CHANNEL ESTIMATION AND WINDOWED DFE FOR OFDM WITH DOPPLER SPREAD

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ABSTRACT

Multicarrier systems are seriously affected by time-varying frequency-selective channels. Recently, windowing and decision-feedback equalization (DFE) have separately been proven to boost the performance of minimum mean-squared error (MMSE) block equalization, while maintaining a very low complexity by capitalizing on a band LDL factorization. This paper jointly considers receiver windowing and DFE, as well as the impact on the performance of a pilot-based frequency-domain channel estimation technique that relies on a basis expansion model (BEM) approach. We show that the combination of windowing and DFE still allows the use of a low-complexity band LDL factorization. Therefore, we can further improve the BER of OFDM systems affected by severe Doppler spread, while preserving linear complexity in the number of subcarriers.

1. INTRODUCTION

Multicarrier systems equipped with a cyclic prefix are particularly attractive due to the simple equalization that is induced by converting a time-invariant (TI) frequency-selective channel in a set of parallel (orthogonal) frequency-flat channels [1]. However, the widespread use of OFDM in several communication standards (e.g., DVB-T, 802.11a, 802.16, etc.) and the increasing request for mobile communication capabilities have pushed researchers to design OFDM equalizers capable to mitigate the effects of significant Doppler spread, which destroys the subcarrier orthogonality and introduces intercarrier interference (ICI) [2]-[8].

In this framework, an MMSE windowed block linear equalizer (W-BLE) and an MMSE block decision-feedback equalizer (BDFE) have been recently proposed in [9]. Exploiting the observation that ICI generated by time-varying (TV) channels is mainly induced by adjacent subcarriers [4], the band LDL factorization approach proposed in [7] is exploited in [9] to show that W-BLE and BDFE exhibit very good trade-offs between performance and complexity with respect to other proposed solutions [4]-[6]. Another possibility is a maximum-likelihood Viterbi approach [8], which however is characterized by a much greater complexity, especially for large constellation sizes. Aim of this paper is to further improve the performance of the banded equalizers in [9] by showing how to incorporate the receiver windowing philosophy into the banded BDFE design, while preserving linear complexity in the number of subcarriers.

Training-based channel estimation techniques are generally preferred in rapidly time-varying environments. Pilot-aided channel estimation techniques for block transmissions over doubly-selective channels have been widely analyzed in [3] [5] [10] [11], where the common idea is to parsimoniously model the time-varying channel by a limited number of parameters that capture the time-variation of the channel within a single data block. Assuming a classical frequency-domain equally-spaced zero-guarded channel sounding technique [12], we show that the parsimonious representation of the channel by means of a basis expansion model (BEM) [13], can also take advantage of windowed bases, and enables accurate channel estimation with fairly high Doppler spread, without significantly affecting the performance of the proposed windowed equalizer.

2. OFDM SYSTEM MODEL

We consider an OFDM system with N subcarriers. We assume time and frequency synchronization, and a cyclic prefix length L greater than the maximum delay spread of the channel. By applying an $N\times 1$ time-domain window $\mathbf{w}$ at the receiver before the FFT, the OFDM input-output relation for the generic OFDM block can be expressed by [6] [9]

$$\underline{\mathbf{z}}_{\mathrm{W}} = \underline{\mathbf{\Lambda}}_{\mathrm{W}}\underline{\mathbf{a}} + \underline{\mathbf{n}}_{\mathrm{W}} = \underline{\mathbf{C}}_{\mathrm{W}}\underline{\mathbf{\Lambda}}\underline{\mathbf{a}} + \underline{\mathbf{n}}_{\mathrm{W}} \,, \tag{1}$$

where $\underline{\mathbf{z}}_{\mathrm{W}}$ is the $N\times 1$ received vector, $\underline{\mathbf{\Lambda}}_{\mathrm{W}} = \mathbf{F}\mathbf{\Delta}_{\mathrm{W}}\underline{\mathbf{H}}\mathbf{F}^H$ is the $N\times N$ frequency-domain windowed channel matrix, with $\mathbf{\Delta}_{\mathrm{W}} = \mathrm{diag}(\mathbf{w})$, $\underline{\mathbf{H}}$ the $N\times N$ time-domain channel matrix, $\mathbf{F}$ the $N\times N$ unitary FFT matrix, $\underline{\mathbf{a}}$ the $N\times 1$ OFDM block that contains the frequency-domain data, $\underline{\mathbf{n}}_{\mathrm{W}} = \mathbf{F}\mathbf{\Delta}_{\mathrm{W}}\underline{\mathbf{v}}$ is the windowed noise, with $\underline{\mathbf{v}}$ the receiver AWGN vector in the time domain, $\underline{\mathbf{\Lambda}} = \mathbf{F}\underline{\mathbf{H}}\mathbf{F}^H$ is the frequency-domain unwindowed channel matrix, and $\underline{\mathbf{C}}_{\mathrm{W}} = \mathbf{F}\mathbf{\Delta}_{\mathrm{W}}\mathbf{F}^H$ is the circulant matrix representing the windowing operation in the frequency domain.

Due to the TV nature of the channel, $\underline{\mathbf{\Lambda}}$ in (1) is not diagonal, but is nearly banded [2], and each diagonal is associated with a discrete Doppler frequency that introduces ICI. Hence, $\underline{\mathbf{\Lambda}}$ can be approximated by the band matrix $\underline{\mathbf{B}}$, thereby neglecting the ICI that comes from faraway subcarriers. We denote with Q the number of subdiagonals and superdiagonals retained from $\underline{\mathbf{\Lambda}}$, so that the total bandwidth of $\underline{\mathbf{B}}$ is $2Q+1$. The parameter Q can be chosen according to the rule of thumb $Q \geq \lceil f_{\mathrm{D}}/\Delta_f \rceil + 1$ [6], where f_{D} is the maximum Doppler frequency and Δ_f is the subcarrier spacing. This leads to very small values of Q, e.g., $1 \leq Q \leq 5$.

In the windowed case, the band approximation is expressed by $\underline{\mathbf{\Lambda}}_{\mathrm{W}} \approx \underline{\mathbf{B}}_{\mathrm{W}}$. Hence, the window design should be tailored to make the channel matrix "more banded," so that $\|\underline{\mathbf{\Lambda}}_{\mathrm{W}} - \underline{\mathbf{B}}_{\mathrm{W}}\| < \|\underline{\mathbf{\Lambda}} - \underline{\mathbf{B}}\|$. In this view, we consider the minimum band approximation error (MBAE) sum-of-exponentials (SOE) window [9], which is expressed by

$$[\mathbf{w}]_n = \sum_{q=-Q}^{Q} b_q \exp(j2\pi qn/N), \quad (2)$$

where the coefficients $\{b_q\}$ are designed in order to minimize $\|\underline{\mathbf{\Delta}}_{\mathrm{W}} - \underline{\mathbf{B}}_{\mathrm{W}}\|$. (Note that this criterion is similar to the *max Average-SINR* criterion of [6].) Thanks to the SOE constraint, the covariance matrix of the windowed noise $\underline{\mathbf{n}}_{\mathrm{W}}$ is banded with total bandwidth $4Q+1$. This leads to linear MMSE equalization algorithms characterized by a very low complexity [9], which is linear in the number of subcarriers.

Due to the band approximation of the channel $\underline{\mathbf{\Delta}}_{\mathrm{W}} \approx \underline{\mathbf{B}}_{\mathrm{W}}$, the ICI has a finite support. Consequently, it is possible to design the transmitted vector $\underline{\mathbf{a}}$ by partitioning training and data in such a way that they will emerge from the channel (almost) orthogonal. Specifically, as proposed in [10] for time-domain training, and in [11] for the frequency-domain counterpart, we can design the transmitted vector as

$$\underline{\mathbf{a}} = [\mathbf{0}_{1\times U}\ s_1\ \mathbf{0}_{1\times 2U}\ \mathbf{d}_1^T\ \mathbf{0}_{1\times 2U}\ s_2\ \mathbf{0}_{1\times 2U}\ \mathbf{d}_2^T\ \ldots\ s_{L+1}\ \mathbf{0}_{1\times 2U}\ \mathbf{d}_{L+1}^T\ \mathbf{0}_{1\times U}]^T, \quad (3)$$

where s_l represents the lth pilot tone, and $\mathbf{d}_l$ is a $D\times 1$ column vector containing the lth portion of the data. The parameter U represents the maximum value of Q that preserves at the receiver the orthogonality between data and pilots, in the banded channel. Thus, the choice of U at the transmitter can be done according to the maximum Doppler spread allowed at the receiver. It is interesting to observe that the transmitted vector in (3) contains equispaced pilots, which is an optimal choice also in channels that are not doubly-selective [12]. Specifically, for $U=0$, the pilot placement of (3) reduces to the optimal pilot placement for OFDM in TI frequency-selective channels [14].

3. PILOT-AIDED CHANNEL ESTIMATION

The basic idea is to express each time-varying channel tap as a linear combination of deterministic time-varying functions defined over a limited time span. Hence, the time variability of each channel tap is captured by its linear combination coefficients. This approach is known in the literature as the BEM, and further details can be found in [13].

The evolution of each channel tap in the time domain during the considered OFDM block is stored diagonally in the matrix $\underline{\mathbf{H}}$ or equivalently in the windowed channel matrix $\underline{\mathbf{H}}_{\mathrm{W}} = \mathbf{\Delta}_{\mathrm{W}}\underline{\mathbf{H}}$. More precisely, the lth tap evolution is contained in the vector $\mathbf{h}_l = \mathbf{\Delta}_{\mathrm{W}}[h[0,l], h[1,l], \ldots, h[N-1,l]]^T$, where $h[n,l]$ represents the lth discrete-time channel path at time n. The BEM expresses each channel tap vector $\mathbf{h}_l$ as

$$\mathbf{h}_l = \mathbf{\Xi}\boldsymbol{\eta}_l = [\boldsymbol{\xi}_0, \boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_{\bar{P}}][\eta_{l,0}, \eta_{l,1}, \ldots, \eta_{l,\bar{P}}]^T, \quad (4)$$

where $\boldsymbol{\xi}_p$ represents the $(p+1)$th deterministic base of size $N\times 1$, which is the same for all taps and all OFDM blocks, $\eta_{l,p}$ is the $(p+1)$th stochastic parameter for the $(l+1)$th tap during the considered OFDM block, and $\bar{P}+1$ is the number of basis functions. Because of the BEM assumption, the possibly windowed channel matrix $\underline{\mathbf{H}}_{\mathrm{W}}$ can be expressed as

$$\underline{\mathbf{H}}_{\mathrm{W}} = \sum_{l=0}^{L} \mathrm{diag}(\mathbf{h}_l)\mathbf{Z}_l = \sum_{l=0}^{L}\sum_{p=0}^{\bar{P}} \eta_{l,p}\, \mathrm{diag}(\boldsymbol{\xi}_p)\mathbf{Z}_l \quad (5)$$

where $\mathbf{Z}_l$ is the $N\times N$ circulant shift matrix with ones in the lth lower diagonal (i.e., $[\mathbf{Z}_l]_{n,(n-l)_{\mathrm{mod}\,N}} = 1$) and zero elsewhere. Thus,

$$\underline{\mathbf{\Delta}}_{\mathrm{W}} = \mathbf{F}\underline{\mathbf{H}}_{\mathrm{W}}\mathbf{F}^H = \sum_{l=0}^{L}\sum_{p=0}^{\bar{P}} \eta_{l,p}\mathbf{X}_p\mathbf{D}_l = \sum_{l=0}^{L}\sum_{p=0}^{\bar{P}} \eta_{l,p}\mathbf{\Gamma}_{l,p} = \mathbf{\Gamma}(\boldsymbol{\eta}\otimes\mathbf{I}_N), \quad (6)$$

where $\mathbf{X}_p = \mathbf{F}\,\mathrm{diag}(\boldsymbol{\xi}_p)\mathbf{F}^H$ is a circulant matrix with circulant vector $N^{-1/2}\mathbf{F}\boldsymbol{\xi}_p$, which represents the discrete spectrum of the $(p+1)$th basis function, $\mathbf{D}_l = \mathbf{F}\mathbf{Z}_l\mathbf{F}^H = \mathrm{diag}(\mathbf{f}_l)$ is a diagonal matrix containing the lth discrete frequency vector $\mathbf{f}_l$, expressed by $[\mathbf{f}_l]_n = \exp(j2\pi l(n-1)/N)$, $\mathbf{\Gamma}_{l,p} = \mathbf{X}_p\mathbf{D}_l = \mathbf{F}\,\mathrm{diag}(\boldsymbol{\xi}_p)\mathbf{Z}_l\mathbf{F}^H$, $\boldsymbol{\eta} = [\boldsymbol{\eta}_0^T, \ldots, \boldsymbol{\eta}_L^T]^T$ contains the $(\bar{P}+1)(L+1)$ BEM parameters, and $\mathbf{\Gamma} = [\mathbf{\Gamma}_{0,0}, \ldots, \mathbf{\Gamma}_{0,\bar{P}}, \mathbf{\Gamma}_{1,0}, \ldots, \mathbf{\Gamma}_{1,\bar{P}}, \ldots\ldots, \mathbf{\Gamma}_{L,0}, \ldots, \mathbf{\Gamma}_{L,\bar{P}}]$. By (1), the received vector becomes

$$\underline{\mathbf{z}}_{\mathrm{W}} = \mathbf{\Gamma}(\boldsymbol{\eta}\otimes\mathbf{I}_N)\underline{\mathbf{a}} + \underline{\mathbf{n}}_{\mathrm{W}} = \mathbf{\Gamma}(\mathbf{I}_{(\bar{P}+1)(L+1)}\otimes\underline{\mathbf{a}})\boldsymbol{\eta} + \underline{\mathbf{n}}_{\mathrm{W}}, \quad (7)$$

which can be rewritten as

$$\underline{\mathbf{z}}_{\mathrm{W}} = \mathbf{\Psi}^{(\underline{\mathbf{a}})}\boldsymbol{\eta} + \underline{\mathbf{n}}_{\mathrm{W}}. \quad (8)$$

where $\mathbf{\Psi}^{(\underline{\mathbf{a}})} = \mathbf{\Gamma}(\mathbf{I}_{(\bar{P}+1)(L+1)}\otimes\underline{\mathbf{a}})$ is the data-dependent matrix that couples the channel parameters with the received vector. Whatever is the choice for the deterministic basis $\{\boldsymbol{\xi}_p\}$, and assuming that the transmitted vector $\underline{\mathbf{a}}$ can be partitioned as the sum of a known training vector $\underline{\mathbf{s}}$ and an unknown data vector $\underline{\mathbf{d}}$, that is $\underline{\mathbf{s}} = [\mathbf{0}_{1\times U}\ s_1\ \mathbf{0}_{1\times 4U+D}\ s_2\ \mathbf{0}_{1\times 4U+D}\ \ldots\ \mathbf{0}_{1\times 4U+D}\ s_{L+1}\ \mathbf{0}_{1\times 3U+D}]^T$ and (see (3)) $\underline{\mathbf{d}} = \underline{\mathbf{a}} - \underline{\mathbf{s}}$, the received vector becomes

$$\underline{\mathbf{z}}_{\mathrm{W}} = \mathbf{\Psi}^{(\underline{\mathbf{s}})}\boldsymbol{\eta} + \underline{\mathbf{\Delta}}_{\mathrm{W}}\underline{\mathbf{d}} + \underline{\mathbf{n}}_{\mathrm{W}}, \quad (9)$$

where $\underline{\mathbf{\Delta}}_{\mathrm{W}}\underline{\mathbf{d}} = \mathbf{\Psi}^{(\underline{\mathbf{d}})}\boldsymbol{\eta}$. Now we introduce the $(2U+1)(L+1)\times N$ matrix $\mathbf{P}_{\mathrm{S}}$ obtained by selecting from $\mathbf{I}_N$ only those rows that correspond to the pilot symbols, i.e., the rows with indices from $(4U+D+1)l+1$ to $(4U+D+1)l+2U+1$, for $l=0,\ldots,L$. We obtain

$$\mathbf{z}_{\mathrm{S}} = \mathbf{P}_{\mathrm{S}}\underline{\mathbf{z}}_{\mathrm{W}} = \mathbf{\Phi}\boldsymbol{\eta} + \mathbf{P}_{\mathrm{S}}\underline{\mathbf{\Delta}}_{\mathrm{W}}\underline{\mathbf{d}} + \mathbf{P}_{\mathrm{S}}\underline{\mathbf{n}}_{\mathrm{W}}, \quad (10)$$

where $\mathbf{\Phi} = \mathbf{P}_{\mathrm{S}}\mathbf{\Psi}^{(\underline{\mathbf{s}})}$ is a $(2U+1)(L+1)\times(\bar{P}+1)(L+1)$ matrix. We observe that if $\underline{\mathbf{\Delta}}_{\mathrm{W}}$ is exactly banded with $Q\le U$, $\mathbf{P}_{\mathrm{S}}\underline{\mathbf{\Delta}}_{\mathrm{W}}\underline{\mathbf{d}}$ in (10) is equal to zero, and hence the interference produced by the data is eliminated. However, in general $\underline{\mathbf{\Delta}}_{\mathrm{W}}$ is not exactly banded, and hence we consider $\mathbf{i} = \mathbf{P}_{\mathrm{S}}\underline{\mathbf{\Delta}}_{\mathrm{W}}\underline{\mathbf{d}} = \mathbf{P}_{\mathrm{S}}\mathbf{\Psi}^{(\underline{\mathbf{d}})}\boldsymbol{\eta}$ in (10) as an interference term. Consequently, we can estimate the BEM parameters in the least squares (LS) sense or in the linear MMSE (LMMSE) sense, as expressed by $\hat{\boldsymbol{\eta}}_{\mathrm{LS}} = \mathbf{\Phi}^{\dagger}\mathbf{z}_{\mathrm{S}}$, and

$$\hat{\boldsymbol{\eta}}_{\mathrm{LMMSE}} = (\mathbf{\Phi}^H(\mathbf{R}_{\mathbf{ii}} + \mathbf{R}_{\mathbf{nn}})^{-1}\mathbf{\Phi} + \mathbf{R}_{\boldsymbol{\eta\eta}}^{-1})^{-1}\mathbf{\Phi}^H(\mathbf{R}_{\mathbf{ii}} + \mathbf{R}_{\mathbf{nn}})^{-1}\mathbf{z}_{\mathrm{S}}, \quad (11)$$

respectively, where the superscript $\dagger$ denotes pseudoinverse, $\mathbf{R}_{\mathbf{nn}} = \mathbf{P}_{\mathrm{S}}E\{\underline{\mathbf{n}}_{\mathrm{W}}\underline{\mathbf{n}}_{\mathrm{W}}^H\}\mathbf{P}_{\mathrm{S}}^H = \sigma_n^2\mathbf{P}_{\mathrm{S}}\underline{\mathbf{C}}_{\mathrm{W}}\underline{\mathbf{C}}_{\mathrm{W}}^H\mathbf{P}_{\mathrm{S}}^H$ is the covariance matrix of the selected windowed noise, $\mathbf{R}_{\mathbf{ii}} = \mathbf{P}_{\mathrm{S}}\mathbf{\Psi}^{(\underline{\mathbf{d}})}\mathbf{R}_{\boldsymbol{\eta\eta}}\mathbf{\Psi}^{(\underline{\mathbf{d}})H}\mathbf{P}_{\mathrm{S}}^H$ is the covariance matrix of the interference, $\mathbf{R}_{\boldsymbol{\eta\eta}} = E\{\boldsymbol{\eta}\boldsymbol{\eta}^H\}$ is the covariance matrix of the $(\bar{P}+1)(L+1)$ channel parameters, composed by square submatrices of size $\bar{P}+1$, expressed by $\mathbf{R}_{\boldsymbol{\eta}_i\boldsymbol{\eta}_j} = E\{\boldsymbol{\eta}_i\boldsymbol{\eta}_j^H\} = \mathbf{\Xi}^{\dagger}E\{\mathbf{h}_i\mathbf{h}_j^H\}\mathbf{\Xi}^{\dagger H}$. After estimating the BEM parameter vector $\boldsymbol{\eta}$, e.g., by (11), we can recover the channel matrix $\underline{\mathbf{\Delta}}_{\mathrm{W}}$ by (6). Depending on the chosen basis matrix $\mathbf{\Xi}$, the channel matrix $\underline{\mathbf{\Delta}}_{\mathrm{W}}$ obtained by (6) could be non-banded but nearly banded. In this case, we select only the $2Q+1$ main diagonals of $\underline{\mathbf{\Delta}}_{\mathrm{W}}$, thus obtaining $\underline{\mathbf{B}}_{\mathrm{W}}$.

4. BANDED MMSE-BDFE WITH WINDOWING

In [9], we presented two low-complexity equalizers that exploit the band approximation and the band LDL factorization algorithm of [7]. The first one is a banded W-BLE with MBAE-SOE windowing. The second one is a banded BDFE with rectangular windowing. Both of them outperform the banded BLE with rectangular windowing of [7], without significant complexity overhead. In this section, we marry banded BDFE and MBAE-SOE windowing.

4.1. Equalizer Design

We assume that the equalizer neglects the frequency guard bands at the edges of the channel bandwidth, since these bands could be affected by adjacent channel interference. Hence, we denote with $N_{\mathrm{A}} = N - 2U$ the number of active subcarriers, which contain both data and pilots. We also assume that the matrix $\mathbf{\Lambda}_{\mathrm{W}}$ is the $N_{\mathrm{A}} \times N_{\mathrm{A}}$ middle block of $\underline{\mathbf{\Lambda}}_{\mathrm{W}}$, $\underset{\sim}{\mathbf{C}}_{\mathrm{W}}$ is the $N_{\mathrm{A}} \times N$ middle block of $\underline{\mathbf{C}}_{\mathrm{W}}$, and that vectors $\mathbf{a}$ and $\mathbf{z}_{\mathrm{W}}$ are the $N_{\mathrm{A}} \times 1$ middle blocks of $\underline{\mathbf{a}}$ and $\underline{\mathbf{z}}_{\mathrm{W}}$, respectively.

With reference to Fig. 1, we design the feedforward filter $\mathbf{F}_{\mathrm{F}}$ and the feedback filter $\mathbf{F}_{\mathrm{B}}$ adopting the MMSE approach [15] [16]. This approach minimizes the quantity $MSE = \mathrm{tr}(\mathbf{R}_{\mathbf{ee}})$, where $\mathbf{R}_{\mathbf{xy}} = E\{\mathbf{x}\mathbf{y}^H\}$, $\mathbf{e} = \tilde{\mathbf{a}} - \mathbf{a}$, and $\tilde{\mathbf{a}} = \mathbf{F}_{\mathrm{F}}\mathbf{z}_{\mathrm{W}} - \mathbf{F}_{\mathrm{B}}\hat{\mathbf{a}}$. We also impose the constraint that $\mathbf{F}_{\mathrm{B}}$ is strictly upper triangular, so that the feedback process can be performed by successive cancellation [16]. By the assumption of correct past decisions $\hat{\mathbf{a}} = \mathbf{a}$, the error vector can be expressed by $\mathbf{e} = \mathbf{F}_{\mathrm{F}}\mathbf{z}_{\mathrm{W}} - (\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})\mathbf{a}$. By the orthogonality principle, it holds $\mathbf{R}_{\mathbf{ez}_{\mathrm{W}}} = \mathbf{0}$, which leads to [16]

$$\mathbf{F}_{\mathrm{F}} = (\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})\mathbf{R}_{\mathbf{az}_{\mathrm{W}}}\mathbf{R}_{\mathbf{z}_{\mathrm{W}}\mathbf{z}_{\mathrm{W}}}^{-1} = (\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})\mathbf{\Lambda}_{\mathrm{W}}^H(\mathbf{\Lambda}_{\mathrm{W}}\mathbf{\Lambda}_{\mathrm{W}}^H + \gamma^{-1}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}^H)^{-1}, \quad (12)$$

where $\gamma = \sigma_s^2 / \sigma_n^2$ is assumed known to the receiver. We now apply $\mathbf{\Lambda}_{\mathrm{W}} \approx \mathbf{B}_{\mathrm{W}}$, thereby obtaining

$$\mathbf{F}_{\mathrm{F}} = (\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})\mathbf{G}_{\mathrm{W}} = (\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})\mathbf{B}_{\mathrm{W}}^H(\mathbf{B}_{\mathrm{W}}\mathbf{B}_{\mathrm{W}}^H + \gamma^{-1}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}^H)^{-1}. \quad (13)$$

This result points out that the feedforward filter is the cascade of the banded W-BLE $\mathbf{G}_{\mathrm{W}} = \mathbf{B}_{\mathrm{W}}^H(\mathbf{B}_{\mathrm{W}}\mathbf{B}_{\mathrm{W}}^H + \gamma^{-1}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}^H)^{-1}$ and an upper triangular matrix $\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}}$ with unit diagonal. To design $\mathbf{F}_{\mathrm{B}}$, we observe that

$$\mathbf{R}_{\mathbf{ee}} = (\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})(\mathbf{R}_{\mathbf{aa}} - \mathbf{R}_{\mathbf{az}_{\mathrm{W}}}\mathbf{R}_{\mathbf{z}_{\mathrm{W}}\mathbf{z}_{\mathrm{W}}}^{-1}\mathbf{R}_{\mathbf{az}_{\mathrm{W}}}^H)(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})^H, \quad (14)$$

and, by the matrix inversion lemma, we obtain

$$\mathbf{R}_{\mathbf{ee}} = \sigma_n^2(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})(\gamma^{-1}\mathbf{I}_{N_{\mathrm{A}}} + \mathbf{\Lambda}_{\mathrm{W}}^H(\underset{\sim}{\mathbf{C}}_{\mathrm{W}}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}^H)^{-1}\mathbf{\Lambda}_{\mathrm{W}})^{-1}(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})^H. \quad (15)$$

We now make the approximation

$$\mathbf{\Lambda}_{\mathrm{W}}^H(\underset{\sim}{\mathbf{C}}_{\mathrm{W}}\underset{\sim}{\mathbf{C}}_{\mathrm{W}}^H)^{-1}\mathbf{\Lambda}_{\mathrm{W}} \approx \underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}}^H(\underline{\mathbf{C}}_{\mathrm{W}}\underline{\mathbf{C}}_{\mathrm{W}}^H)^{-1}\underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}}, \quad (16)$$

where $\underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}} = \mathbf{F}\underline{\mathbf{H}}_{\mathrm{W}}\underset{\sim}{\mathbf{F}}^H$ is the $N \times N_{\mathrm{A}}$ middle block of $\underline{\mathbf{\Lambda}}_{\mathrm{W}}$ and $\underset{\sim}{\mathbf{F}}$ is the $N_{\mathrm{A}} \times N$ middle block of $\mathbf{F}$, thus obtaining

$$\mathbf{R}_{\mathbf{ee}} = \sigma_n^2(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})(\gamma^{-1}\mathbf{I}_{N_{\mathrm{A}}} + \underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}}^H(\underline{\mathbf{C}}_{\mathrm{W}}\underline{\mathbf{C}}_{\mathrm{W}}^H)^{-1}\underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}})^{-1}(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})^H. \quad (17)$$

Since $\underline{\mathbf{C}}_{\mathrm{W}}$ is circulant,

$$\begin{aligned}\underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}}^H(\underline{\mathbf{C}}_{\mathrm{W}}\underline{\mathbf{C}}_{\mathrm{W}}^H)^{-1}\underset{\sim}{\mathbf{\Lambda}}_{\mathrm{W}} &= (\underset{\sim}{\mathbf{F}}\underline{\mathbf{H}}^H\mathbf{\Delta}_{\mathrm{W}}^H\mathbf{F}^H)(\mathbf{F}\mathbf{\Delta}_{\mathrm{W}}^{-1}\mathbf{\Delta}_{\mathrm{W}}^{-H}\mathbf{F}^H)(\mathbf{F}\mathbf{\Delta}_{\mathrm{W}}\underline{\mathbf{H}}\underset{\sim}{\mathbf{F}}^H) \\ &= \underset{\sim}{\mathbf{F}}\underline{\mathbf{H}}^H\underline{\mathbf{H}}\underset{\sim}{\mathbf{F}}^H = \underset{\sim}{\mathbf{F}}\underline{\mathbf{H}}^H\mathbf{F}^H\mathbf{F}\underline{\mathbf{H}}\underset{\sim}{\mathbf{F}}^H = \underset{\sim}{\mathbf{\Lambda}}^H\underset{\sim}{\mathbf{\Lambda}}, \end{aligned} \quad (18)$$

where $\underset{\sim}{\mathbf{\Lambda}}$ is the $N \times N_{\mathrm{A}}$ middle block of the unwindowed channel matrix $\underline{\mathbf{\Lambda}}$. Consequently, Eq. (17) reduces to

$$\mathbf{R}_{\mathbf{ee}} = \sigma_n^2(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})(\gamma^{-1}\mathbf{I}_{N_{\mathrm{A}}} + \underset{\sim}{\mathbf{\Lambda}}^H\underset{\sim}{\mathbf{\Lambda}})^{-1}(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})^H. \quad (19)$$

Henceforth, we can exploit the computational advantages given by the LDL factorization algorithm in [7] by making the band approximation $\underset{\sim}{\mathbf{\Lambda}}^H\underset{\sim}{\mathbf{\Lambda}} \approx \underset{\sim}{\mathbf{B}}^H\underset{\sim}{\mathbf{B}}$, where $\underset{\sim}{\mathbf{B}}$ is the $N \times N_{\mathrm{A}}$ middle block of $\underline{\mathbf{B}}$, and $\underline{\mathbf{B}}$ is the banded version of $\underline{\mathbf{\Lambda}}$. Consequently, since Eq. (19) becomes

$$\mathbf{R}_{\mathbf{ee}} = \sigma_n^2(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})(\gamma^{-1}\mathbf{I}_{N_{\mathrm{A}}} + \underset{\sim}{\mathbf{B}}^H\underset{\sim}{\mathbf{B}})^{-1}(\mathbf{F}_{\mathrm{B}} + \mathbf{I}_{N_{\mathrm{A}}})^H, \quad (20)$$

$\mathrm{tr}(\mathbf{R}_{\mathbf{ee}})$ can be minimized by using the band LDL factorization of $\mathbf{M} = \gamma^{-1}\mathbf{I}_{N_{\mathrm{A}}} + \underset{\sim}{\mathbf{B}}^H\underset{\sim}{\mathbf{B}}$, expressed by $\mathbf{M} = \mathbf{LDL}^H$ and setting

$$\mathbf{F}_{\mathrm{B}} = \mathbf{L}^H - \mathbf{I}_{N_{\mathrm{A}}}. \quad (21)$$

By (21), (13), and $\mathbf{M} = \gamma^{-1}\mathbf{I}_{N_{\mathrm{A}}} + \underset{\sim}{\mathbf{B}}^H\underset{\sim}{\mathbf{B}} = \mathbf{LDL}^H$, we obtain

$$\mathbf{F}_{\mathrm{F}} = \mathbf{L}^H\mathbf{G}_{\mathrm{W}}. \quad (22)$$

We observe that the design of the feedforward and feedback filters does not consider the presence of pilot symbols. However, we can always reinsert the known pilot symbols when performing the successive cancellation in the feedback path. This partially prevents the error propagation, because the pilots are equispaced. Alternatively, we can design $(L+1)$ smaller DFEs, each one for a single portion $\mathbf{d}_l$ of the data.

4.2. Complexity Analysis

We now compute the number of complex operations necessary to perform the proposed banded windowed BDFE (W-BDFE). By (21) and (22), the soft output of the W-BDFE, expressed by $\tilde{\mathbf{a}} = \mathbf{F}_{\mathrm{F}}\mathbf{z}_{\mathrm{W}} - \mathbf{F}_{\mathrm{B}}\hat{\mathbf{a}}$, can be rewritten as

$$\tilde{\mathbf{a}} = \mathbf{L}^H\mathbf{G}_{\mathrm{W}}\mathbf{z}_{\mathrm{W}} - (\mathbf{L}^H - \mathbf{I}_{N_{\mathrm{A}}})\hat{\mathbf{a}}. \quad (23)$$

The computation of $\mathbf{G}_{\mathrm{W}}\mathbf{z}_{\mathrm{W}}$ is equivalent to applying the banded W-BLE presented in [9], which requires roughly $(8Q^2 + 24Q + 5)N_{\mathrm{A}}$ complex operations. The band LDL factorization of $\mathbf{M}$ needs $(8Q^2 + 10Q + 2)N_{\mathrm{A}}$ complex operations. To perform $\mathbf{L}^H\mathbf{G}_{\mathrm{W}}\mathbf{z}_{\mathrm{W}}$, we need $2QN_{\mathrm{A}}$ complex multiplies (CM) and $2QN_{\mathrm{A}}$ complex adds (CA). To perform $(\mathbf{L}^H - \mathbf{I}_{N_{\mathrm{A}}})\hat{\mathbf{a}}$, $2QN_{\mathrm{A}}$ CM and $(2Q-1)N_{\mathrm{A}}$ CA are required. Moreover, N_{A} CA are necessary to perform the subtraction between $\mathbf{L}^H\mathbf{G}_{\mathrm{W}}\mathbf{z}_{\mathrm{W}}$ and $(\mathbf{L}^H - \mathbf{I}_{N_{\mathrm{A}}})\hat{\mathbf{a}}$. As a result, the proposed banded W-BDFE requires approximately $(16Q^2 + 42Q + 7)N_{\mathrm{A}}$ complex operations. Hence, the complexity of the banded W-BDFE with MBAE-SOE windowing is nearly doubled with respect to the banded W-BLE with MBAE-SOE windowing [9].

It is worth noting that, thanks to the banded approach, the complexity of the proposed banded W-BDFE is linear in the number of subcarriers. Therefore, the proposed equalizer is less complex than other non-banded DFE schemes, e.g., the serial DFE [4], whose complexity is $O(N_{\mathrm{A}}^2)$.

5. SIMULATION RESULTS

We consider an OFDM system with $N = 256$, $U = Q$, $Q = 2$ unless otherwise stated, $L = 4$, and QPSK modulation. We assume Rayleigh fading channels with uniform power delay profile and Jakes' Doppler spectrum with $f_{\mathrm{D}} / \Delta_f = 0.256$. As far as channel estimation is concerned, we choose $\overline{P} + 1 = 2Q + 1$ generalized complex exponential (GCE) basis functions with oversampling factor $K = 2$ [17]. The channel is estimated by using the LMMSE criterion (11). The power ratio $\rho \approx 3.316$ between data and pilots has been chosen according to [18]. The SNR is defined as the ratio between total signal power (including pilot power) and noise power.

Fig. 2 illustrates the normalized MSE of the estimated channel matrices $\underline{\mathbf{H}}$ and $\underline{\mathbf{H}}_{\mathrm{W}}$, by using orthogonalized GCE (O-GCE) (i.e., $\mathbf{\Xi}$ is obtained by QR decomposition of GCE basis matrix) and orthogonalized windowed GCE (OW-GCE), as basis functions. Specifically, with O-GCE, we first estimate $\underline{\mathbf{H}}$ and then reconstruct $\underline{\mathbf{H}}_{\mathrm{W}} = \mathbf{\Delta}_{\mathrm{W}}\underline{\mathbf{H}}$ by the MBAE-SOE window, whereas with OW-GCE we first estimate $\underline{\mathbf{H}}_{\mathrm{W}}$ and then reconstruct $\underline{\mathbf{H}} = \mathbf{\Delta}_{\mathrm{W}}^{-1}\underline{\mathbf{H}}_{\mathrm{W}}$. It is shown that it is better to estimate the windowed channel rather than the unwindowed channel.

Fig. 3 compares the BER performance of the banded W-BDFE with the banded W-BLE and the banded BDFE. It is evident that the W-BDFE outperforms the other two equalizers. Specifically, the W-BDFE is able to reduce the error floor. This reduction is more pronounced for high values of Q. It is also worth noting that the degradation produced by channel estimation is quite small for both W-BLE and W-BDFE, especially at high SNR. Because of the good channel estimation, the BER floor is caused mainly by the band approximation. Similar conclusion can be drawn for different normalized Doppler spreads.

We remark that the values of Q used in the band approximations (13), (20), and (2), could also be different. However, due to space constraints, we used the same Q for all the band approximations. A deeper analysis of the impact of different Q's could be the subject of future work.

6. CONCLUSIONS

We have shown how to incorporate a receiver windowing technique into the design of a banded BDFE to boost the BER performance of OFDM systems affected by high Doppler spread, while preserving linear complexity in the number of subcarriers. By exploiting a BEM approach and a common frequency-domain training, we have also illustrated that reliable channel estimation is possible without sacrificing the BER performance.

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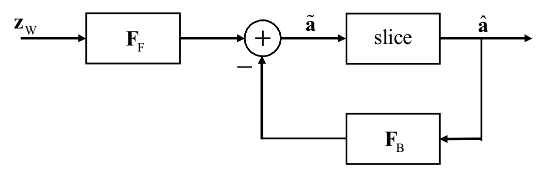

Fig. 1. Structure of the BDFE.

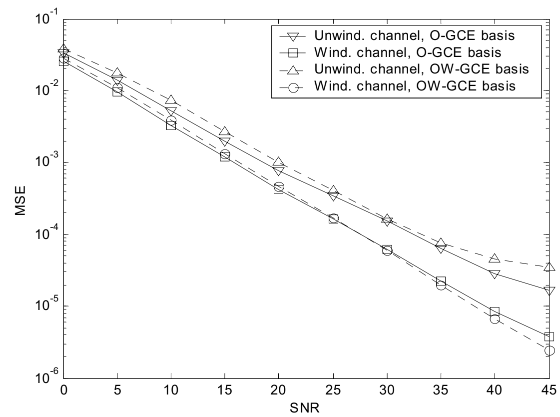

Fig. 2. MSE of different channel estimations.

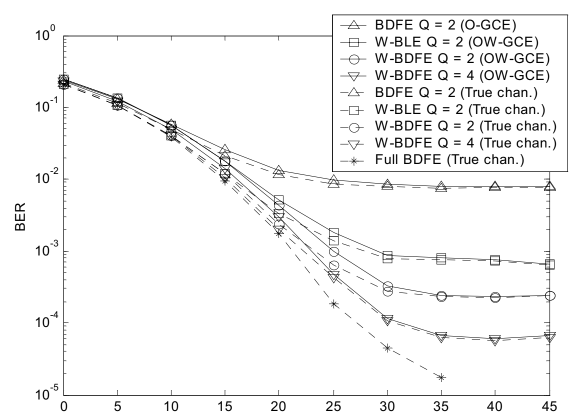

Fig. 3. BER comparison of banded MMSE equalizers.