Estimation Errors Sensitivity of MMSE Multiuser Receivers in DS-CDMA

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Abstract – **The Minimum Mean Square Error (MMSE) receiver is a widely known linear detector that reduces the Multiple Access Interference (MAI) in DS-CDMA systems. Its optimal implementation in frequency selective scenarios requires the knowledge of the received signal covariance matrix and of the channel signature waveform that impairs the communication of each user. These quantities are generally not known and therefore they must be estimated by some techniques. This paper considers a chip-based MMSE receiver with a blind channel estimation technique characterised by a subspace approach. The aim is to investigate by simulations the BER degradation caused by the estimation errors of the covariance matrix and of the channel signature waveform, in order to point out problems and limitations in a realistic environment.**

Keywords – **Multiuser Detection, MMSE, Multiple Access Interference, Spectral Decomposition, Channel Estimation.**

I. Introduction

In DS-CDMA systems the near-far problem and the presence of MAI make attracting the use of multiuser receivers [1]. The detector known as *linear MMSE receiver* jointly minimises the effects of thermal noise, intersymbol interference (ISI) and MAI on the BER performance degradation, and it can be obtained multiplying the inverse of the covariance matrix of the received signal by the desired user's channel-affected signature waveform [2]. This kind of receiver, obtained by spectral decomposition of the covariance matrix, is analysed by Wang and Poor in [3] where random sequences and random channels are used. The present paper, differently from [3], considers deterministic low cross correlation sequences and severe frequency selective fading channels in order to test the MMSE receiver performance in a more realistic scenario. Furthermore, in practical situations, the covariance matrix of the received signal and the channels are not known and hence they must be estimated. The estimation errors of the covariance matrix and of the user channel introduce BER performance degradation. This problem worsens for time varying channels, because the estimates can be obtained by averaging the received signal information on a few symbols.

Object of this paper is to investigate the BER performance degradation due to such estimation errors by comparing the estimated detector performance with the ideal one. The sensitivity to estimation errors of the channel and of the covariance matrix is examined separately also. Moreover, the impairment due to the imperfect knowledge of the signal subspace dimension is considered. Extensive simulations are presented in both power control and near-far scenarios.

II. System model

The Multiple-Input Multiple-Output (MIMO) baseband channel model introduced in [3] is herein summarised. The transmitted signal of the *k*th user, in a DS-CDMA system with K active users, is expressed by

$$x_k(t) = A_k \sum_{i=0}^{I-1} b_k(i) s_k(t - iT - \tau_k)\,, \tag{1}$$

where T is the symbol duration, A_k and $s_k(t)$ are the amplitude and the spreading waveform of the *k*th user respectively, τ_k is the *k*th user relative delay ($0 \le \tau_k < T$), *I* is the number of transmitted symbols, and $b_k(i)$ is the *i*th symbol of the *k*th user. It is assumed that $b_k(i)$ belongs to a set of indipendent equiprobable $\{\pm 1\}$ random variables. The spreading waveform $s_k(t)$ can be expressed by

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} c_k(j) \psi(t - jT_c) \qquad 0 \le t < T \, , \tag{2}$$

where *N* is the processing gain, $T_c = T / N$ is the chip duration, $\psi(t)$ is the normalised rectangular chip waveform of duration T_c and $c_k(j)$ is the $\{\pm 1\}$ *j*th value of the *k*th user binary code sequence. We suppose to deal with slowly time varying channels such that they can be considered constant during the transmission of *P* symbols. The *k*th user channel is denoted by

$$g_k(t) = \sum_{q=1}^{Q} \alpha_{q,k} \delta(t - \tau_{q,k}) \, , \tag{3}$$

where *Q* is the number of paths, $\tau_{q,k}$ and $\alpha_{q,k}$ are respectively the delay and the complex amplitude of the *q*th path, $\Delta_k = \max_{q1,q2} \left| \tau_{q1,k} - \tau_{q2,k} \right|$ is the maximum delay spread and $\delta(t)$ is the Dirac function. When the channels are supposed to be constant, the received signal component of the *k*th user is

$$y_k(t) = x_k(t) * g_k(t) = \sum_{q=1}^{Q} \alpha_{q,k} x_k(t - \tau_{q,k}) \, . \tag{4}$$

The total received signal is the superposition of the *K* users' signals $y_k(t)$ with a complex zero-mean white Gaussian noise $v(t)$ of power spectral density σ^2. The received signal $r(t)$, expressed by (5), is first filtered by a chip-matched filter and then sampled at the chip rate $1/T_c$, obtaining (6).

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t) = y(t) + v(t) \tag{5}$$

$$r_n(l) = y_n(l) + v_n(l) = \int_{lT + nT_c}^{lT + (n+1)T_c} r(t) \psi(t - lT - nT_c) dt \tag{6}$$

The sampling instant is selected arbitrarily, leading to a mean power loss of 1.76 dB with respect to a symbol matched filter-bank architecture characterised by a perfect time synchronisation [4]. The discrete-time signal component due to the *k*th user is expressed by

$$y_{nk}(l) = \int_{lT + nT_c}^{lT + (n+1)T_c} y_k(t) \psi(t - lT - nT_c) dt = A_k \sum_{i=0}^{L_k - 1} b_k(l - i) \sum_{j=0}^{N-1} c_k(j) \sum_{q=1}^{Q} \alpha_{q,k} R_\psi(iT + (n - j)T_c - \tau_k - \tau_{q,k}) \tag{7}$$

where $R_\psi(t)$ is the autocorrelation function of $\psi(t)$. By the following expressions

$$h_k(w) = A_k \sum_{j=0}^{N-1} c_k(j) \sum_{q=1}^{Q} \alpha_{q,k} R_\psi[(w - j)T_c - \tau_k - \tau_{q,k}] \quad , 0 \le w \le L_k N$$

$$h_{nk}(i) = h_k(iN + n) \quad , 0 \le i \le L_k - 1 \qquad 0 \le n \le N - 1$$

it is possible to obtain (8), where L_k is the duration in symbol intervals of the *k*th channel taking into account of the delay τ_k.

$$y_n(l) = \sum_{k=1}^{K} y_{nk}(l) = \sum_{k=1}^{K} b_k(l) * h_{nk}(l) = \sum_{k=1}^{K} \sum_{i=0}^{L_k - 1} b_k(l - i) h_{nk}(i) \tag{8}$$

If the discrete-time channel coefficients $h_{nk}(l)$ are grouped in $[N \times K]$ matrices $\underline{H}(l)$, the received samples $r_n(l)$ in $[N \times 1]$ vectors $\underline{r}(l)$, and the symbols $b_k(l)$ in $[K \times 1]$ vectors $\underline{b}(l)$, by the notation in [3], the following MIMO relation (9) is obtained

$$\underline{r}(l) = \underline{y}(l) + \underline{v}(l) = \underline{H}(l) * \underline{b}(l) + \underline{v}(l) \, . \tag{9}$$

If *m* successive vectors $\underline{r}(l)$ are stacked to form a $[Nm \times 1]$ vector $\mathbf{r}_m(l)$, the expression (10) is obtained

$$\mathbf{r}_m(l) = H_m \mathbf{b}_m(l) + \mathbf{v}_m(l), \tag{10}$$

where H_m is the generalised block Sylvester matrix of dimension $Nm \times K(m+L-1)$ and $L = \max\{L_k\}$ (see [3] for more details). The received signal covariance matrix is defined by

$$\mathbf{C}_r = E\left\{\mathbf{r}_m(l)\mathbf{r}_m(l)^H\right\}. \tag{11}$$

Finally, it is useful to define $\mathbf{h}_k = [h_k(0) \ \cdots \ h_k(LN-1)]^T$ and the $[Nm \times 1]$ vector

$$\underline{h}_k = \begin{cases} [h_k(0) \cdots h_k(LN-1) \ 0 \ \cdots \ 0]^T / \left\|\mathbf{h}_k\right\| & , m > L \\ [h_k(0) \cdots\cdots h_k(mN-1)]^T / \left\|\mathbf{h}_k\right\| & , m \le L \end{cases} \tag{12}$$

III. MMSE MULTIUSER RECEIVER

III.1- Ideal MMSE receiver

If the transmitted data are BPSK mapped, the receiver decision rule is expressed by

$$\hat{b}_k(l) = \operatorname{sgn}\left[\operatorname{Re}(\mathbf{m}_k^H \mathbf{r}_m(l))\right], \tag{13}$$

where the $\mathbf{m}_k$ vector represents the detector. The MMSE receiver is obtained by minimising $E\left[| \, b_k(l) - \hat{b}_k(l) \, |^2\right]$, which leads to the expression

$$\mathbf{m}_k = \mathbf{C}_r^{-1} \underline{h}_k. \tag{14}$$

The subspace decomposition of the $\mathbf{C}_r$ matrix is shown in the following expression

$$\mathbf{C}_r = \mathbf{U}\Lambda\mathbf{U}^H = \begin{bmatrix}\mathbf{U}_{S,\chi} & \mathbf{U}_{N,\chi}\end{bmatrix} \begin{bmatrix}\Lambda_{S,\chi} & \mathbf{0} \\ \mathbf{0} & \Lambda_{N,\chi}\end{bmatrix} \begin{bmatrix}\mathbf{U}_{S,\chi}^H \\ \mathbf{U}_{N,\chi}^H\end{bmatrix} = \mathbf{U}_{S,\chi}\Lambda_{S,\chi}\mathbf{U}_{S,\chi}^H + \mathbf{U}_{N,\chi}\Lambda_{N,\chi}\mathbf{U}_{N,\chi}^H = \mathbf{C}_{r,S,\chi} + \mathbf{C}_{r,N,\chi} \tag{15}$$

where $\mathbf{U}_{S,\chi} = \begin{bmatrix}\mathbf{u}_1 & \cdots & \mathbf{u}_\chi\end{bmatrix}$, $\mathbf{U}_{N,\chi} = \begin{bmatrix}\mathbf{u}_{\chi+1} & \cdots & \mathbf{u}_{Nm}\end{bmatrix}$, $\Lambda_{N,\chi} = \sigma^2 \mathbf{I}_{Nm-\chi}$, $\Lambda_{S,\chi} = diag\left\{\lambda_1, \ldots, \lambda_\chi\right\}$, $\lambda_i \ge \lambda_{i+1} \ge \sigma^2$, $i = 1, \ldots, \chi - 1$, and $\mathbf{C}_{r,S,\chi}$ is the component of the covariance matrix on the signal subspace, which dimension χ should be equal to the rank ζ of the matrix H_m. If H_m is a full rank matrix, thus satisfying the sufficient condition of channel identifiability given in [3], the dimension of the signal subspace is $\chi = d = K(m+L-1)$. The inverse of $\mathbf{C}_r$ can be easily obtained by the spectral decomposition (15) as expressed by

$$\mathbf{C}_r^{-1} = \mathbf{U} \, \Lambda^{-1} \mathbf{U}^H. \tag{16}$$

Since the channel $\underline{h}_k$ lies in the signal subspace it is clear that

$$\mathbf{U}_{N,\chi}^H H_m = \mathbf{0}, \tag{17}$$

and therefore only the signal subspace is effective to express (14) that becomes

$$\mathbf{m}_k = \mathbf{C}_r^{-1} \underline{h}_k = \mathbf{C}_{r,S,\chi}^{-1} \underline{h}_k = \mathbf{U}_{S,\chi} \Lambda_{S,\chi}^{-1} \mathbf{U}_{S,\chi}^H \underline{h}_k = \sum_{i=1}^{\chi} \frac{\mathbf{u}_i^H \underline{h}_k}{\lambda_i} \mathbf{u}_i. \tag{18}$$

III.2-Estimated receivers

In practical situations, neither $\underline{h}_k$ nor $\mathbf{C}_r$ are known. The channel $\underline{h}_k$ can be estimated either by the aid of training sequences or by blind techniques [3] [5]. In the present paper the blind method to estimate $\underline{h}_k$ proposed in [3] is considered, because the $\mathbf{C}_{r,S,\chi}^{-1}$ (and the receiver expression $\mathbf{m}_k$) can be obtained almost without extra computational cost. When $\underline{h}_k$ and $\mathbf{C}_r$ are replaced by their estimated versions $\hat{\underline{h}}_k$ and $\hat{\mathbf{C}}_r$, the estimation errors effects on the system performance have to be considered. In this paper the covariance matrix is estimated by

$$\hat{\mathbf{C}}_r = \frac{1}{P-m+1} \sum_{l=0}^{P-m} \mathbf{r}_m(l) \mathbf{r}_m(l)^H, \tag{19}$$

because the subspace decomposition of the estimated matrix $\hat{\mathbf{C}}_r$ gives the maximum likelihood estimate of the eigenvalues λ_i and of the eigenvectors $\mathbf{u}_i$ of $\mathbf{C}_r$ [4]. Consequently, when only the estimated version $\hat{\mathbf{U}}_{N,\chi}$ is available by subspace decomposition of $\hat{\mathbf{C}}_r$, an estimate $\underline{\hat{h}}_{k,\chi}$ of the channel $\underline{h}_k$ can be obtained by solving (17) in the least square sense making use of the code sequence $c_k(j)$ of the user of interest [3]. Since the estimate $\hat{\mathbf{C}}_r^{-1}$ is the inverse of the estimate $\hat{\mathbf{C}}_r$, the MMSE receiver expression becomes

$$\hat{\mathbf{m}}_k = \hat{\mathbf{C}}_r^{-1}\underline{\hat{h}}_{k,\chi} = \hat{\mathbf{C}}_{r,S,\chi}^{-1}\underline{\hat{h}}_{k,\chi} + \hat{\mathbf{C}}_{r,N,\chi}^{-1}\underline{\hat{h}}_{k,\chi} = \sum_{i=1}^{Nm}\frac{\hat{\mathbf{u}}_i^H\underline{\hat{h}}_{k,\chi}}{\hat{\lambda}_i}\hat{\mathbf{u}}_i . \qquad (20)$$

It is noteworthy that in the estimated situation the relation (17) is not satisfied, because the estimated channels do not lie completely in the estimated signal subspace. Therefore the signal subspace receiver $\hat{\mathbf{m}}_{k,\chi} = \hat{\mathbf{C}}_{r,S,\chi}^{-1}\underline{\hat{h}}_{k,\chi}$ is not equivalent to $\hat{\mathbf{m}}_k$. Furthermore, the H_m matrix may be rank deficient because the users are asynchronous and characterised by different maximum delay spreads [6]. As a consequence, the signal subspace dimension $\zeta = rank(H_m)$ is lower than d. If both the covariance matrix and the channel are known, it is easy to prove that

$$\mathbf{m}_{k,d} = \mathbf{C}_{r,S,d}^{-1}\underline{h}_k = \mathbf{m}_{k,\zeta} = \mathbf{C}_{r,S,\zeta}^{-1}\underline{h}_k = \mathbf{m}_k = \mathbf{C}_r^{-1}\underline{h}_k . \qquad (21)$$

Differently, in the estimated case, the signal subspace receiver performance depends also on the estimated signal subspace dimension and consequently

$$\hat{\mathbf{m}}_{k,\zeta} = \hat{\mathbf{C}}_{r,S,\zeta}^{-1}\underline{\hat{h}}_{k,\zeta} \neq \hat{\mathbf{m}}_{k,d} = \hat{\mathbf{C}}_{r,S,d}^{-1}\underline{\hat{h}}_{k,d} , \qquad (22)$$

where $\underline{\hat{h}}_{k,\zeta}$ and $\underline{\hat{h}}_{k,d}$ are the estimated $[Nm \times 1]$ channel vectors obtained by (17) setting $\chi = \zeta$ and $\chi = d$ respectively. The simulations shown in the next section verify if some BER performance degradation is introduced using the $\hat{\mathbf{m}}_{k,d}$ receiver instead of $\hat{\mathbf{m}}_{k,\zeta}$, which requires the estimation of ζ.

IV. SIMULATIONS AND COMMENTS

Gold sequences of length $N = 31$ have been chosen for the short spreading codes $c_k(j)$. The chip rate has been fixed to $1/T_c = 8.192$ Mcps and consequently the users bit rate is approximately equal to 264 kbps. The channels model is compliant with the *pedestrian B* channel defined in [7]. The chosen chip rate $1/T_c$ and the processing gain N lead to a maximum delay spread Δ_k that is little longer than one symbol duration. The uplink situation is considered with delays τ_k uniformly distributed in $[0,T)$. However, both τ_k and Δ_k of each user are assumed known by the receiver within a chip period, and therefore the maximum channel order *L* is assumed known. The window size m has been selected considering the identifiability conditions (23) [3]; most of the simulated situations lead to $m = 2$ or $m = 3$.

$$m \geq \lceil K(L-1)/(N-K) \rceil \quad , \quad (N-K)m^2 + (L-1)(N-2K)m \geq K(L-1)^2 + NL . \qquad (23)$$

It is well known that all blind estimation techniques can recover the channels up to a complex scalar factor [4]. In this paper, the phase of the maximum magnitude coefficient of the discrete-time channel is supposed to be known.

The SNR shown in all the following figures is the one of the weakest user (user 1) and it is defined as $SNR = A_1^2/\sigma^2$. The Normalised Root Mean Square Error (NRMSE) is defined by

$$NRMSE = \sqrt{\frac{1}{N_{runs}}\sum_{i=1}^{N_{runs}}\left\|\underline{\hat{h}}_{1,\chi}^{(i)} - \underline{h}_1^{(i)}\right\|^2 / \left\|\underline{h}_1^{(i)}\right\|^2} \qquad (24)$$

where $N_{runs} = 300$ is the number of independent simulations.

The Fig.1 shows the simulated NRMSE and points out the near-far resistance of the channel estimation (the NRMSE does not depend on the amount of MAI for a given number K of users), as already shown in [4] in a small delay spread situation. The channel estimation performance is highly influenced by the number of active users and by the averaging factor P. It is noteworthy

to remind that, in practical applications, it is not possible to increase P as we like in order to reduce the estimation errors, because the channels cannot be considered constant due to the mobility of the users. Consequently this algorithm can not be used if the users mobility is too high. The Fig.1 and the Figs.3-4 show only the BER curves obtained by setting $\chi = d$ because those obtained using the true dimension ζ are practically the same (see Fig.2). Indeed, the subspace channel estimation can be done using only the partial knowledge of the noise subspace [6].
The Fig.3 compares the BER performance of:

- ❑ the ideal receiver (both $\mathbf{C}_r^{-1}$ and $\underline{h}_k$ are known)
- ❑ the estimated receivers (both $\mathbf{C}_r^{-1}$ and $\underline{h}_k$ are estimated)
  - EIn-ECh (**E**stimated **In**verse – **E**stimated **Ch**annel)
- ❑ the partial-estimated receivers:
  - TIn-ECh (**T**rue **In**verse – **E**stimated **Ch**annel)
  - EIn-TCh (**E**stimated **In**verse – **T**rue **Ch**annel)

It is noteworthy that:

1) When the signal subspace matrix $\mathbf{C}_{r,S,\chi}^{-1}$ is used in the detector, in power control situations (Fig.3), the two kind of partial-estimated receivers show BER performance characterised by a different behaviour. Indeed, at low SNR, where the channel estimation errors are higher (Fig.1), the TIn-ECh detector has a significant BER degradation with respect to the EIn-TCh detector. On the contrary, for increasing SNR, the TIn-ECh detector significantly outperforms the EIn-TCh one. Indeed, when SNR increases, the eigenvalues spread (condition number) of $\mathbf{C}_{r,S,\chi}$ increases and consequently the estimation errors on $\mathbf{C}_{r,S,\chi}^{-1}$ are amplified by the inversion. Furthermore, for increasing SNR, the TIn-ECh receiver shows BER performance that tends to the ideal one, while the EIn-TCh receiver BER tends to exhibit a noise floor whose BER value depends on the averaging factor P. Obviously the EIn-ECh detector BER is imposed by the performance of the two partial-estimated ones. Therefore an improved detector should have a better inverse matrix estimation at high SNR and a better channel estimation at low SNR. The inverse matrix estimation error can be reduced in many ways: the first choice may consist in adding a positive quantity to the signal eigenvalues in order to reduce the eigenvalues spread of $\hat{\mathbf{C}}_{r,S,\chi}$ [8]; the obtained detector is a CMOE-type receiver, which has been introduced in [9] in order to reduce only the channel estimation errors. A second choice is to eliminate some signal eigenvectors in a similar fashion to the method proposed in [10], obtaining a reduced-rank detector. The third [8] is to restore the estimated covariance matrix by using the estimated channel of all users, which are available if an uplink situation is considered. The channel estimation may instead be improved by using a different algorithm [5] or reapplying the subspace method at the restored covariance matrix.
2) In near-far situations (Fig.3 (c)) the TIn-ECh receiver outperforms the EIn-TCh one at low SNR also, because the channel estimation algorithm exploits the higher power of the interfering users.
3) Obviously, as shown in Fig.4, the BERs of the detectors are highly influenced by the averaging factor P, with better performance obtained for higher values of P thanks to the estimation noise reduction. However unacceptable performance emerges if the high mobility of one user forces to work with low P values.
4) When both the covariance matrix and the channel are estimated, the full inverse matrix receiver $\hat{\mathbf{m}}_k$ defined in (20) has different performance with respect to the signal subspace inverse matrix receiver $\hat{\mathbf{m}}_{k,d}$ defined in (22) (Fig.3). In all the situations the full inverse receiver $\hat{\mathbf{m}}_k$ has a higher BER performance degradation, unacceptable if the averaging factor P is low, because for $i > \zeta$ the values $\left|\hat{\mathbf{u}}_i^H \hat{\underline{h}}_{k,\chi}\right|$ in (20) are not negligible with respect to the $\left|\hat{\lambda}_i\right|$ values. An analogous situation occurs when either the covariance matrix or the channel is estimated. It

can also be noted that if P is not too low the estimated detector tends to outperform the partial-estimated ones. This fact, even if unexpected, may be explained by means of the quasi-orthogonal relation between $\hat{\mathbf{U}}_{N,\chi}$ and $\underline{\hat{h}}_{k,\chi}$, that is $\left\|\hat{\mathbf{U}}^H_{N,\chi}\underline{\hat{h}}_{k,\chi}\right\| < \left\|\hat{\mathbf{U}}^H_{N,\chi}\underline{h}_{k}\right\|$ and $\left\|\hat{\mathbf{U}}^H_{N,\chi}\underline{\hat{h}}_{k,\chi}\right\| < \left\|\mathbf{U}^H_{N,\chi}\underline{\hat{h}}_{k,\chi}\right\|$.

V. CONCLUSIONS

The effects of the covariance estimation errors and of the channel estimation errors on the BER performance degradation of MMSE multiuser detector have been analysed. It has been shown that channel estimation errors are predominant at low SNR, while the estimation errors of the inverse covariance matrix becomes remarkable at high SNR. The effect of the covariance estimation errors can be reduced by opportune techniques as proposed in [8] and in [10]. The analysis and the potential improvements of different channel estimation algorithms, as proposed in [5] and in [11], will be the object of future works.

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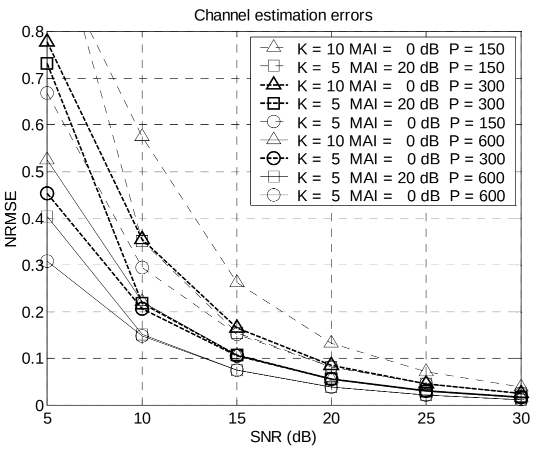


Fig.1 NRMSE in different scenarios (K, MAI, P).

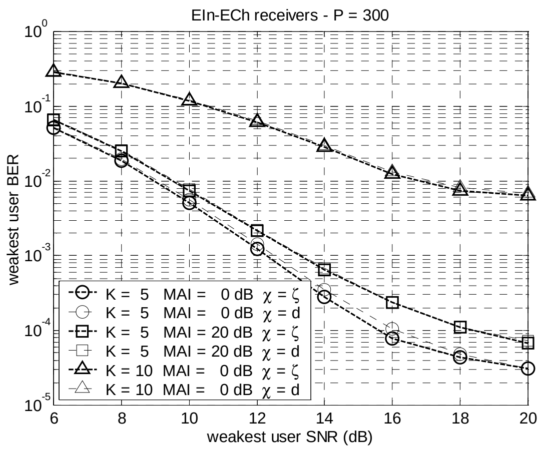
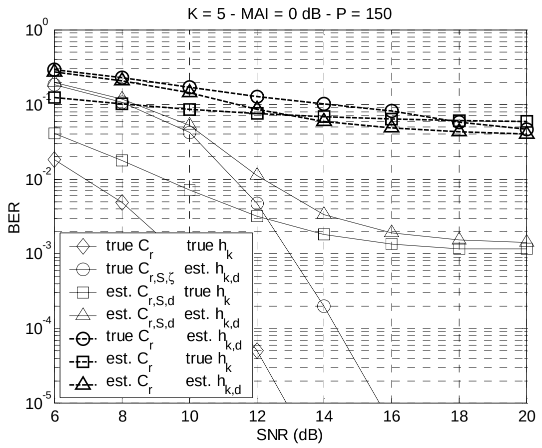
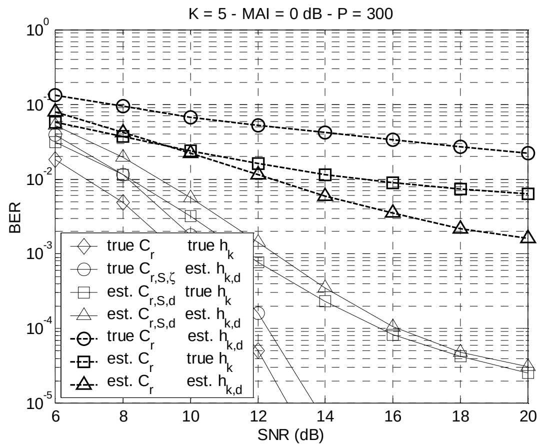


Fig.2 BER comparison of the estimated receivers using the true signal subspace dimension ζ and the approximated one d.



(a)



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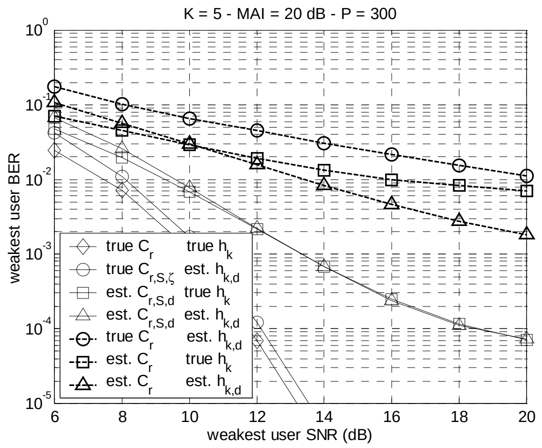
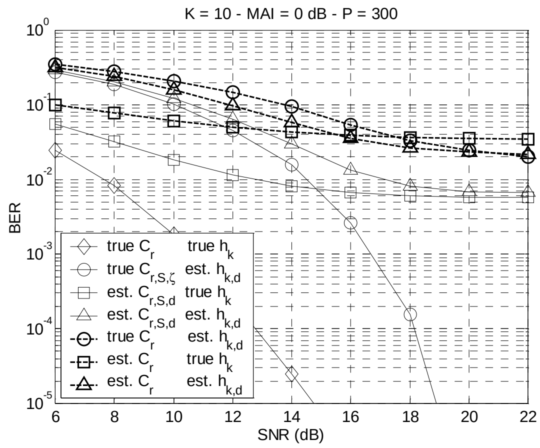
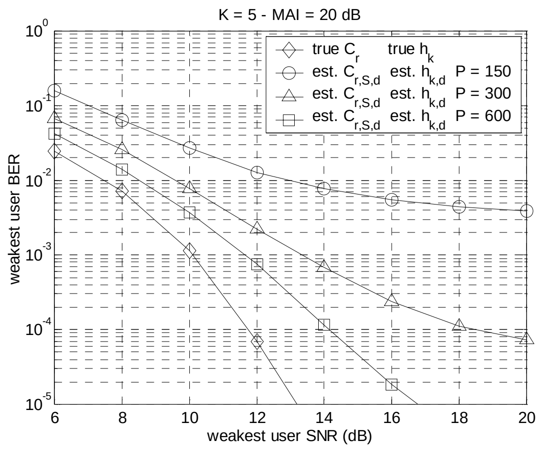


(c)

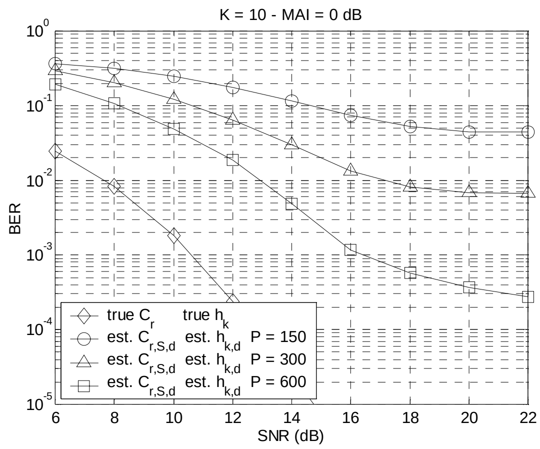


(d)

Fig.3 (a)-(d) BER comparison of different receivers (ideal, TIn-ECh, EIn-TCh, EIn-ECh). Different scenario (K, MAI, P) in each figure.



(a)



(b)

Fig.4 (a)(b) Effect of the averaging factor P on the estimated receiver BER performance. Different scenario (K, MAI) in each figure.