Effects of HPA Non Linearity on Frequency Multiplexed OFDM Signals

Paolo Banelli, Giuseppe Baruffa, and Saverio Cacopardi

***Abstract*—The paper analyzes the performance of the downlink channel of a multimedia interactive service system which transmits the desired information by the frequency multiplexing of several OFDM signals compliant with the DVB-T standard. The effects of the nonlinear distortions introduced by a High Power Amplifier on the system performance are evaluated both in terms of the Bit Error Rate (BER) degradation in AWGN channels and of the spectral regrowth. The performance comparison to the case of a single DVB-T signal as well as the benefits of an ideal predistortion is also considered by comparing analytical results to computer simulations.**

***Index Terms*—CABSINET, communication system nonlinearities, DVB-T, non linear distortions, OFDM, predistortion.**

I. Introduction

THE INCREASING demand of multimedia services and the impressive growth of Internet related contents directly leads the interest to communication systems that are capable to offer a broadband access to such services. Different solutions to this task have been proposed in the last years such as x-DSL (Digital Subscriber Line), HFC (Hybrid Fiber Coaxial), FITL (Fiber In The Loop) and WITL (Wireless In The Loop) technologies [1]. The WITL schemes offer the possibility to rapidly develop, at reasonable costs, a physical network also to those providers that have not cable terminations at the end-user premises. Moreover, this technique offers to broadcasters a stimulating opportunity to move toward a new concept about the diffusion of their services, realizing both the integration with cellular topology networks and the Internet world. In such a context great attention has received the LMDS (Local Multipoint Distribution Systems) architecture: it generally provides asymmetrical broadband services, characterized by RF downlink channels in frequency bands varying from 2.5–2.7 GHz up to 40.5–42.5 GHz, and by a cellular (GSM, IS-95, CDMA, etc.) or cordless (DECT) return channel, depending on the mobility requirements. These topics have been also the object of two ACTS European projects named CRABS [2] and CABSINET [3]. The CABSINET project has considered a double layer network, with the first layer characterized by 1 Km radius macrocells dedicated to the fixed users, and the second layer characterized by hundred meter microcells employed to serve mobile-nomadic receivers [4].

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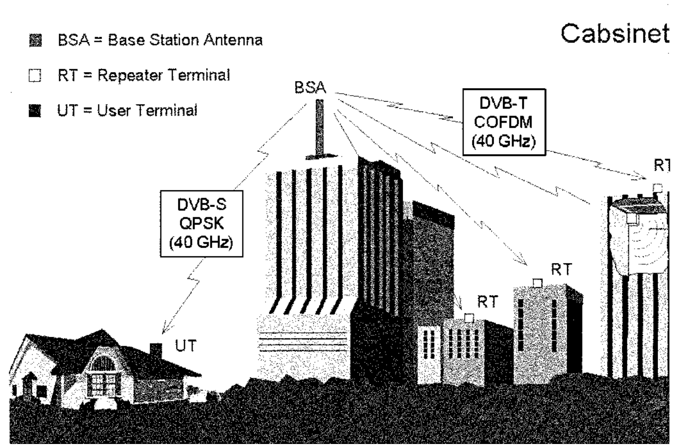


Fig. 1. Downlink channel for the two layer architecture of the CABSINET project.

The first layer macro-cells irradiate the multimedia services by a Base Station (BS) transmitter in the 40 GHz frequency band, where Line Of Sight (LOS) condition is required in order to make the communication effective. The BS transmitter receives the signal either by optical fiber/coaxial connections or by satellite links. The second layer microcell, on the contrary, irradiates the signal in the 5.8 GHz frequency band. The first layer is manifestly thought to serve fixed users with directional rooftop antennas in LOS with the transmitter, while the second layer is dedicated to portable receivers with integrated omni-directional antennas. CABSINET project has proposed to provide multimedia interactive services in indoor environment, by transmitting in the microcell downlink channel a COFDM DVB-T compatible signal located in the 5.8 GHz band. The macrocell downlink channel for fixed users adopt a QPSK modulation, compliant to DVB-S standardization, on a single carrier system located in the 40–42 GHz band. Local repeaters grant the distribution of the DVB-T [5] signals in the indoor environment (see Fig. 1). The OFDM processing however is accomplished at the main transmitter, that broadcasts the downlink signal to the same repeaters used for the QPSK outdoor services. In particular, each transmitter realizes the frequency multiplexing of 4 independent DVB-T signals, each one characterized by the following parameters:

- 2K mode (FFT size 2048): Subcarrier Spacing $=$ 4.464 KHz, Useful Symbol Duration $= 224\ \mu$s
- Constellation: 4 QAM or 16-QAM

OFDM signals, like DVB-T, are made up by the sum of a large number of carriers, and consequently they are very sensitive to nonlinear distortions because of their greatly variable envelope. The aim of this paper is to briefly summarize the distortion effects introduced by a nonlinear High Power Amplifier (HPA) modeled by AM/AM and AM/PM distortion curves, in a single DVB-T signal [6]–[8] and to compare the results to the situation in which the same amplifier handles a Multiplexed DVB-T signal.

Due to the Central Limit Theorem, a baseband OFDM signal can be modeled (for a high number of independently modulated carriers) as a complex Gaussian process with Rayleigh envelope distribution. This fact allows the analytical treatment of nonlinear distortion of OFDM signals making use of the more general results obtained for nonlinear distortions of Gaussian signals [9], [10]. If a signal is distorted by nonlinearities, the resulting signal exhibits a spectral regrowth that can be distinguished as In Band Intermodulation and Out Band Intermodulation. The former is responsible for the BER degradation of the system, while the latter causes adjacent channel interference (ACI). The nonlinear distortion introduced by a power amplifier becomes as higher as more the input signal works in the saturation zone. The Output Back Off (OBO) from the maximum power of the amplifier is the parameter that controls how much the amplifier works in its nonlinear region. The increase of the OBO in general produces the reduction of the nonlinear distortions introduced by the power amplifier but at the same time, also a reduction of the coverage area of the transmitter. The Total Degradation (or the Power Penalty) introduced in the following, is a parameter that takes account of these two competitive effects and it will be used to calculate the optimum OBO for the DVB-T system, both in single and in multiplexed signal configuration. Moreover, the ACI will be considered as another key parameter to evaluate the practical value for the amplifier OBO. The obtained results will be compared with simulations in order to validate the correctness of the analytical approach.

II. Analytical Approach

The frequency multiplexing of several independent OFDM (DVB-T) signals is expressed by

$$x_{mx}(t) = \sum_{i=1}^{N_s} x_i(t) \cdot e^{j2\pi f_i t} \tag{1}$$

where f_i is the central frequency for the ith OFDM signal. If each one of the N_s OFDM signals is characterized by a zero mean value and by the same power σ_x^2, then the multiplexed signal $x_{mx}(t)$ is also a zero mean Gaussian process with a power expressed by (2)

$$P_{mx} = \sigma_{mx}^2 = \sum_{i=1}^{N_s} P_{x_i} = \sum_{i=1}^{N_s} \sigma_{x_i}^2 = N_s \cdot \sigma_x^2. \tag{2}$$

As a consequence, it is possible to analyze the non linear distortion effect introduced by the HPA on single or frequency multiplexed DVB-T signals by means of the same analytical approach used for any Gaussian input signal. By generalizing the Bussghang Theorem to complex non linearity [10], [7], it is possible to state that the AM/AM and AM/PM distortions introduced by a HPA on the DVB-T signal, generate a distorted output signal which is the sum of two different components. One represents an amplified replica of the useful input signal while the other is a nonlinear distortion noise uncorrelated with the useful one, as expressed by the following relation (3)

$$s_d(t) = s_u(t) + n_d(t) = \alpha \cdot s(t) + n_d(t), \tag{3}$$

being $s_u(t)$ and $n_d(t)$ the useful and distorted part of the output signal $s_d(t)$ respectively.

The output correlation function is expressed by

$$R_{s_d s_d}(\tau) = |\alpha|^2 \cdot R_{ss}(\tau) + R_{n_d n_d}(\tau) \tag{4}$$

where the complex multiplicative coefficient a of the useful component is expressed by

$$\begin{aligned} \alpha &= \frac{R_{S_d S}(0)}{R_{SS}(0)} = \frac{E\{s_d(t)s^*(t)\}}{2\sigma^2} \\ &= \frac{E\{f(r)r\}}{2\sigma^2} = \frac{1}{2\sigma^2} \int_0^\infty f(r) \cdot r \cdot p(r)\, dr. \end{aligned} \tag{5}$$

In the last expression, $R_{S_d S}(\tau)$ denotes the input–output cross-correlation function, $f(r)$ represents the nonlinear distorting function, 2 σ^2 the input signal power, and $p(r)$ the Rayleigh probability density function of the input envelope.

The nonlinear distortion noise $n_d(t)$ is distributed over a wider bandwidth than the signal one, and it is responsible for the spectral regrowth of the output signal $s_d(t)$. The Power Spectrum Density (PSD) of the output signal is obtained as the Fourier Transform (FT) of the output correlation function expressed in (4).

It is possible to prove that, for any $f(r)$ nonlinear distortion function, the expression (4) can be reduced to (6) [6]

$$\begin{aligned} R_{s_d s_d}(\tau) &= \sum_{n=0}^{\infty} c_n \cdot \left[\frac{R_{ss}(\tau)}{2\sigma^2}\right]^{2n+1} \\ &= \frac{c_0}{(2\sigma^2)} \cdot R_{ss}(\tau) + \sum_{n=1}^{\infty} c_n \cdot \left[\frac{R_{ss}(\tau)}{2\sigma^2}\right]^{2n+1} \end{aligned} \tag{6}$$

and that the output Power Spectrum Density (PSD) can be expressed as (7)

$$\begin{aligned} S_{s_d s_d}(\nu) &= FT\{R_{s_d s_d}(\tau)\} = \alpha^2 \cdot S_{ss}(\tau) + S_{n_d n_d}(\tau) \\ &= \frac{c_0}{(2\sigma^2)} \cdot S_{ss}(\tau) + \sum_{n=1}^{\infty} \frac{c_n}{(2\sigma^2)^{2n+1}} \\ &\quad \cdot [S_{ss}(\tau) \otimes_1 L \otimes_{2n+1} S_{ss}(\tau)], \end{aligned} \tag{7}$$

with c_n coefficients given by (8)

$$\begin{aligned} c_n &= \frac{1}{2\sigma^2} \frac{1}{n+1} \\ &\quad \cdot \left\| \int_{D(r)} f(r) \cdot \frac{r^2}{\sigma^2} \cdot e^{-(r^2/2\sigma^2)} \cdot L_n^{(1)}\left(\frac{r^2}{2\sigma^2}\right) dr \right\|^2 \end{aligned} \tag{8}$$

$L_n^{(k)}(x)$ being the Laguerre function (9)

$$L_n^{(k)}(x) = \frac{x^{-k}e^x}{n!}\left(\frac{d}{dx}\right)^n (x^{n+k}\cdot e^{-x}). \quad (9)$$

It is noteworthy that (6) represents the expansion of (4), with $c_0/2\sigma^2$ equal to the $|a|^2$ multiplication of the useful signal power and the series equal to the correlation function $R_{n_dn_d}(\tau)$ for the nonlinear distortion noise, as expressed by (10)

$$R_{n_dn_d}(\tau) = \sum_{n=1}^{\infty} c_n \cdot \left[\frac{R_{ss}(\tau)}{2\sigma^2}\right]^{2n+1}. \quad (10)$$

This expression has a simple physical interpretation [the Fourier transform of (6) is the output PSD]. The Fourier transform of the nth term in the series represents the PSD obtained by a $(2n+1)$ convolution of the input PSD with itself. This situation is similar to the case of a deterministic signal spectrum at the output of an odd nonlinearity. An easily manageable expression for the c_n coefficients of (8) can be obtained when the nonlinear distortion $f(r)$, introduced by an instantaneous Power Amplifier, is represented by a Bessel series expansion [12]

$$\begin{aligned} f(r) = h(r)e^{j\phi(r)} &= \sum_{m=1}^{L} b_m J_1\left(\beta_m \frac{r}{\sqrt{2}\sigma}\right) \\ &= \sum_{m=1}^{L} b_m J_1\left(\beta_m \frac{\sqrt{ibo}}{A} r\right) \end{aligned} \quad (11)$$

with $h(r)$ and $f(r)$ being the AM/AM and the AM/PM distortion curves respectively. The b_m coefficients can be derived by a fitting procedure on the measured data, while $b_m = b_m(ibo, m)$ is expressed by (12)

$$b_m(ibo, m) = \frac{(2m-1)p}{R_{\max}} \times \frac{A}{\sqrt{ibo}} \quad (12)$$

where $R_{\max}$ is a normalization parameter [6]. The ibo term in (11) and (12) is the Input Back Off and represents the ratio between the Input Saturation Power and the Input Mean Power as expressed by

$$ibo = \frac{A^2}{2\sigma^2} \quad (13)$$

where A is the input voltage to which corresponds the maximum amplifier output power.

Under this hypothesis the c_n coefficients are obtained by substituting (11) in (8) and are expressed by [6]

$$c_n = \frac{1}{n!(n+1)!}\cdot\left\|\sum_{m=0}^{L} b_m \cdot \left(\frac{\beta_m}{2}\right)^{2n+1} \cdot e^{-(\beta_m/2)^2}\right\|^2. \quad (14)$$

Differently, if the RF amplifier is ideally predistorted, $f(r)$ is reduced to be a real function expressed by (15)

$$f(r) = h(r) = \begin{cases} r, & r < A \\ A, & r > A. \end{cases} \quad (15)$$

In this situation it is possible to express the c_n coefficients by means of an exact expression [6] as reported in the Appendix.

III. OFDM PERFORMANCE IN NONLINEAR AWGN CHANNEL

In the time domain, the nonlinear distortion of the transmitted signal generates a noise distortion signal spread in the frequency domain over each carrier. In order to understand the role of this non linear noise contribution, it is useful to remind that the samples of a complex baseband OFDM signal, transmitted during each period $T_b = L_{fft}T_s$, are expressed by (16) [11]

$$s(nT_s) = \sum_{k=0}^{L_{fft}-1} z[k]\cdot e^{j\omega_k nT_s}, \quad (16)$$

where

$z_k = a_k + jb_k$ represents the complex information symbols;
$\omega_k = k\cdot 1/T_b$ is the kth carrier;
L_{fft} is the carriers number.

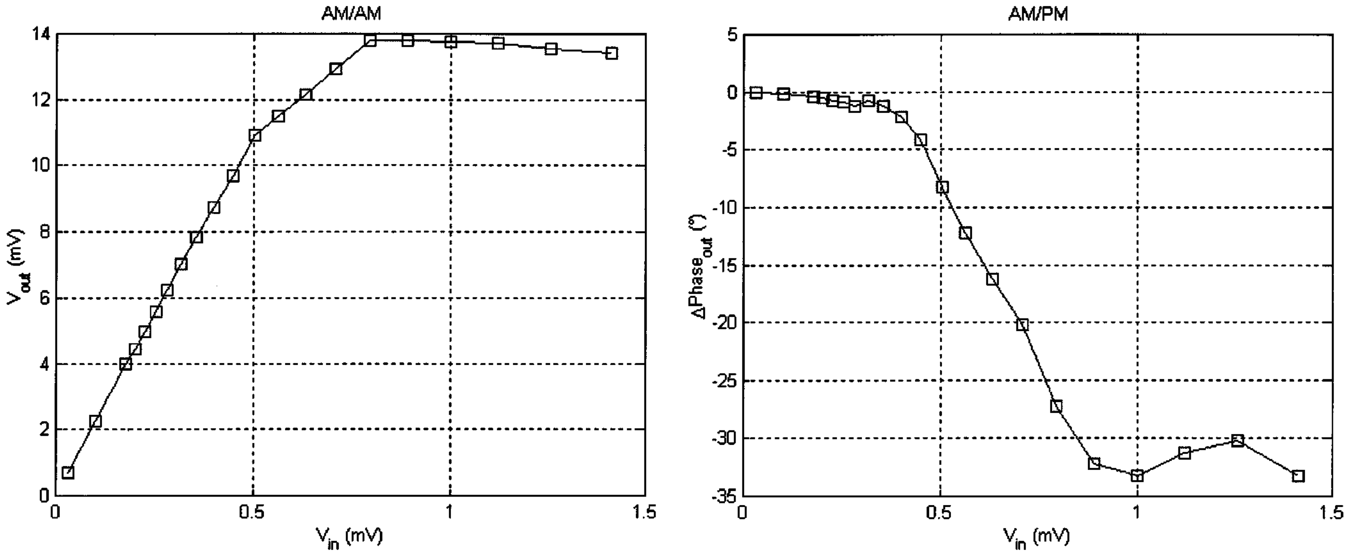
The received signal is expressed by (17)

$$s_r(t) = s_d(t) + n_r(t), \quad (17)$$

being $n_r(t)$ the thermal Additive White Gaussian Noise (AWGN) at the receiver side. The complex symbols, received through the non linear AWGN channel are expressed, after the cyclic prefix removal and the DFT processing, by (18)

$$\begin{aligned} z_r[k] &= \sum_{n=0}^{L_{fft}-1} s_r(nT_s)\cdot e^{-j(2\pi/L_{fft})nk} \\ &= \alpha\cdot z[k] + \sum_{n=0}^{L_{fft}-1}[n_d(nT_s) + n_r(nT_s)] \\ &\quad \cdot e^{-j(2\pi/L_{fft})nk} \\ &= \alpha\cdot z[k] + e_{nd}[k] + e_{nr}[k] = \alpha\cdot z[k] + e_n[k]. \end{aligned} \quad (18)$$

The terms $e_{nd}[k]$ and $e_{nr}[k]$ are two uncorrelated noise contributions that distort the received symbols. The contribution $e_{nd}[k]$ is obtained by linear combination of samples of a stationary process and, if the number of the distortion errors in the time domain is high enough, it can be modeled as a complex Gaussian process because of the Central Limit Theorem. In this sense, the distortion term $e_{nd}[k]$ affects the constellation mapped over each carrier, in the same way as a Gaussian noise added to the $e_{nr}[k]$ thermal noise contribution of the receiver. As a consequence, the system performance of an OFDM system can be derived making use of the same relationship used for AWGN channels, with the attention of redefining the effective signal to noise ratio at the receiver input [6].



(a) (b)

Fig. 2. Amplifier AM/AM and AM/PM non linear distorting function.

The power of the noise term that affects each constellation symbol is expressed by

$$\begin{aligned} P_{e_n}[k] &= E\left\{e_n[k]\cdot e_n^*[k]\right\} \\ &= L_{fft}\cdot S_{n_d n_d}\left(\frac{k}{T_b}\right) + L_{fft}\cdot S_{n_r n_r}\left(\frac{k}{T_b}\right) \end{aligned} \tag{19}$$

while the power of the useful term is expressed by

$$\begin{aligned} P_{z_u}[k] &= E\left\{z_u[k]\cdot z_u^*[k]\right\} \\ &= |\alpha|^2\cdot E\left\{z[k]\cdot z^*[k]\right\} = |\alpha|^2\cdot L_{fft}\cdot S_{ss}\left(\frac{k}{T_b}\right) \end{aligned} \tag{20}$$

where $S_{n_d n_d}$, $S_{n_r n_r}$ and S_{ss} represent PSD's of the corresponding signals.

As shown in the previous equations, the two power terms respectively represent the sampling on the kth carrier position of the PSDs of the noise term $n_d(t)$ and of the useful term $s_u(t)$ in (3).

The uncoded system performance in AWGN channels are generally expressed by (21)

$$\text{SER } = P_{err}\left(\frac{E_b}{N_0}\right), \tag{21}$$

where (E_b/N_o) is the signal to noise ratio per bit for each carrier as expressed by (22)

$$\frac{E_b}{N_o} = \frac{(\text{SNR})}{n_{bit}}. \tag{22}$$

with n_{bit} being the number of bits per symbol transmitted over each carrier.

In the time domain, the nonlinear distortion of the transmitted signal generates a noise distortion signal that is spread in the frequency domain over each carrier. As a consequence the system performance of an OFDM system can be derived making use of relation (21) with the attention of redefining the effective signal to noise ratio at the receiver. The nonlinear distortion noise is uncorrelated with the thermal noise: they can be added in power to estimate the BER performance by (21). Defining $(E_b/N_o)_{\text{eff}}$ as the effective signal to noise ratio at the receiver in presence of non linear distortion, $(E_b/N_o)_{app}$ as the apparent value at the receiver input and $(E_b/N_o)_{nl}$ as the value at the nonlinearity output, it is easy to obtain [6]

$$\begin{aligned} \left(\frac{E_b}{N_0}\right)_{\text{eff}} &= \frac{(E_b)_{use}}{(N_0)_{the} + (N_0)_{nl}} \\ &= \left\{\frac{1}{\left(\frac{E_b}{N_0}\right)_{app}} + \left[1+\frac{1}{\left(\frac{E_b}{N_0}\right)_{app}}\right]\cdot\frac{1}{\left(\frac{E_b}{N_0}\right)_{nl}}\right\}^{-1} \end{aligned} \tag{23}$$

where $(E_b/N_o)_{nl}$ and $(E_b/N_o)_{app}$ for the kth carrier are expressed by

$$\begin{aligned} \left[\left(\frac{E_b}{N_o}\right)_{nl}\right]_k &= \frac{(E_b)_{use}}{(N_o)_{nl}} = \frac{c_o^2}{n_{bit}\cdot(2\sigma^2)}\cdot\frac{S_{ss}\left(\frac{k}{T_b}\right)}{S_{n_d n_d}\left(\frac{k}{T_b}\right)} \\ \left[\left(\frac{E_b}{N_0}\right)_{app}\right]_k &= \frac{(E_b)_{use}+(N_0)_{nl}}{(N_0)_{the}} = \frac{S_{s_d s_d}\left(\frac{k}{T_b}\right)}{(N_0)_{the}} \end{aligned} \tag{24}$$

and the subscripts the, use and nl mean respectively thermal, useful and nonlinear. The calculation of the above quantity depends on the nonlinearity $f(r)$ by means of (7) and (8). Expression (21) becomes, for M-QAM mapping [13]

$$\text{SER} = 1 - \left(2 \cdot \left(\sqrt{\frac{M-1}{M}} \right) \cdot Q \left(\sqrt{\frac{3 \cdot n_{bit} \cdot \left(\frac{E_b}{N_o} \right)}{M-1}} \right) \right)^2, \tag{25}$$

where $M = 2^{n_{bit}}$ is the QAM constellation size.

In multicarrier systems like DVB-T, the average BER is obtained by averaging the BER of each subcarrier. It is computed by (25), making use of (23) to express the effective signal to noise ratio that impose the performance and it is expressed as

$$\text{BER} = \frac{\text{SER}}{n_{bit}} = \frac{1}{n_{bit}} \frac{\sum_{k=1}^{L_{fft}} (\text{SER})_k}{L_{fft}}. \tag{26}$$

IV. Amplifier Characterization

The CABSINET project has considered employing at the base transmitter a 24 dBm BOSCH HPA [3] to transmit 4-multiplexed DVB-T signals in the 40–42 GHz band. The complex function $f(r)$, summarizing the AM/AM and AM/PM distortion curves represented in Fig. 2, has been fitted by Bessel series expansion by means of expression (11). The coefficients b_n and the $R_{\max}$ parameter, obtained by the fitting procedure, are reported in Table I.

The system Link Budget [3] from the macrocell transmitter to the microcell repeaters has specified a required TX power of about 14.22 and 22.22 dBm for the 4-QAM and the 16-QAM carriers mapping, respectively. The 24 dBm output power at 1 dB compression point of the BOSCH amplifier seems to be a good compromise value for the 4-QAM modulation, whereas more care must be used for the 16-QAM modulation.

V. System Performance

As previously said, CABSINET project decided to employ an OFDM signal compliant with the DVB-T standard. In particular, the proposed system makes use of the 2K-Mode with 4-QAM or 16-QAM modulation for each carrier. This signal is obtained by a 2048 IFFT of the M-QAM symbols to be modulated over each carrier. The 2048 carriers corresponding to each one of the IFFT inputs are not totally used to transmit information. A part of them are switched off to implement a guard band from adjacent channels, while other carriers, named “pilot carriers,” are used to deliver system configuration parameter, synchronization information, and channel sounding signals in order to help the receiver in the demodulation process. The 2048 carriers are distributed as reported in Table II.

The BER performance only depends on the SNR (or E_b/N_o) of the Data Carriers. It means that the BER performance can be exactly calculated by expressions (23)–(26) selecting the values of the k indices that correspond to the data carriers. Table III is computed by (27) and summarizes how to scale the (E_b/N_o) values, shown in the simulation results, for the purpose to obtain the SNR of the signal computed in its useful bandwidth:

$$\begin{aligned} \text{SNR} &= n_{bit} \left(\frac{E_b}{N_o} \right)_{Tot} \\ &= n_{bit} \cdot \frac{(16/9) \cdot (N_{Cont} + N_{Scatt}) + (N_{Data} + N_{TPS})}{(N_{cont} + N_{Scatt} + N_{TPS} + N_{Data})} \\ &\quad \cdot \left(\frac{E_b}{N_o} \right)_{Data}. \end{aligned} \tag{27}$$

Figs. 3–6 show the system performance obtained for different value of the HPA Output Back Off (OBO). The comparison between analytical results and computer simulations confirm the correctness of the analytical approach. The DVB-T, as most

TABLE I
BESSEL COEFFICIENTS OBTAINED BY AM/AM AND AM/PM COMPLEX FITTING

	$R_{\max} = 3.5$	
n	**Real(β_n)**	**Imaginary(β_n)**
1	1.6635840616971718e+000	-9.5677930460009741e-001
2	4.6978526075253912e-001	4.7634285430000860e-002
3	7.0392488346885010e-002	2.6747179128139936e-001
4	-5.8794381179369158e-002	1.5697162876720908e-002
5	-4.8767377845625959e-002	-1.1098175834217154e-001
6	3.8987712060308294e-002	5.7903703564138549e-002
7	-3.3628774026267245e-002	-3.4397163054712639e-002
8	1.5955461060820832e-002	-5.2817398619022238e-003
9	2.3498156521932070e-002	-4.1332308280155978e-003
10	-8.1668814061384397e-003	3.4398981665960893e-002
11	-2.6906228557785936e-002	8.3184849852572847e-003
12	1.3992022790639160e-002	-3.8537770522655029e-002
13	6.3029613241409521e-003	1.5847324528844214e-002

TABLE II
2K-DVBT CARRIERS DISTRIBUTION

CARRIERS TYPE	N°	Relative Mean Power	Relative Mean Power (dB)
Data	1512	1	0
Continuous	45	16/9	2.498
Scattered	131	16/9	2.498
TPS	17	1	0
Guard Band (Switched Off)	343	0	-
Active	1705	1.0802	0.335
Total	2048	0.89935	- 0.461

TABLE III
(E_b/N_o) TO (SNR) SCALING FACTOR

Mapping	$(SNR) - (E_b/N_o)$
4-QAM	3.35 dB
16-QAM	6.36 dB
64 – QAM	8.12 dB

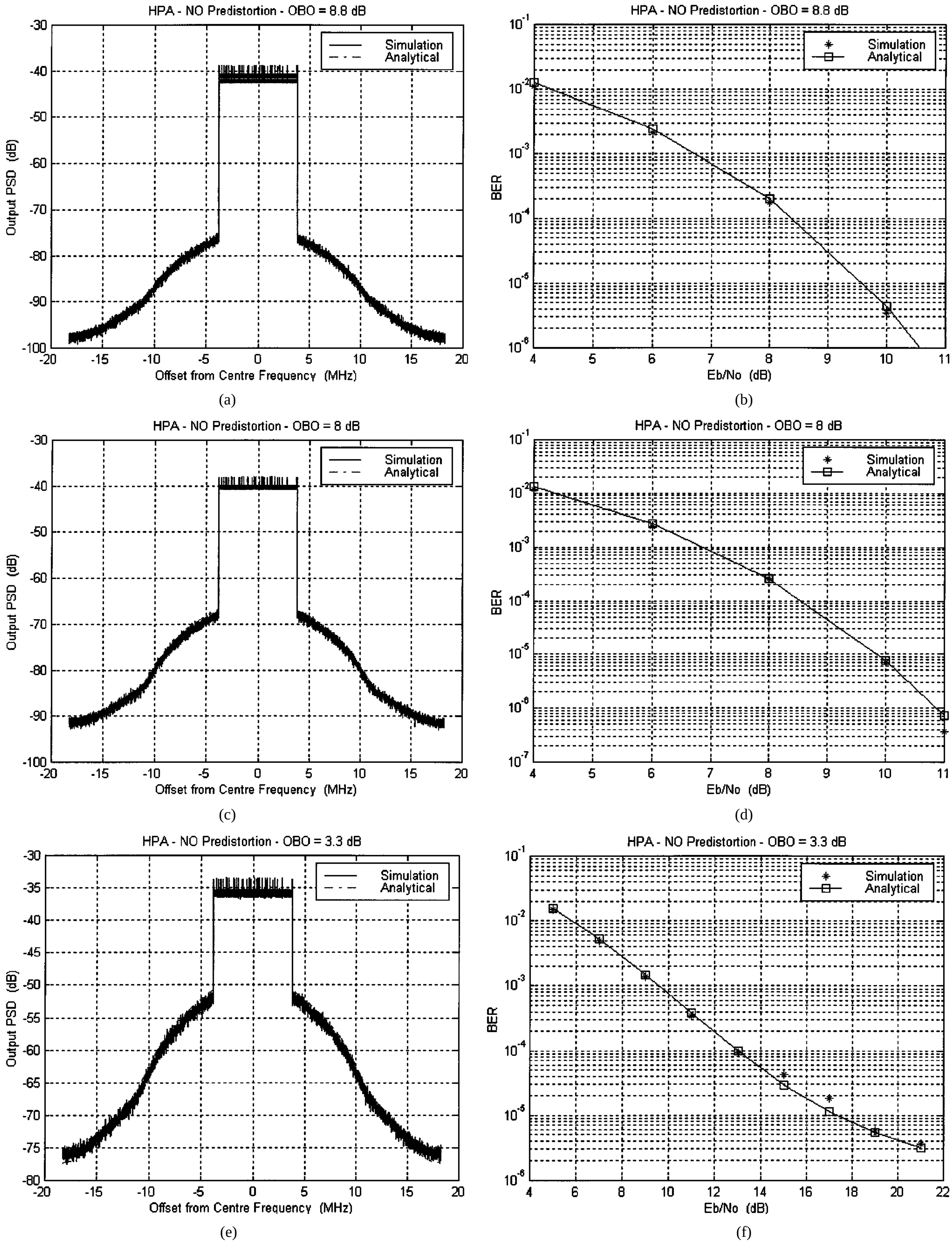


Fig. 3. Single 2K DVB-T signal amplified by the non linear HPA.

OFDM systems, uses a cyclic extension of the IFFT output in order to compensate for multipath at the receiver. This aspect is not considered, in our analysis, for the estimation of the output PSD, because it does not impact on the system

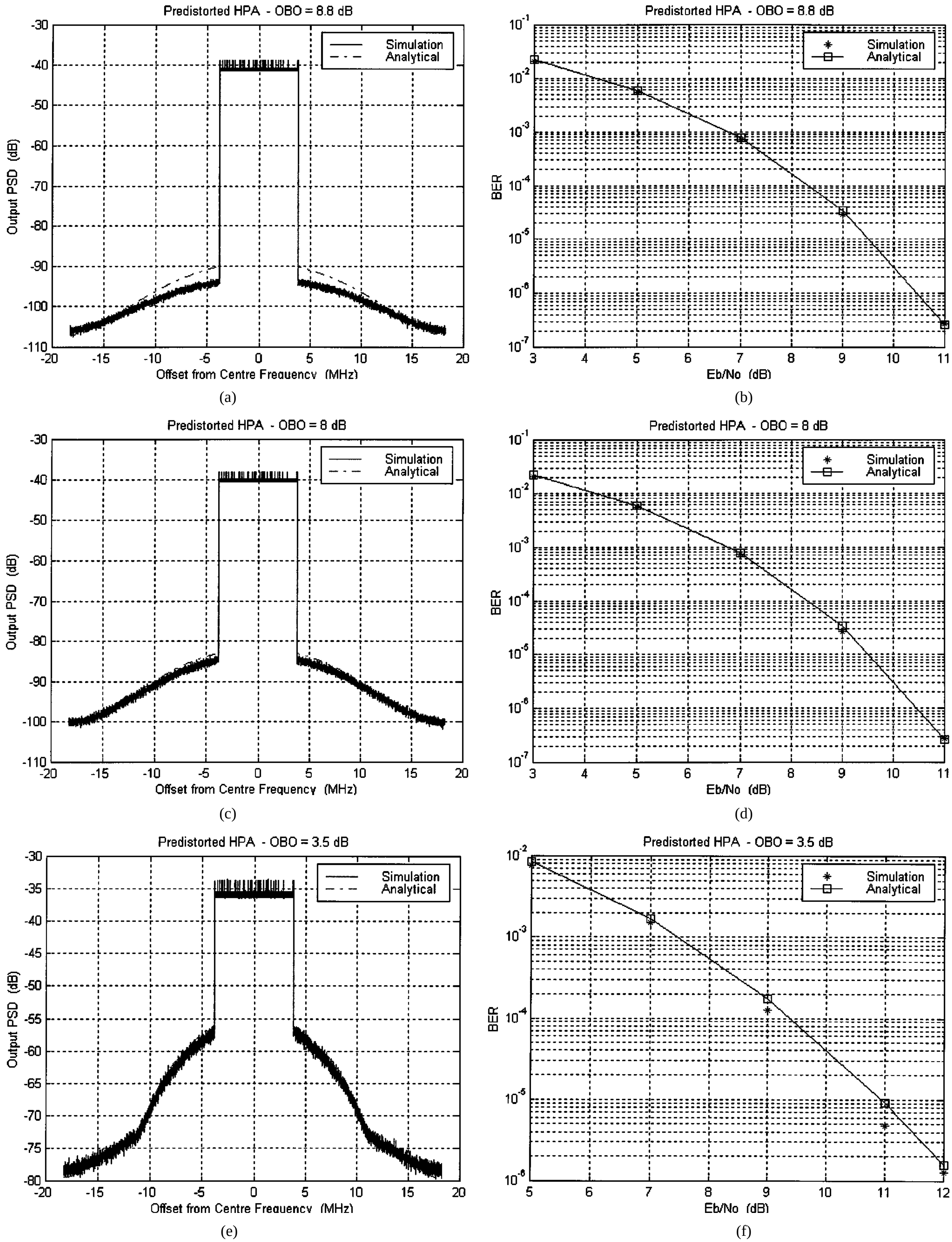


Fig. 4. Single 2K DVB-T signal amplified by predistorted HPA.

performance in nonlinear AWGN channels. As a consequence, a rectangular PSD has been considered for undistorted OFDM signals. In the simulation approach, the PSD was estimated as the mean of the power spectrum density, calculated by DFT,

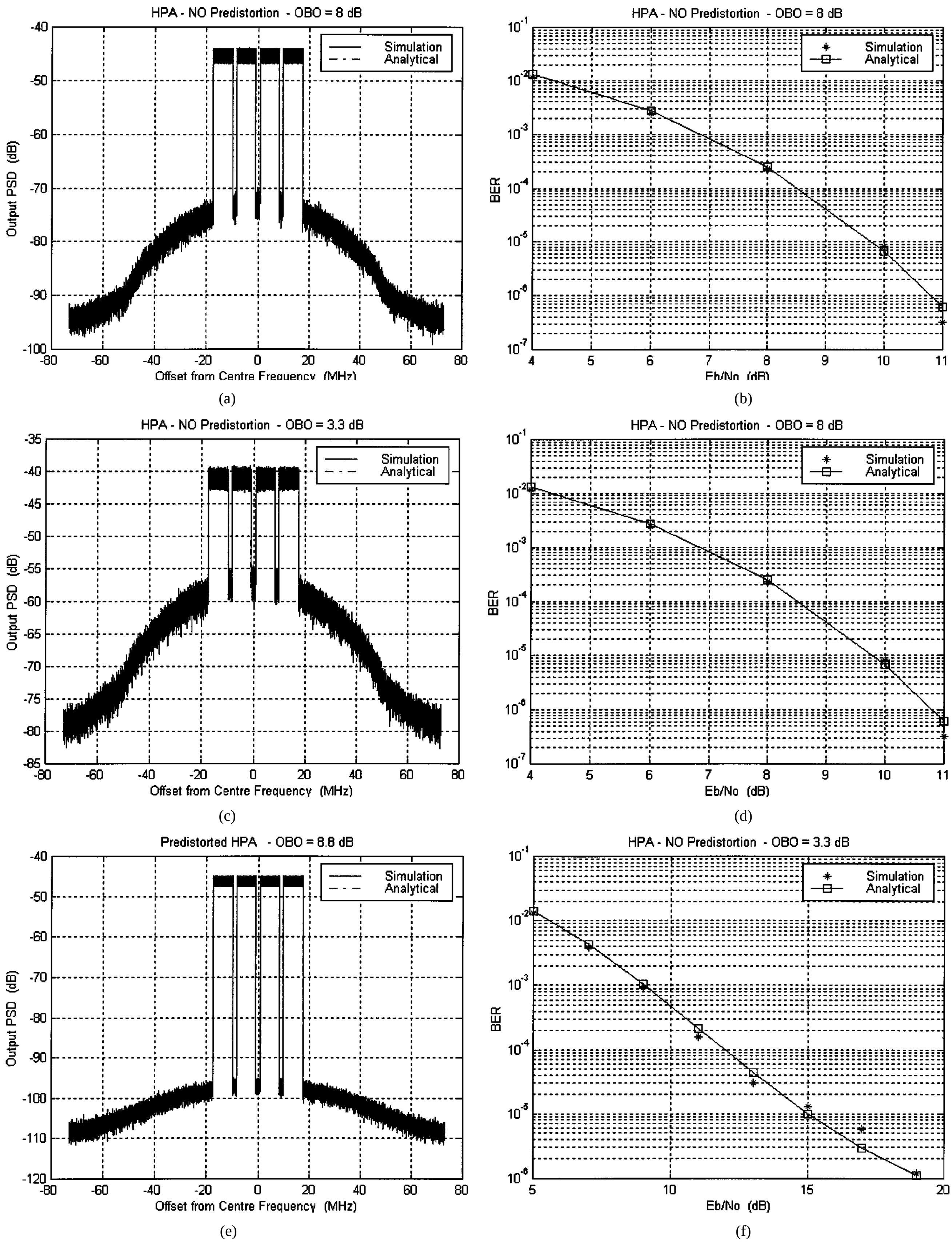


Fig. 5. Four Multiplexed 2K DVB-T signals amplified by the non linear HPA.

of each OFDM block. The PSD obtained in this way is not comparable with the one measured by a spectrum analyzer in a real system, because it does not take into account the phase discontinuities between adjacent OFDM blocks. Moreover,

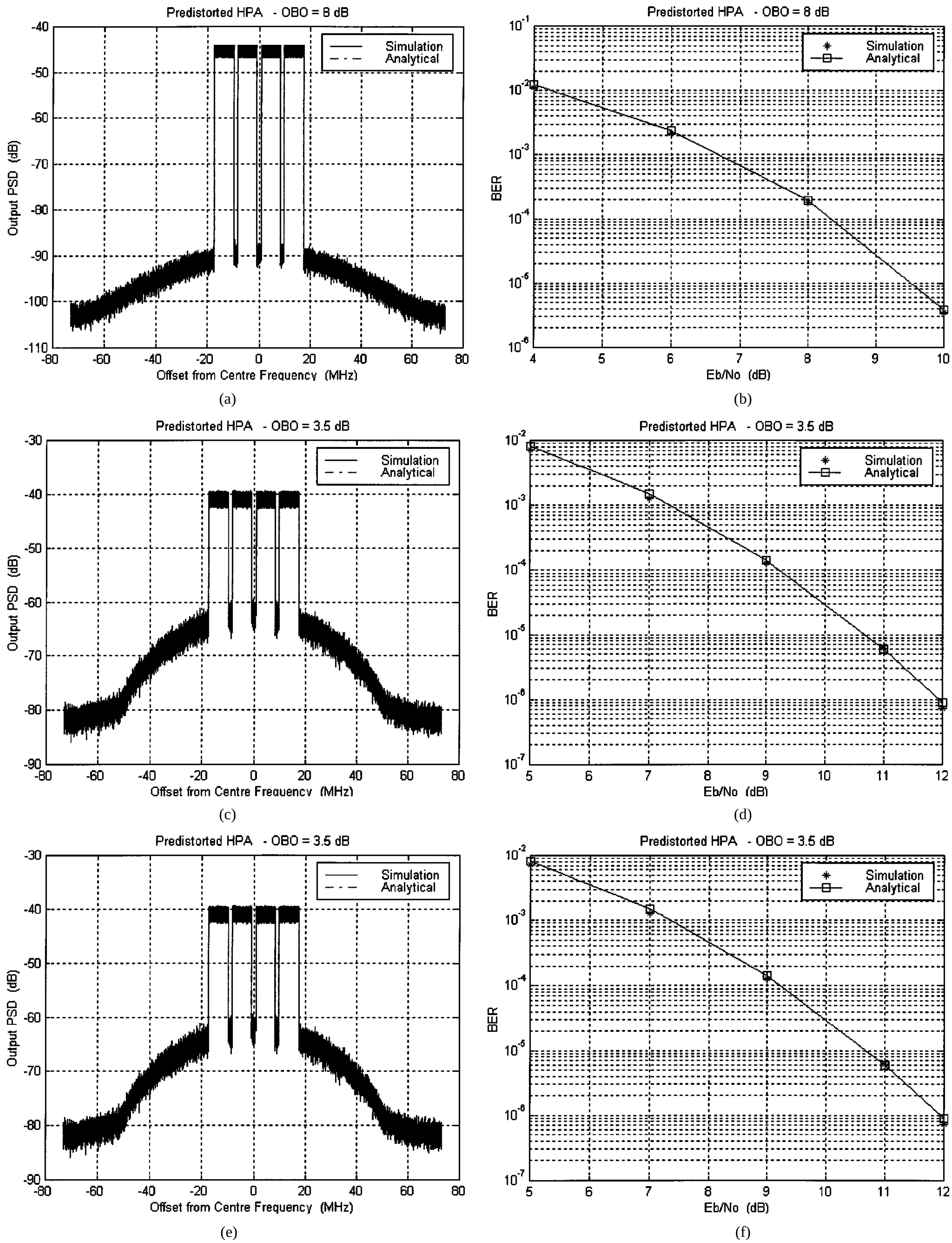


Fig. 6. Four Multiplexed 2K DVB-T signals amplified by predistorted HPA.

the PSD measured by a spectrum analyzer strongly depends on the used video bandwidth, resolution bandwidth and video averaging. However, it is possible to compare the analytical results with the real measures by substituting the PSD measured

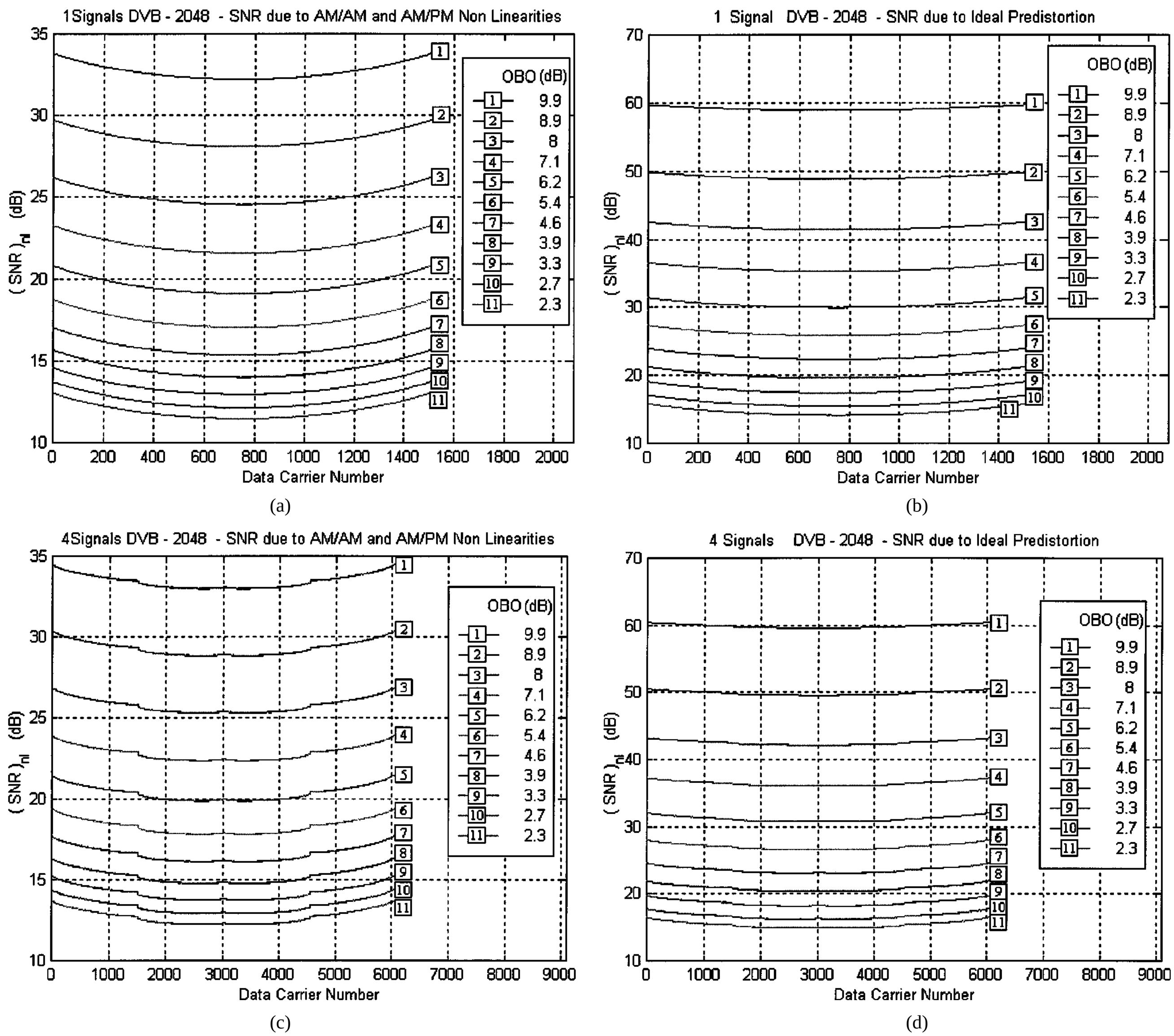


Fig. 7. In band $(\mathrm{SNR})_{nl}$ at the output of the HPA with and without predistortion.

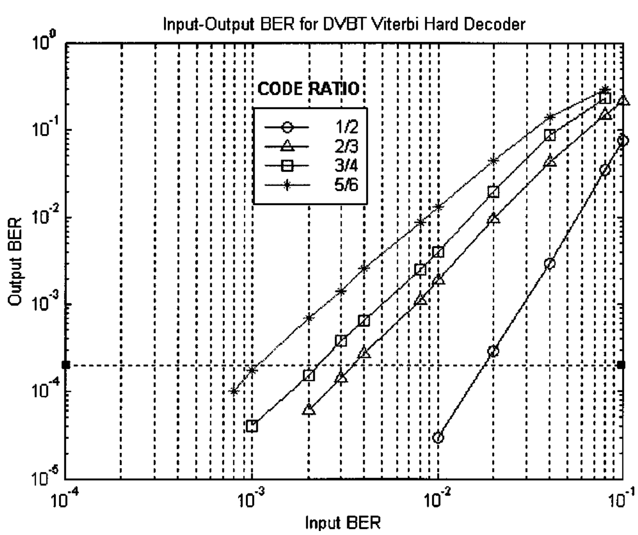


Fig. 8. BER at the output of Viterbi hard decoder as function of the BER at the input of the decoding process (depuncturing included).

in linear conditions in right hand side of (7) or, equivalently, substituting its inverse Fourier transform in (6).

Twenty OFDM blocks have been used to estimate the PSD by simulation. Both the PSD regrowth and the corresponding BER degradation are outlined. The same results are shown also for the ideal predistortion condition expressed by (15). All the analytical expressions used to obtain the system performance depend on the Input Back-Off (ibo) value. However, it is important to estimate the Output Back-Off (obo) for a meaningful comparison of the predistorted and not- predistorted conditions. The obo is defined as the ratio between the maximum and the mean output power.

Figs. 5 and 6 show that almost the same performance is achievable when many DVB-T signals are frequency multiplexed. The frequency-multiplexed signals have a little advantage in terms of nonlinear distortion introduced. This fact is not surprising, because the input PSD of the multiplexed signal has the same shape of the PSD of the single signal, with some holes representing the guard bands between adjacent

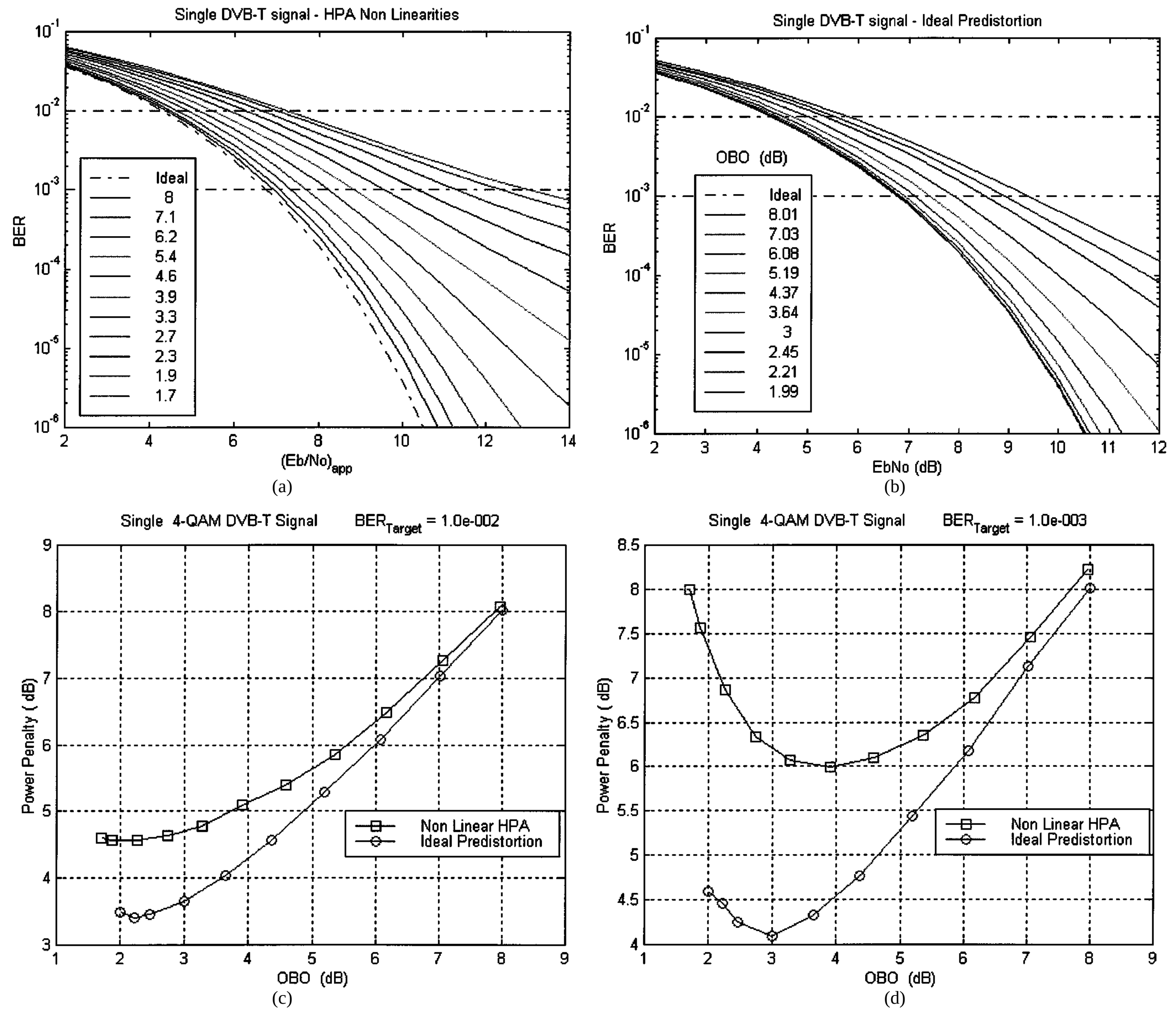


Fig. 9. Power penalty (total degradation) for 4-QAM 2K-DVBT in AWGN channel.

channels. These holes in the rectangular PSD shape represents an advantage in terms of lower intermodulation produced by the amplifier, analytically represented by the n-times convolution of the input PSD with itself as stated in (7).

Fig. 7 shows the SNR at the output of the amplifier for each data carrier, which is almost the same for the 4-QAM, 16-QAM and 64-QAM modulation, and it is related to the $(E_b/N_o)_{nl}$ ratio by (28)

$$(\mathrm{SNR})_{nl} = n_{bit} \cdot \left(\frac{E_b}{N_o}\right)_{nl}. \tag{28}$$

The DVB-T standard requires a BER of 2.10^{-4} at the output of the Viterbi decoding; to guarantee a quasi bit error free transmission at the output of the Reed Solomon decoder (BER $< 10^{-11}$). These values correspond to an uncoded BER of the system at the input of the Viterbi decoder, that can change between 1.10^{-3} and 1.10^{-2}, depending on the code rate imposed by the puncturing, as shown in Fig. 8.

The BER performance at different OBO values is summarized in Figs. 9–11 for 4-QAM, 16-QAM and 64-QAM modulation, in the AWGN channel that represents the 40 GHz free propagation channel between the Macrocell transmitter and the Microcell repeater. The HPA output power increases, as the OBO parameter becomes lower. A part of the increased output power, however, is a nonlinear distortion power that degrades the system performance. As a consequence, if the OBO decreases, a higher (E_b/N_o) is required at the receiver side is order to guarantee the same BER performance. The Total Degradation *(TD)* is a well known parameter used in literature [14] to take count of these two competitive effects: it is defined as the sum (in dB) of the OBO with the (E_b/N_o) that is required at the receiver side, for that OBO value, to guarantee a certain BER, as expressed by (29)

$$\mathrm{TD(BER,\, OBO)} = \mathrm{OBO} + \left(\frac{E_b}{N_o}\right)_{\mathrm{OBO,\, BER}}. \tag{29}$$

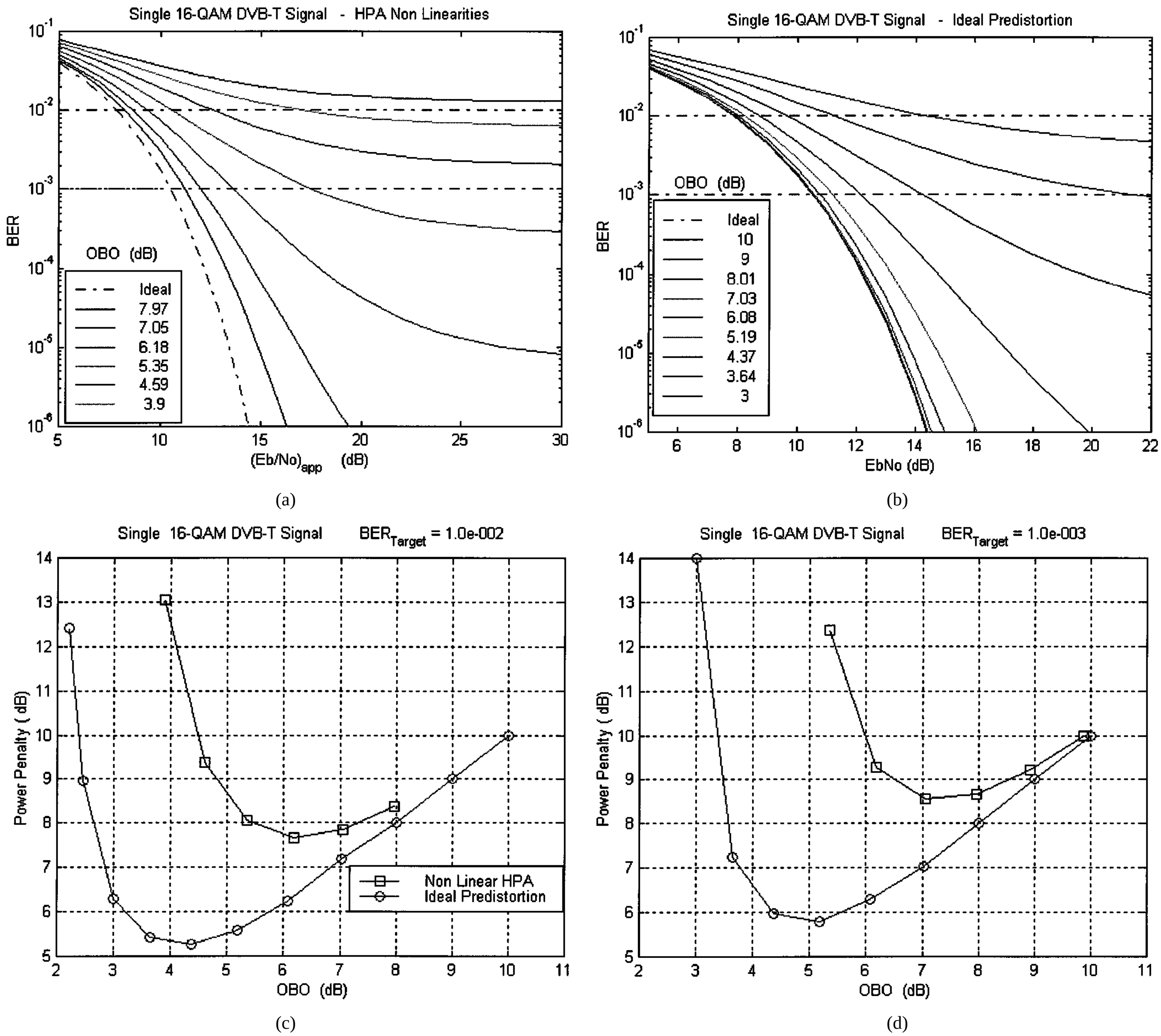


Fig. 10. Power penalty (total degradation) for 16-QAM 2K-DVBT in AWGN channel.

In the following we will use as figure of merit a parameter, named Power Penalty *(PP)*, that is obtained normalizing the D by the (E_b/N_o) value required in linear environment to reach the desired BER, as expressed by (30)

$$\begin{aligned}\text{PP(BER, OBO)} \\ = \text{OBO} + [(E_b/N_o)_{\text{OBO}} - (E_b/N_o)_{lin}]_{\text{BER}}. \quad (30)\end{aligned}$$

The first term in the PP expression is a power penalty from the maximum output power of the HPA while the second term is a penalty for an higher power of the received signal, in order to guarantee the same performance as in the linear situation. The optimum OBO is the one that minimizes the PP or equivalently the TD function. The same results are shown also for an Ideal Predistortion of the HPA, in order to outline the potential improvement of the system performance by adequate predistortion strategies. In conclusion the parameters to consider in evaluating the system performance are:

- the Power Penalty, which represents the penalty we have to introduce in the Link Budget calculation with respect to the ideal situation, where the HPA is perfectly linear and transmits at its maximum output power;
- the Optimum OBO, defined as the one that minimizes the Power Penalty;
- the Outband Intermodulation Power @ 4 MHz from the DVBT central frequency, which is a measure of the Adjacent Channel Interference in the RF spectrum.

The optimum OBO varies depending on the Code Rate used in the convolutional channel encoder, which is imposed by the puncturing level at the DVB-T modulator. Indeed, in order to guarantee a quasi bit error free transmission at the output of the Reed Solomon Decoder, it is necessary a target BER at the output of the IFFT Demapping that varies between $1e^{-2}$ (Code Rate = 1/2) and $1e^{-3}$ (Code Rate 5/6). As a consequence, also the Power Penalty depends on the protection level used in the channel coding. Table IV summarizes the results shown in the

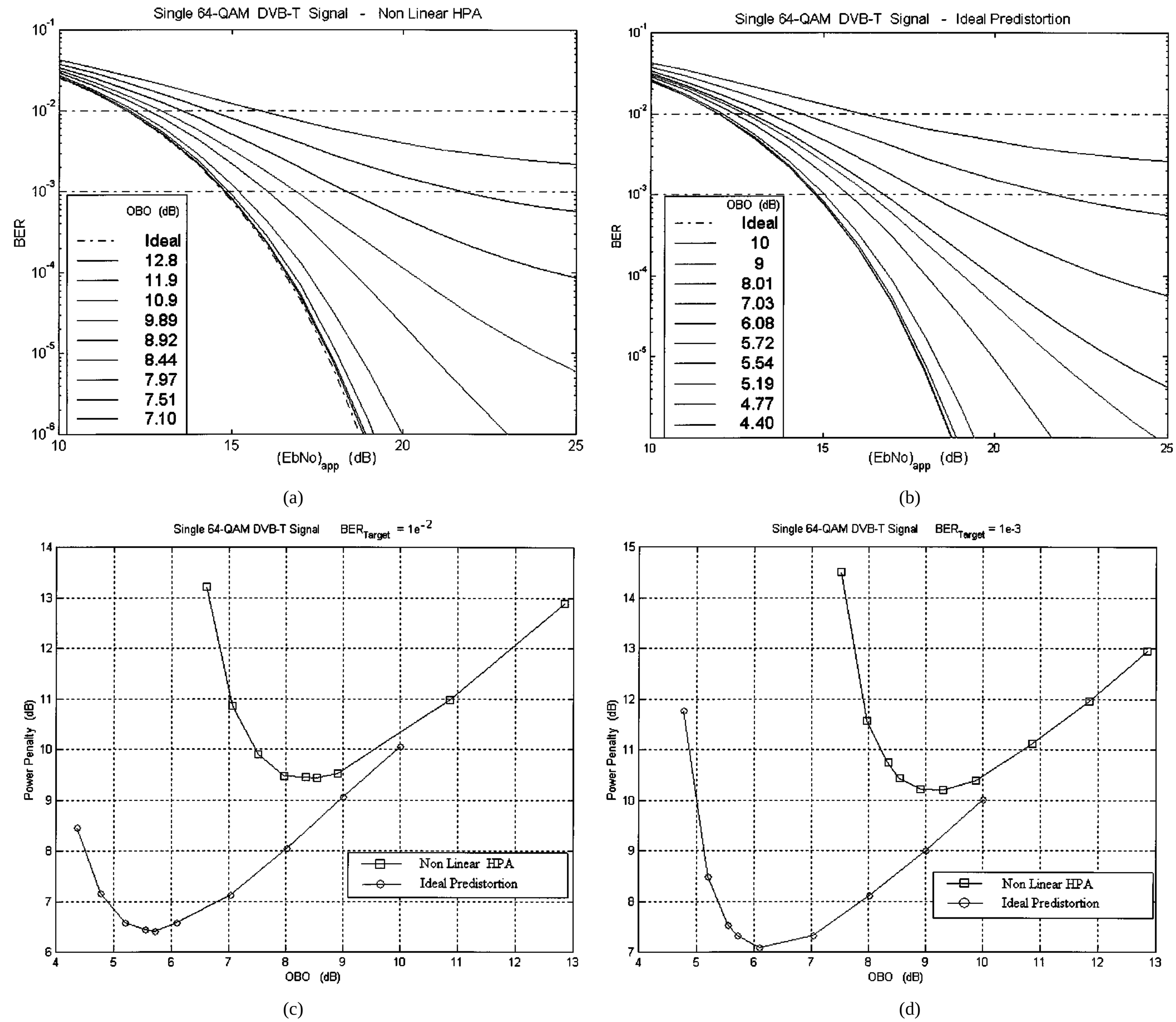


Fig. 11. Power penalty (total degradation) for 64-QAM 2K-DVBT in AWGN channel.

previous figures and shows the Optimum Output Back Off, the Minimum Power Penalty and the Adjacent Channel Interference power for the proposed HPA, with and without Predistortion. The worst Power Penalty of about 10 dB is obtained without any predistortion for a 64 QAM-5/6 Coded—DVBT signal. The Power Penalty must be included in the 40 GHz macrocell to microcell link budget for the signal distribution to the local repeater. Another penalty factor, equal to the number of multiplexed signal, must be considered and, consequently, a further 6-dB margin in the link budget must be considered to transmit 4 multiplexed DVB-T signals, because each signal uses a quarter of the total transmitted power. The power saving in the link-budget, offered by a predistortion technique, becomes more significant as the QAM modulation size increases and varies from a minimum value of about 1.2 dB, for the 4-QAM—1/2 encoded DVB-T signal, to a maximum value of about 3.1 dB, for the 64-QAM—5/6 situation. However, great care must be taken in selecting the optimum OBO as the one at which the PP is minimized. Indeed, especially for the 4-QAM situation, the ACI values can be significantly high and not compliant to the spectrum emission mask imposed by the international regulatory committees.

VI. Conclusions

The impact of non linear distortion phenomena on the downlink channel of an LMDS system, such as the one proposed in CABSINET, has been analyzed in AWGN channels. The optimum Output Back Off, the corresponding Adjacent Channel Interference and the related Power Penalty due to non linear distortion has been evaluated. It has been proved, by both analytical consideration and computer simulation, that the Frequency Multiplexing of several DVB-T signals is not problematic from the nonlinear distortion point of view, and that experiences almost the same performance degradation of a single OFDM signal for the same amplifier OBO. The power penalty on the link budget only depends on the fact that the amplifier output power is shared among the signals ensemble. The same conclusions

TABLE IV
OPTIMUM OUTPUT BACK OFF WITH AND WITHOUT PREDISTORTION

HPA (without Predistortion)

Mapping	OBO Optimum Range (Code Rate ½ - 5/6)	POWER PENALTY (Link Budget)	Outband Intermodulation Power @4Mhz
4—QAM	2.2 – 4.0 (dB)	4.6 – 6.0 (dB)	15 – 18 (dBc)
16-QAM	6.0 – 7.0 (dB)	7.7 – 8.5 (dB)	23 – 26 (dBc)
64-QAM	8.2 – 9.0 (dB)	9.4 – 10.2 (dB)	31 – 38 (dBc)

PREDISTORTED HPA

Mapping	OBO Optimum Range (Code Rate 1/2 - 5/6)	POWER PENALTY (Link Budget)	Outband Intermodulation Power @4Mhz
4—QAM	2.2 – 3.0 (dB)	3.4 – 4.1 (dB)	18 – 20 (dBc)
16-QAM	4.1 – 5.0 (dB)	5.2 – 5.8 (dB)	23 – 26 (dBc)
64-QAM	5.7 – 6.2 (dB)	6.4 – 7.1 (dB)	35 – 38 (dBc)

can be analytically extended to frequency selective channels [15].

APPENDIX

The coefficients c_n for expression (6) when the non linear distorting function $f(r)$ is the envelope clipping of (15), which represent the ideal predistortion condition, are reported in the following expression (30)

$$\begin{aligned} c_0 &= \left[1 - e^{-ibo} + \tfrac{1}{2}\sqrt{\pi \cdot ibo} \cdot \operatorname{erfc}\left(\sqrt{ibo}\right)\right]^2 \\ c_1 &= \tfrac{1}{2}\left[\tfrac{1}{2}\, ibo \cdot e^{-ibo} + \tfrac{1}{4}\sqrt{\pi \cdot ibo} \cdot \operatorname{erfc}\left(\sqrt{ibo}\right)\right]^2 \\ c_n &= \frac{1}{n!(n+1)!} \\ &\quad \cdot \left[\left(\frac{(2n)!}{2^{2n} \cdot n!} - \sum_{i=0}^{n-2} d_{i,n} \cdot ibo^{i+1}\right) \cdot ibo \right. \\ &\quad \left. \cdot\, e^{-ibo} + \frac{(2n)!}{2^{2n+1} \cdot n!}\sqrt{\pi \cdot ibo} \cdot \operatorname{erfc}\left(\sqrt{ibo}\right)\right]^2, \\ &\quad n = 2, K, \infty \end{aligned} \tag{30}$$

where erfc (x) is the complementary error function, that is erfc $(x) \equiv 1 - (2/\sqrt{\pi}) \int_0^x e^{-t^2}\, dt$.

The $d_{i,n}$ coefficients in (30) are recursively obtained by (31) [6]

$$\begin{aligned} &\begin{cases} p_{o,n} = \dfrac{(n+1)!}{2}, & i = 0 \\ p_{i,n} = p_{i-1,n-1} + (n+1) \cdot p_{i,n-1}, & i = 1, L, n-2 \end{cases} \\ &\begin{cases} q_{0,n} = \dfrac{1}{3}\left(\dfrac{(n+1)!}{2} - \dfrac{(2n)!}{2^{2n-1} \cdot n!}\right), & i = 0 \\ q_{i,n} = \frac{1}{3}\left(p_{i,n} - 2q_{i-1,n}\right), & i = 1, L, n-2 \end{cases} \\ &d_{i,n} = \frac{1}{i!} \cdot \sum_{p=i}^{n-2} q_{p,n} \cdot \left[\sum_{m=1}^{i} (-1)^m \binom{i}{m} m^p\right], \\ &\quad i = 0, \ldots, n-2. \end{aligned} \tag{31}$$

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