## OPTIMUM OUTPUT POWER BACK-OFF in NON-LINEAR CHANNELS for OFDM BASED WLAN

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#### Abstract

Cyclically extended OFDM based WLAN systems have the capability to easily counteract frequency selective fading channels by one-tap equalizers in the frequency domain. Conversely, the sensitivity to nonlinear distortions, mainly introduced by power amplifiers, is one of the drawbacks of this modulation technique. Usually, the optimum Output power Back Off (OBO) is the one that minimizes the Total Degradation. Aim of this work is to show that the optimum OBO is a function of the channel behavior (e.g. AWGN, frequency selective Rician or Rayleigh fading channels). The effectiveness of amplifier predistortion and the gain introduced in the system link budget is also considered for different M-QAM constellation size. Computer simulations are shown and compared to an analytical approach.

#### **1. INTRODUCTION**

OFDM based WLAN systems like HYPERLAN/2 [2] or IEEE 802.11a [3] have the capability to easily counteract frequency selective fading channels by onetap equalizers in the frequency domain [1],[4]. Conversely, the sensitivity to non-linear distortions mainly introduced by power amplifiers is one of the drawbacks of this modulation technique. Indeed, an OFDM signal is generally obtained by the sum of a high number of carriers and consequently is characterized by a highly variable envelope, which makes the technique sensitive to non-linear distortions introduced by real hardware (power amplifiers, A/D-D/A converters, etc). Such sensitivity to non-linear distortions forces to use predistortion techniques as well as to introduce some power back-off at the transmitter side in order to reduce the SER degradation and the spectral re-growth.

A complex baseband OFDM signal, with a high number of carriers, can be modeled as a complex Gaussian process due to the Central Limit Theorem and, consequently, the distortions introduced by non-linear amplifiers can be computed by means of a complex extension of the Bussgang Theorem [5] [7]. Under this assumption, by modeling as Gaussian the non-linear distortion noise after the receiver FFT, it is possible to derive analytical BER performance in closed form in AWGN channels [5] as well as in integral form in Rice or Rayleigh frequency selective fading channels [6].

The Gaussian distribution for the OFDM signal is quite accurate also for a number of carriers not very high such as the 52 generally adopted for proposed WLAN systems such as [2] and [3]. On the contrary, the Gaussian assumption of the non-linear distortion noise after the FFT processing at the receiver side is less accurate especially when the constellation size is very high (e.g. for M-QAM with  $M \ge 64$ ).

Usually, the optimum Output power Back Off (obo) for a non-linear amplifier is the one that minimizes the Total Degradation [12]. This work will derive such optimum obo for OFDM-WLAN systems for Rician or Rayleigh frequency selective fading channels, and different M-QAM constellation size.

Moreover, the paper will consider the gain introduced in the system link budget by amplifier predistortion [8].

Computer simulations for the SER performance are also shown to investigate when the proposed analytical approach is accurate or not. Particularly, we will show that the analytical SER for 64-QAM are optimistic compared to what happens in reality.

The typical OFDM system architecture, the nonlinear amplifier model and the fading channel model are analyzed in section 2. The analytical evaluation for the system SER is derived in section 3, the Total Degradation concept is revised in section 4 while the overall results are presented in section 5.

#### 2. SYSTEM MODEL

The complex baseband samples of an OFDM signal (without the guard time interval) transmitted during an OFDM block duration  $T_b = NT_c$ , are expressed by [1]

$$z_m[k] = \sum_{n=0}^{N-1} a_m[n] e^{j \omega_n k T_c} \quad ,k = 0, ..., N-1, (1)$$

where  $a_m[n]$  represents the complex information symbol (M-QAM, M-DPSK, M-PSK mapped) transmitted during the  $m^{th}$  OFDM block on the  $n^{th}$ subcarrier with angular frequency  $\omega_n = 2\pi n/T_b$ 

The time continuous complex signal  $z(t) = r(t) \exp[j\theta(t)]$ , which is obtained from  $z_m[k]$  after parallel-to-serial and digital-to-analog conversions, is transmitted by a power amplifier whose instantaneous non-linear distortions are usually modeled by means of a complex non-linear distorting function  $f(r) = g(r) \cdot e^{j\phi(r)}$ , where g(r) and  $\phi(r)$  are the AM-AM and AM-PM distorting curves, respectively [8]. Thus, the output signal  $z_d(t)$  can be expressed by

$$z_{d}(t) = f[r(t)] \cdot e^{j\theta(t)} = g[r(t)] \cdot e^{j\{\theta(t) + \phi[r(t)]\}}.$$
 (2)

The Bussgang theorem for a complex Gaussian

input z(t) [7] allows expressing the non-linear output  $z_d(t)$  as the sum of a complex-scaled useful input replica and an uncorrelated non-linear distortion noise  $n_d(t)$ , as expressed by

$$z_{d}(t) = z_{u}(t) + n_{d}(t) = \alpha \cdot z(t) + n_{d}(t)$$
(3)

where  $\alpha$  is time-invariant [7] for OFDM signals obtained by a rectangular pulse shaping of the IFFT outputs, as assumed in the current paper.

For a slowly-varying channel, a cyclic extension of the OFDM block is generally adopted in order to reduce the equalizer complexity at the receiver side [4]. Indeed, for the  $m^{th}$  OFDM block, each FFT output at the receiver side is expressed by [1], [4]

$$R_m[n] = \sum_{k=0}^{N-1} r_m[k] e^{-j\frac{2\pi}{N}kn} , \qquad (4)$$

where  $r(t) = \int_{-\infty}^{+\infty} h(t,\tau) x(t-\tau) d\tau$  is the signal received through the time-varying channel  $h(t,\tau)$  and  $r_m[k] = r(mT_b' + \Delta T_b)$  is the sampled receiver input after the cyclic extension removal. It is straightforward from (1) to derive that

$$R_{m}[n] = \alpha H_{m}[n] a_{m}[n] + H_{m}[n] N_{dm}[n] + N_{tm}[n], \quad n = 0, \dots, N - 1, (5)$$

where the generic signal  $'X_m [n]'$  represents the FFT of  $x_m [k] = x \left( mT_b' + kT_c \right)$  during the  $m^{th}$  OFDM block,  $H(t,f) = \int_{-\infty}^{+\infty} h(t,\tau) e^{-j2\pi f\tau} d\tau$  is the time varying channel transfer function,  $H_m [n] = H \left( mT_b', n/T_b \right)$  and  $1/T_b$  is the frequency separation between each subcarrier.

Expression (5) outlines that each subcarrier receives a complex distorted replica of the transmitted symbol, corrupted by two noise terms. The  $H_m[n]N_{dm}[n]$ term represents the channel-filtered non-linear distortion noise, while  $N_{t,m}$  [n] represents the receiver Gaussian noise. The non-linear distortion noise  $N_{d,m}$  [n] can be modeled as a zero-mean uncorrelated complex Gaussian random variable with power  $\left(2\sigma_{NL}^2\right)_n$  if the power Input Back Off  $\gamma$  is not too high and if the OFDM block size is not too small. The Gaussian nature of the non-linear distortion noise is motivated by the fact that  $N_{d,m}$  [n] is a linear combination, through the FFT coefficients, of the distortion noise introduced in the time domain on a block of samples  $r_m[k]$  [5], thus generating a Gaussian-like clustering of the constellation points transmitted on each carrier. The Gaussian assumption of  $N_{d,m}$  [n] will be the basis for the performance analysis in Section 3, where its validity is also discussed with greater details.

#### 2.1 Channel Model

A popular and general representation of the timevarying channel  $h(t, \tau)$  in (4) is

$$h(t,\tau) = \sum_{i=0}^{L-1} \beta_i(t) \delta[\tau - \tau_i(t)],$$
(6)

where a multipath fading channel is modeled as the sum of L frequency flat channels with a complex Gaussian distribution for each path.

Under these hypotheses, the channel coefficients  $H_m [n]$  are expressed by

$$H_m[n] = \sum_{i=0}^{L-1} \beta_i [m] e^{-j2\pi\tau_i mn/T}, \qquad (7)$$

Clearly,  $H_m[n]$  is a complex Gaussian random variable and it is easy to proof that, if  $\beta_i[m]$  has uncorrelated real and imaginary components with the same variance  $\sigma_{\beta_i}^2$ , then  $H_m[n]$  has uncorrelated components too, and its envelope  $\rho_{H_m}[n] = |H_m[n]|$  is characterized by a Rice probability density function *(pdf)*. Specifically, if the channel has a single LOS path, then the Rician *pdf* of  $\rho_{H_m}[n]$  is independent of the block index *m* and the carrier index *n*, and it is expressed by

$$p_{\rho_{H}}(\rho_{H}) = \frac{\rho_{H}}{\sigma_{H}^{2}} e^{-\frac{(\rho_{H}^{2} + A_{H}^{2})}{2\sigma_{H}^{2}}} I_{0}\left(\frac{A_{H}\rho_{H}}{\sigma_{H}^{2}}\right)$$
(8)

$$\sigma_{H}^{2} = \sum_{i=0}^{L-1} \sigma_{\beta_{i}}^{2} , \quad A_{H} = \sqrt{\overline{H}_{R}^{2} + \overline{H}_{I}^{2}} = \sqrt{\overline{\beta}_{o_{R}}^{2} + \overline{\beta}_{o_{I}}^{2}} ,$$

where  $(\overline{H}_R + j\overline{H}_I)$  is the mean-value of the channel transfer function and  $(\overline{\beta}_{o_R} + j\overline{\beta}_{o_I})$  is the mean-value of the LOS coefficient  $\beta_0$ . The envelope  $\rho_H$  is still Rice distributed even with multiple LOS paths, but the Rician *pdf* will be time-invariant only if the delays  $\tau_i$  associated to the LOS paths are time-invariant. The previous hypotheses, which are encountered in most realistic scenarios, greatly simplify the analytical evaluation of the system performance and allow averaging the SER performance over a single *pdf* of the signal-to-noise ratio rather than over a number of *pdfs* equal to the number of subcarriers.

#### 2.2 Non-linearity characterization and PSD

The PSD evaluation at the non-linearity output is not the main subject of this paper and the interested reader is referred to [5]. Anyway, some concepts developed in [5] that are necessary to evaluate the SER performance are briefly summarized in this paragraph to assist the reader. The analytical computation of the output PSD, and its separation in useful and non-linear noise components by means of (3) and (9), allow to exactly compute the system performance. Indeed, only the inband frequency components of the non-linear distortion noise have to be taken into account to precisely compute the SER performance. This is different from [7], where all the non-linear noise power was considered in AWGN channels.

The PSD of the non-linear output  $z_d(t)$  in (3) can be expressed by

$$S_{z_{d}z_{d}}(\nu) = \frac{c_{0}}{(2\sigma^{2})} S_{zz}(\nu) + \sum_{n_{0}=1}^{\infty} \frac{c_{n_{0}}}{(2\sigma^{2})^{2n_{0}+1}} \left[ S_{zz}(\nu) \otimes_{1} \cdots \otimes_{2n_{0}+1} S_{zz}(\nu) \right],$$
(9)

where the  $n_o^{th}$  term in the series represents the  $(2n_o+1)^{th}$  auto-convolution of the input PSD  $S_{zz}$  ( $\nu$ ).

The coefficients  $c_{n_o}$  are expressed by [5]

$$c_{n_o} = \frac{\left\| \int_{D(r)} f(r) \frac{r^2}{\sigma^2} e^{-\frac{r^2}{2\sigma^2} L_{n_o}^{(1)} \left( \frac{r^2}{2\sigma^2} \right) dr \right\|^2}{2\sigma^2 (n_o + 1)}, \quad (10)$$

where  $D(r) = \{r : 0 \le r \le \infty\}$  is the integration domain and  $L_{n_o}^{(1)}(x)$  is the Laguerre polynomial of the first kind and  $n_o^{th}$  order. Closed form expressions of the coefficients  $c_{n_o}$  are derived in [5], either when the complex non-linearity f(r) represents the amplifier AM/AM and AM/PM curves expressed by a Bessel series expansion

$$f(r) = \sum_{m_o=1}^{N_b} b_{m_o} J_1\left(\beta\left(m_o,\gamma\right)\frac{\sqrt{\gamma}}{A}r\right),\tag{11}$$

or when f(r) represents the ideally predistorted amplifier as expressed by

$$f(r) = \begin{cases} r & ,r \le A \\ A & ,r > A \end{cases}$$
(12)

where  $R_{\text{max}}$  and A are normalization coefficients [8].

The power amplifier model assumed in this paper is the classical Saleh model [10] where the AM/PM curve has been considered as null for a WLAN amplifier.

# 3. THEORETICAL OFDM PERFORMANCE IN NON-LINEAR FADING CHANNELS

This section derives the analytical SER performance of the WLAN-OFDM systems in non-linear frequency selective fading channels. The transmission of the information symbols  $a_m$  [n] over each subcarrier is impaired by the non-linear channel as expressed by (5), which can be rewritten as

$$R_m [n] = \eta_m [n] a_m [n] + N_m [n],$$
(13)

where  $\eta_m [n] = \alpha H_m [n]$  and the additive noise term is  $N_m [n] = H_m [n] N_{d,m} [n] + N_{t,m} [n]$ . Thus, each OFDM sub-channel is modeled as a classical Rice flat fading channel where, in the non-linear scenario, the fading process influences also the noise term. The amplifier contribution  $\alpha$  and the channel contribution  $H_m [n]$  to the distortion of the useful signal will be indistinguishable to any channel estimation technique. Consequently, with perfect channel state information at the receiver side, the estimated symbol  $\hat{a}_m [n]$ , obtained by Zero Forcing (ZF) equalization, is given by

$$\hat{a}_{m}[n] = a_{m}[n] + \frac{N_{d,m}[n]}{\alpha} + \frac{N_{t,m}[n]}{\alpha \cdot H_{m}[n]}.$$
 (14)

It is well known that in a single-tap scenario there is no reason to use a MMSE equalizer because it is equivalent to a scaled version of the ZF, and consequently it performs as the ZF if the decision thresholds after the equalization are accordingly scaled. In the non-linear fading scenario, the Signal-to-Noise Ratio (SNR) for a given  $H_m$  [n] is expressed by

$$(\chi_{_{TOT}})_{n} = \frac{E_{a}}{\frac{\left(\sigma_{d}^{2}\right)_{n}}{|\alpha|^{2}} + \frac{\sigma_{t}^{2}}{|\alpha|^{2} \cdot |H_{m}[n]|^{2}}}.$$
 (15)

where  $E_a = E\{|a_m|^2\}$  and  $\sigma_t^2 = E\{|N_t|^2\}$  are the constellation mean power and the thermal noise power, respectively.

The SNR on the  $n^{th}$  subcarrier  $(\chi_{TOT})_n$  can be expressed as a function of the SNR  $\chi_n$  in linear environment (when  $\alpha = 1$  and  $N_{d,m}$  [n] = 0) as expressed by

$$(\chi_{_{TOT}})_n = \left[\frac{1}{(\chi_{_{NL}})_n} + \frac{1}{|\alpha|^2 \cdot \chi_n}\right]^{-1}.$$
 (16)

 $(\chi_{NL})_n$  in (16) represents the non-faded SNR at the non-linearity output, or equivalently, the SNR in the absence of thermal noise on the  $n^{th}$  subcarrier and it is expressed by [5]

$$(\chi_{NL})_n = \frac{c_o^2}{2\sigma^2} \cdot \frac{S_{zz} \left(n / T_b\right)}{S_{n_d n_d} \left(n / T_b\right)},\tag{17}$$

where  $S_{zz}(\nu)$  and  $S_{n_d n_d}(\nu)$  are used to denote the PSD of the signal z(t) and  $n_d(t)$ , respectively.

The SNR in linear environment, conditional to a channel realization, is clearly expressed by

$$\chi_n \, = \, |\, H_m \, [\, n \, ] \, |^2 \, E_a \, / \, \sigma_t^2 \, = \, |\, \rho_H \, |^2 \, E_a \, / \, \sigma_t^2 \, ,$$

and its  $pdf \ p(\chi_n)$  is either exponential or chi-squared with two degrees of freedom, if  $\rho_m[n] = |H_m[n]|$  is either a Rayleigh or a Rice random variable, respectively. Specifically,  $p(\chi_n)$  is given by

$$p(\chi_n) = \begin{cases} \frac{1}{\chi_o} \exp\left(-\chi_n/\chi_o\right) & , \chi_n \ge 0\\ 0 & , \chi_n < 0 \end{cases}$$
(18)

or

$$p(\chi_{n}) = \begin{cases} \frac{1+Q^{2}}{\chi_{o}} e^{-Q^{2}} \exp\left[-\left(1+Q^{2}\right)\frac{\chi_{n}}{\chi_{o}}\right] \\ I_{o}\left(2Q\sqrt{\left(1+Q^{2}\right)\frac{\chi_{n}}{\chi_{o}}}\right) &, \chi_{n} \ge 0 \end{cases}, (19) \\ 0 &, \chi_{n} < 0 \end{cases}$$

The parameter  $\chi_o$ , which represents the mean signal-to-noise ratio, is independent of the  $n^{th}$  subcarrier if the channel is characterized by a single LOS (as assumed in the following) and is expressed by

$$\chi_o = \begin{cases} 2\sigma_H^2 \cdot E_a \big/ \sigma_t^2 & ,|H| \text{ Rayleigh} \\ (1+Q^2)2\sigma_H^2 \cdot E_a \big/ \sigma_t^2 & ,|H| \text{ Rice} \end{cases}, (20)$$

where  $Q^2 = A_H^2 / 2\sigma_H^2$  is the power ratio between the

LOS and the NLOS contributes.

The uncoded mean SER for the  $n^{th}$  subcarrier results computable by

$$(SER)_n = \int_{-\infty}^{\infty} P_{err} \left[ \left( \frac{1}{(\chi_{NL})_n} + \frac{1}{|\alpha|^2 \chi_n} \right)^{-1} \right] p(\chi_n) d\chi_n . (21)$$

where  $P_{err}(\chi)$  is the error probability, which depends on the  $a_m[n]$  mapping.

The expression of  $P_{err}(\cdot)$  for an AWGN channel can be used if the channel-scaled non-linear noise  $H_m [n] N_{d,m} [n]$  can be modeled as Gaussian, in such a way that the noise  $N_m [n] = H_m [n] N_{d,m} [n] + N_t [n]$ is Gaussian too, being the sum of the two independent Gaussian contributions.

This assumption is as much correct as more the size (i.e. the numbers of carriers) of the OFDM block increases, as well as the input power back-off (*ibo*)  $\gamma$  to the non-linearity decreases. Indeed, as long as the *ibo* increases the distortion events become rarer and consequently the number of distortion errors in the time domain is small (in short blocks) and, therefore, the central limit theorem after the FFT processing cannot be invoked.

Consequently, when the ibo is too high and/or the block size is too small, the distribution of the received noise on each subcarrier is no longer Gaussian and the expression of  $P_{err}$  ( $\rightarrow$ ) in (21) should be evaluated taking into account the real distribution of the error  $N_m$  [n]. This subject is beyond the scope of the present paper even if consideration about the ibo values, and the constellation size M that require such an analysis will be outlined in the next section by simulation results.

Expression (14) and (21) state the well known fact that the SER performance for each  $n^{th}$  subcarrier of an OFDM system, in a frequency selective fading channel, is the same SER of a single carrier system in a frequency flat fading channel.

The mean SER for the OFDM system is simply obtained by averaging the mean SERs of all the subcarriers, as expressed by

$$\overline{SER} = \frac{1}{N_a} \sum_{n=1}^{N_a} (SER)_n , \qquad (22)$$

where  $N_a$  represents the number of active carriers used to transmit information within the total number N of carriers. In order to reduce the computing time involved in the numerical evaluation of (21) for all the  $N_a$ subcarriers, the SER can be approximated by

$$\overline{SER} \approx \int_{-\infty}^{\infty} P_{err} \left[ \left( \frac{1}{\overline{\chi}_{NL}} + \frac{1}{|\alpha|^2 \chi} \right)^{-1} \right] p(\chi) d\chi , (23)$$

where

$$\bar{\chi}_{NL} = \frac{1}{N_a} \sum_{n=1}^{N_a} (\chi_{NL})_n$$
 (24)

is the mean non-linear SNR for all the  $N_a$  active subcarriers and  $\chi = \chi_n$  for any n in a single LOS scenario.

The use of (23) and (24) instead of (21) produces a little inaccuracy in the evaluation of the SER performance of the OFDM system because the nonlinear SNR  $(\chi_{NL})_n$  expressed by (17) is quite the same for most of the  $N_a$  active carriers. Indeed, the PSD  $S_{zz} (n/T_b)$  of the useful OFDM signal is usually constant for each subcarrier while the non-linear distortion noise  $n_d(t)$  is characterized by a quasi-constant PSD inside the useful OFDM bandwidth, with monotonically decreasing values starting from the band-center up to the band-edges [5].

## 4. OPTIMUM OUTPUT POWER BACK-OFF

The *obo* is the key parameter for a meaningful comparison between the predistorted and the nonpredistorted scenario. The *obo* is defined as the ratio between the maximum and the mean amplifier output power. It generally depends on both the *ibo* and the non-linear distortion f(r) [5].

The so called "Total Degradation" (TD) of the power link budget in the presence of non linear amplifiers, is defined as the sum of the *obo* with the excess SNR that guarantees at this *obo* the same target SER with respect to the linear situation [12]. These two power penalties are clearly in competition with one another. Indeed, an *obo* reduction, which represents a power gain at the transmitter, results in a power waste at the receiver because of the higher signal power that is required by the receiver to compensate for the increased non-linear distortion noise.

The optimum obo, as far as performance is concerned, is consequently defined as the one that minimizes the TD and it is a function of the target SER.

Usually OFDM systems make use of channel coding to improve SER, by exploiting the system diversity in the frequency domain. The relationship between the target coded SER and the corresponding target uncoded SER changes as a function of applications, channel coding techniques and scenarios. A reasonable uncoded SER for WLAN, which can exploit coding and retransmission, lies in the range  $[1 \cdot 10^{-2} \div 1 \cdot 10^{-3}]$ 

It should be pointed out that the choice of the optimum *obo* value for an OFDM system could also consider the signal spectral re-growth in the adjacent channels. Consequently, also the PSD as a function of the *obo* value should be evaluated by means of (9) [5].

#### 5. SIMULATION RESULTS

In this section, we show performance comparisons between analytical and simulation results to verify the applicability of the proposed analytical approach. The IEEE 802.11a physical layer [2], which is formed by 64-length blocks (N = 64) and  $N_a = 52$  active carriers, has been chosen as a reference. Each OFDM block has been over-sampled by four in the time-domain in order to obtain an adequate signal representation in a non-linear environment. The interpolated signal has been successively distorted during the simulations by the curves (11) and (12), which represent the AM/AM distorting amplifier and the ideally predistorted amplifier, respectively. The values of the  $N_b = 13$ coefficients  $b_{m_o}$  used in (11) to represent the distorting amplifier, have been obtained by fitting the Saleh model with null AM/PM, by using  $R_{\rm max} = 7.9$  and A = 1.

The 802.11a standard [3] maps the information bits on a M-QAM constellation with M = 4,16, or 64. Consequently, the expression of the symbol error probability  $P_{err}$  (·) that must be used in (21) or (23) to calculate the SER for each subcarrier is given by [9]

$$P_{err}(\chi) = 1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{M}}\right)Q_o\left(\sqrt{\frac{3}{M-1}\chi}\right)\right]^2, (25)$$

where  $\chi$  is the average SNR per symbol and  $Q_o(x) = 0.5 \cdot erfc(x/\sqrt{2})$ .

The relationship between the SNR  $\chi_{TOT}$  that establishes the SER performance in (21) and the "apparent" one  $\chi_{App}$  that is measured at the receiver side, is expressed by

$$\frac{1}{\chi_{TOT}} = \frac{1}{\chi_{NL}} \left[ 1 + \frac{1}{\chi_{App}} \right] + \frac{1}{\chi_{App}} \,. \tag{26}$$

The simulations have been performed with perfect channel inversion and perfect ISI elimination as supposed in the theoretical analysis. Typical channels [11], compliant with the general model of section 2.2, have been employed for the WLAN environment.

Figures 1-2 show (with and without ideal predistortion, respectively) that the analytical model is quite accurate to predict the simulation results in Rayleigh fading channels for SER and *obo* of practical interest, when 16-QAM is employed. On the contrary, Figures 3-4 point out that the analytical model is optimistic for 64-QAM constellations. Such behavior is not evident when the OFDM system uses a higher number of carriers (as the ones examined in [5] and [6]) and it worsen when the *obo* increases. This is due, as explained in section 3, to the failure of the Gaussian modeling for the non-linear distortion noise after the FFT processing at the receiver side.

This result could be misleading. Indeed, the simulation results may suggest that the sensitivity of OFDM systems to non-linear distortions increases when the carriers number decreases, which is counter-intuitive and basically wrong (e.g. a single carrier system is less sensitive to non linear distortion rather than a multicarrier system). However, the analysis of the output PSD for OFDM-WLAN systems, which is not shown herein, reveals that the non-linear distortion noise power on each subcarrier is lower than or equal to the case when more carriers are employed. The higher performance degradation with few carriers is due to the noise statistic, which is no longer Gaussian and consequently a decision device based on a classical distance metric is no longer optimum (e.g. it is not the

one that minimizes the SER). Similar behaviors are observed for AWGN and Rice channels, which are not reported for lack of space.

Figures 5-6 show the analytical TD for a target  $BER \simeq SER / \log_2 M = 1 \cdot 10^{-3}$  for different channel behaviors (AWGN, Rice and Rayleigh) with and without predistortion, respectively. Generally it is possible to conclude that the optimum *obo* in Rayleigh fading channels is lower than the one suggested by the classical AWGN analysis, because the non linear distortion noise at the receiver side is partially masked by the channel power statistic.

The gain obtained in the system link budget by amplifier predistortion [8], (e.g. the TD reduction at the optimum *obo*) suggests using a predistortion technique either for 16 or 64-QAM constellations (1 dB and 2.3 dB of power gain, respectively), while the negligible power gain for 4-QAM does not justify such a choice, if other effects like out-band spectral re-growth can also be neglected.

### 6. CONCLUSIONS

We have analyzed the effect of non-linear amplifications with and without predistortion in OFDM WLAN systems. The optimum *obo*, which minimizes the power loss in the system link budget, has been shown to be a function of the channel condition. It is lower for Rayleigh fading with respect to the AWGN scenarios, and it also suggests using predistortion for 16 and 64 QAM. The analytical approach based on the Bussgang theorem is still accurate to predict the output PSD while the Gaussian approximation of the distortion noise in the frequency domain is sufficiently accurate for 4 and 16-QAM for uncoded SER of practical interest. Further analysis is required for 64-QAM in order to analytically evaluate the SER as well as to identify a robust detector that takes into account the real statistic of the non-linear distortion noise.

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Figure 1. SER for Predistorted Amplifier and 16-QAM.



Figure 3. SER for Predistorted Amplifier and 64-QAM



Figure 5TD for Predist. Amplifier with 4,16,64-QAM

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Figure 2. SER for Saleh Amplifier and 16-QAM



Figure 4. SER for Saleh Amplifier and 64-QAM



Figure 6. TD for Saleh Amplifier with 4,16,64-QAM