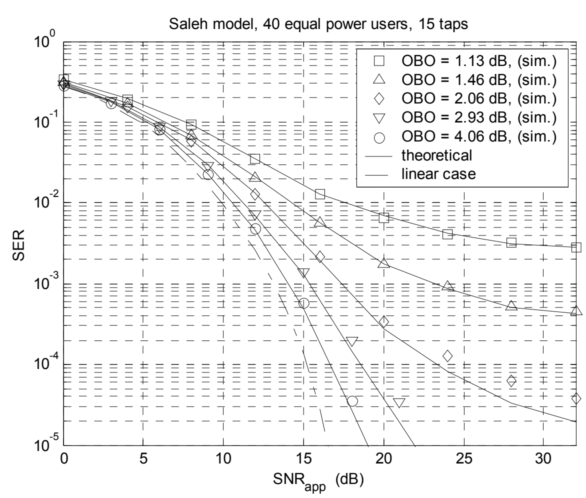

Fig. 4. SER of the MMSE in frequency-selective channels.