Spectrum Sensing Using Energy Detectors with Performance Computation Capabilities

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Abstract—**We focus on the performance of the energy detector for cognitive radio applications. Our aim is to incorporate, into the energy detector, low-complexity algorithms that compute the performance of the detector itself. The main parameters of interest are the probability of detection and the required number of samples. Since the exact performance analysis involves complicated functions of two variables, such as the regularized lower incomplete Gamma function, we introduce new low-complexity approximations based on algebraic transformations of the one-dimensional Gaussian Q-function. The numerical comparison of the proposed approximations with the exact analysis highlights the good accuracy of the low-complexity computation approach.**

I. INTRODUCTION

In cognitive radio networks, spectrum sensing plays a crucial role, since it allows to determine if a given frequency band is available or not for the signal transmission of a secondary (unlicensed) user [1], [2]. Among the various criteria for spectrum sensing, energy detection is perhaps the most famous [1], [2], and is particularly appropriate for low-complexity applications [3]. This paper examines the possible incorporation of some performance computation capabilities into an energy detector (ED). This way, an ED can be able to self-estimate its own probability of detection and to automatically select the sample size that is required for a given performance level.

Although the exact performance of the ED is well established [4], the mathematical relation among the various performance parameters usually involves complicated functions of two variables [4], [5], such as the incomplete Gamma function [6], whose implementation into a low-complexity device is prohibitively expensive. As a consequence, some approximations have been proposed in the literature [5], [7]–[10], with the aim of reducing the computational complexity while preserving a sufficient accuracy. These approximations differ in two main aspects. First, the assumed model for the signal of the primary (licensed) user may be either deterministic [5], [8]–[10], or random Gaussian [5], [9] or unknown [7]. Herein, we focus on the random Gaussian model for the primary signal [4], [5], [9]. Second, different functions have been proposed for approximating the probability density function of the test statistic [5], [7]–[10].

The conventional way to approximate the test statistic of the ED invokes the central limit theorem and considers the decision variable as Gaussian [4], [5]. Although this approach works quite well when the number of samples is large, the accuracy of the Gaussian approximation worsens significantly when the sample size is low [9]. Note that the reduced sample-size scenario is of practical interest for spectrum sensing applications, because the sample size is constrained by time-bandwidth product considerations. As a consequence of the reduced accuracy of the Gaussian approximation, [9] proposed an improved approximation based on the cube transformation of a Gaussian random variable. This transformation belongs to a wider class of approximations, known as power (or root) transformations [11]–[13], where the power exponent can be different from that used in [9].

This paper investigates power transformations with generic exponents in the context of low-complexity approximation of the performance of the ED. In addition, linear combinations of power transformations are also considered [13], [14], because of their potentially increased accuracy. Specifically, this paper derives new closed-form expressions for the probability of detection as a function of the probability of false alarm, of the signal-to-noise ratio (SNR) and of the sample size, for power transformations and for suitable linear combinations of power transformations. In addition, in the context of power transformations, new closed-form expressions for the required sample size are also derived (as a function of the probability of detection, of the probability of false alarm, and of the SNR). For linear combinations of power transformations, we propose a fast algorithm that exactly finds the required sample size with logarithmic complexity. In order to validate the accuracy of the proposed approximations, we include some numerical results that highlight the superior accuracy of the linear combination approaches.

II. ENERGY DETECTOR

A. System Model

We consider a cognitive radio network, where secondary (unlicensed) users perform spectrum sensing in a preselected frequency band in order to detect the possible presence of one or more primary (licensed) users. We assume that the aggregate signal of the primary users is random and zero-mean Gaussian distributed, and that the noise at the input of the secondary-user receiver is zero-mean Gaussian as well. After baseband conversion and sampling, the complex-valued received signal $\mathbf{y} = [y_1, ..., y_N]^T$ can be expressed by

$$\mathbf{y} = \alpha \mathbf{s} + \mathbf{w}\,, \tag{1}$$

where $\mathbf{s} = [s_1, ..., s_N]^T$ represents the primary-user signal, assumed complex Gaussian with zero mean and covariance $\mathbf{\Sigma}_{\mathrm{S}} = \sigma_{\mathrm{S}}^2 \mathbf{I}_N$, N is the number of samples, $\mathbf{w} = [w_1, ..., w_N]^T$ is the secondary-user receiver noise, assumed complex Gaussian with zero mean and covariance $\mathbf{\Sigma}_{\mathrm{W}} = \sigma_{\mathrm{W}}^2 \mathbf{I}_N$, independent from the primary-user signal $\mathbf{s}$, and $\alpha \in \{0,1\}$ denotes the absence or presence of $\mathbf{s}$, referred to as the H_0 or H_1 hypothesis, respectively. Using (1), the ED calculates the test statistic

$$T(\mathbf{y}) = \|\mathbf{y}\|^2 = \sum_{i=1}^{N} |y_i|^2 \tag{2}$$

and then compares it to a threshold t: if $T(\mathbf{y}) \geq t$ the ED decides that a primary user is present, whereas if $T(\mathbf{y}) < t$ the ED assumes that primary users are absent.

B. Exact Performance Analysis

The exact performance of the ED can be determined by statistical analysis of the test (2). Under the H_0 hypothesis, $2T(\mathbf{y})/\sigma_{\mathrm{W}}^2$ is a chi-squared random variable with $2N$ degrees of freedom (DOF): this leads to a probability of false alarm [4]

$$P_{\mathrm{FA}} = \Pr\{T(\mathbf{y}) > t \mid \alpha = 0\} = 1 - F_{2N}\left(2t / \sigma_{\mathrm{W}}^2\right), \tag{3}$$

$$F_{2N}(x) = [\Gamma(N)]^{-1} \int_0^{x/2} \nu^{N-1} e^{-\nu} d\nu\,, \tag{4}$$

where $\Gamma(N)$ is the Gamma function [6]. Under the H_1 hypothesis, $2T(\mathbf{y})/(\sigma_{\mathrm{S}}^2 + \sigma_{\mathrm{W}}^2)$ is a chi-squared random variable with $2N$ DOF: hence, the probability of detection is [4]

$$P_{\mathrm{D}} = \Pr\{T(\mathbf{y}) > t \mid \alpha = 1\} = 1 - F_{2N}\left(2t / (\sigma_{\mathrm{S}}^2 + \sigma_{\mathrm{W}}^2)\right). \tag{5}$$

By eliminating the threshold t from (3) and (5), the receiver operating characteristic (ROC) is expressed by

$$P_{\mathrm{D}} = 1 - F_{2N}\left((1+\gamma)^{-1} F_{2N}^{-1}(1 - P_{\mathrm{FA}})\right), \tag{6}$$

where $\gamma = \sigma_{\mathrm{S}}^2 / \sigma_{\mathrm{W}}^2$ is the SNR and $x = F_{2N}^{-1}(p)$ is the inverse of $p = F_{2N}(x)$ with respect to x.

The ROC (6), which summarizes the relation among the four parameters P_{FA}, P_{D}, N, and γ, can be inverted in order to find either the probability of false alarm P_{FA} or the SNR γ as a function of the other three parameters, as expressed by

$$P_{\mathrm{FA}} = 1 - F_{2N}\left((1+\gamma) F_{2N}^{-1}(1 - P_{\mathrm{D}})\right), \tag{7}$$

$$\gamma = \frac{F_{2N}^{-1}(1 - P_{\mathrm{FA}})}{F_{2N}^{-1}(1 - P_{\mathrm{D}})} - 1\,. \tag{8}$$

Unfortunately, the equations (6)–(8) are not suitable for implementation into a low-complexity device, since they depend on complicated functions, such as the regularized lower incomplete Gamma function $F_{2N}(x)$ in (4) and its inverse $F_{2N}^{-1}(x)$. Moreover, since both $F_{2N}(x)$ and $F_{2N}^{-1}(x)$ are functions of two variables, storing their values into a lookup table (LUT) would require significant memory overhead. In addition, the equations (6)–(8) cannot be inverted with respect to N: therefore, if a sensing ED device wants to calculate the minimum number of samples N as a function of P_{FA}, P_{D}, and γ, an iterative numerical approach is required for the multiple evaluations of $F_{2N}^{-1}(x)$. As a consequence, when a low-complexity device wants to automatically select either P_{FA}, or P_{D}, or N, some approximations are necessary.

III. Approximated Performance Computation

A. Gaussian Approximation

In the spectrum sensing literature, there exist different approximations for the performance of the ED [5], [8]–[10]. For both cases of deterministic signals and random signals, the conventional approach approximates a chi-squared random variable with a Gaussian random variable characterized by a suitable mean and variance [5], [8]. Using the statistical signal model of Section II.A, the conventional Gaussian approximation corresponds to

$$F_{2N}(x) \approx \hat{F}_{2N}(x) = 1 - Q\left(\frac{x - 2N}{2\sqrt{N}}\right), \tag{9}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-\nu^2/2} d\nu\,. \tag{10}$$

As summarized in [5] and [9], the Gaussian approach of (9) produces both simple approximations of (6)–(8) and an analytical expression for the required sample size N. Therefore, these approximated expressions can be easily implemented into a low-complexity ED, provided that a LUT is available for the evaluation of the one-dimensional Q-function in (10) (and of its inverse). On the other hand, the accuracy of the Gaussian approximation is quite low [5], especially for small sample sizes [9]. Noteworthy, the sample size N cannot be arbitrarily large, due to time and bandwidth constraints.

B. Power Transformation

In order to increase the approximation accuracy, [9] and [10] discuss different options that are valid for the cases of deterministic signals [9], [10], and random signals [9]. Specifically, the cube-of-Gaussian approach of [9] approximates a chi-squared random variable with the cube of a Gaussian random variable with suitable mean and variance [11]. This cube-of-Gaussian approach is quite promising, since the obtained expressions are more accurate than the Gaussian approximations, but with similar complexity [9]. In this section, we generalize the cube-of-Gaussian approach of [9] to accommodate different power exponents r. The motivation for this generalization lies in the statistical investigation done in [12], which analyzes the Kullback-Leibler (KL) divergence of a power-transformed chi-squared random variable from a Gaussian random variable with suitable mean and variance: the results of this analysis clearly show that the KL divergence is minimized for power exponents ranging from $r = 3$ to $r = 4$. A similar result is obtained by [13], which compares the cumulants of a power-transformed chi-squared random variable with the cumulants of a Gaussian random variable: if we match either the skewness or the kurtosis of the two random variables, we obtain integer power exponents $2 \leq r \leq 4$. As a consequence, herein we propose to approximate a chi-squared random variable x, normalized by the number of DOF $2N$, with the rth power of a Gaussian random variable, as expressed by

$$F_{2N}(x) \approx \hat{F}_{2N}(x) = 1 - Q\left(\frac{\sqrt[r]{x/(2N)} - m_r(N)}{\sqrt{V_r(N)}}\right), \tag{11}$$

where $m_r(N)$ and $V_r(N)$ are the mean and the variance of the Gaussian variable, expressed respectively by [11], [13]

$$m_r(N) \approx 1 - \frac{r-1}{2r^2 N}, \qquad V_r(N) \approx \frac{1}{r^2 N}. \quad (12)$$

For $r = 1$, the approximation (11)–(12) corresponds to the conventional Gaussian approximation (9), while for $r = 3$ corresponds to the cube-of-Gaussian approximation of [9]. However, (11)–(12) can be used also for other values of r, which could also be noninteger. For the special cases $r = 2$ and $r = 4$, other approximations for the mean have been suggested in [11] and [12], as expressed by

$$m_2(N) \approx \sqrt{1-(4N)^{-1}}, \qquad m_4(N) \approx \sqrt[4]{1-(4N)^{-1}}. \quad (13)$$

We now derive new simplified expressions for the probability of detection P_{D}, the probability of false alarm P_{FA}, and the required SNR γ. From (11), we obtain

$$F_{2N}^{-1}(x) \approx \hat{F}_{2N}^{-1}(x) = 2N\left[\sqrt{V_r(N)}\; Q^{-1}(1-x) + m_r(N)\right]^r, \quad (14)$$

which together with (11), (6) and (8) leads to

$$P_{\text{D}} \approx Q\left(\frac{Q^{-1}(P_{\text{FA}})}{\sqrt[r]{1+\gamma}} - \left(1 - \frac{1}{\sqrt[r]{1+\gamma}}\right)\frac{m_r(N)}{\sqrt{V_r(N)}}\right), \quad (15)$$

$$P_{\text{FA}} \approx Q\left(\sqrt[r]{1+\gamma}\, Q^{-1}(P_{\text{D}}) + \left(\sqrt[r]{1+\gamma} - 1\right)\frac{m_r(N)}{\sqrt{V_r(N)}}\right), \quad (16)$$

$$\gamma \approx \left(\frac{\sqrt{V_r(N)}\; Q^{-1}(P_{\text{FA}}) + m_r(N)}{\sqrt{V_r(N)}\; Q^{-1}(P_{\text{D}}) + m_r(N)}\right)^r - 1. \quad (17)$$

Note that the results in (17) are valid only when the fraction is larger than one. In addition, using (11)–(12) and (14)–(16), we can express the sample size in closed form as

$$N \approx \left\lceil \frac{1}{4r^2}\left(b_r + \sqrt{b_r^{\,2} + 2(r-1)}\right)^2 \right\rceil, \quad (18)$$

$$b_r = b_r(\gamma, P_{\text{FA}}, P_{\text{D}}) = \left(\frac{Q^{-1}(P_{\text{FA}})}{\sqrt[r]{1+\gamma}} - Q^{-1}(P_{\text{D}})\right)\left(1 - \frac{1}{\sqrt[r]{1+\gamma}}\right)^{-1}. \quad (19)$$

For $r = 1$ and $r = 3$, the expressions (11)–(12) and (15)–(19) coincide with those already obtained for the conventional Gaussian approximation [5] and for the cube-of-Gaussian approximation [9], respectively. Therefore, the proposed rth power transformation approach generalizes the previously proposed approximations to a broader range of values of the power exponent r. For the special cases $r = 2$ and $r = 4$ that use the mean (13) instead of that in (12), the expression (18) is not valid and can be replaced, respectively, by

$$N \approx \left\lceil \frac{b_2^{\,2}+1}{4} \right\rceil, \qquad N \approx \left\lceil \frac{1}{4} + \sqrt{\left(\frac{b_4^{\,2}}{4}\right)^4 + \frac{1}{16}} \right\rceil. \quad (20)$$

Interestingly, the expressions (15)-(20) are characterized by a low complexity, especially when r is integer, since these expressions only require few algebraic computations and one LUT for the (inverse) Q-function. As a consequence, (15)–(20) can form the basis for incorporating some performance computation capabilities into a low-complexity ED, in place of the complexity-demanding exact equations (6)–(8). The accuracy of the different approximations (15)–(20) will be evaluated in the numerical section.

C. *Linear Combination of Power Transformations*

In order to further reduce the approximation error of the simplified performance analysis, we propose other new transformations obtained by linearly combining different power transformations [13], [14]. Basically, this second proposed approach approximates as Gaussian the linear combination of different powers of a chi-squared random variable. Aiming at good accuracy, suitable linear combinations can again be chosen by using KL divergence minimization approaches or cumulant-matching methods. Aiming at low-complexity expressions, we focus on simple linear combinations obtained using few power exponents and simple coefficients.

Among the possible choices, two interestingly simple expressions have been suggested in [13] and [14], respectively:

$$L_{2,4}(x) = \sqrt{x/(2N)} + 4\sqrt[4]{x/(2N)}, \quad (21)$$

$$L_{2,3,6}(x) = \frac{1}{3}\sqrt{x/(2N)} - \frac{1}{2}\sqrt[3]{x/(2N)} + \sqrt[6]{x/(2N)}, \quad (22)$$

where x is again a chi-squared random variable with $2N$ DOF; in (21)–(22), $L_{2,4}(x)$ and $L_{2,3,6}(x)$ are approximated as Gaussian with mean and variance expressed respectively by

$$m_{2,4}(N) \approx 5 - \frac{1}{N}, \qquad V_{2,4}(N) \approx \frac{9}{4N}, \quad (23)$$

$$m_{2,3,6}(N) \approx \frac{5}{6} - \frac{1}{18N}, \qquad V_{2,3,6}(N) \approx \frac{1}{36N}. \quad (24)$$

This corresponds to the approximations

$$F_{2N}(x) \approx \hat{F}_{2N}(x) = 1 - Q\left(\frac{L_{2,4}(x) - m_{2,4}(N)}{\sqrt{V_{2,4}(N)}}\right), \quad (25)$$

$$F_{2N}(x) \approx \hat{F}_{2N}(x) = 1 - Q\left(\frac{L_{2,3,6}(x) - m_{2,3,6}(N)}{\sqrt{V_{2,3,6}(N)}}\right). \quad (26)$$

Since both (21) and (22) are monotonic increasing functions of x, the inverse functions of (25) and (26) exists, and, due to the simple form of (21) and (22), these inverse functions can be found analytically. By substituting $y = [x/(2N)]^{1/4}$ in (21), we obtain the quadratic equation $L_{2,4} = y^2 + 4y$, whose unique positive solution for y can be found as $y = (L_{2,4}+4)^{1/2} - 2$. Hence, by (21) and (25), we obtain $F_{2N}^{-1}(x) \approx \hat{F}_{2N}^{-1}(x)$, where

$$\hat{F}_{2N}^{-1}(x) = 2N\left[\left(\sqrt{V_{2,4}(N)}\; Q^{-1}(1-x) + m_{2,4}(N) + 4\right)^{\frac{1}{2}} - 2\right]^4, \quad (27)$$

and $m_{2,4}(N)$ and $V_{2,4}(N)$ are expressed by (23). Analogously, by substituting $z = [x/(2N)]^{1/6}$ in (22), we obtain the cubic equation $6L_{2,3,6} = 2z^3 - 3z^2 + 6z$, which has a unique real solution for z (since this cubic function is strictly increasing). This real solution can be calculated as in [6] using Cardano's formula, leading to

$$z = \frac{1}{2} + \left[\sqrt{\Delta(L_{2,3,6})} + a(L_{2,3,6})\right]^{\frac{1}{3}} - \left[\sqrt{\Delta(L_{2,3,6})} - a(L_{2,3,6})\right]^{\frac{1}{3}}, \quad (28)$$

$$\Delta(L_{2,3,6}) = [a(L_{2,3,6})]^2 + (3/4)^3, \quad (29)$$

$$a(L_{2,3,6}) = (12L_{2,3,6} - 5)/8. \quad (30)$$

From (22), (26), (28)–(30), we obtain $F_{2N}^{-1}(x) \approx \hat{F}_{2N}^{-1}(x)$, where

$$\hat{F}_{2N}^{-1}(x) = 2N\left\{\frac{1}{2}+\left[\sqrt{[A(x,N)]^2+(3/4)^3}+A(x,N)\right]^{1/3} -\left[\sqrt{[A(x,N)]^2+(3/4)^3}-A(x,N)\right]^{1/3}\right\}^6, \quad (31)$$

$$A(x,N) = \frac{3}{2}\left[\sqrt{V_{2,3,6}(N)}\; Q^{-1}(1-x)+m_{2,3,6}(N)\right]-\frac{5}{8}, \quad (32)$$

and $m_{2,3,6}(N)$ and $V_{2,3,6}(N)$ are expressed by (24). The inverse approximation functions (27) and (31)–(32) can be used in order to derive approximated expressions for the probability of detection P_{D}, the probability of false alarm P_{FA}, and the required SNR γ. Hence, the ROC can be approximated as

$$P_{\mathrm{D}} \approx Q\left(\frac{[B(P_{\mathrm{FA}},N)]^2}{\sqrt{(1+\gamma)V_{2,4}(N)}}+\frac{4B(P_{\mathrm{FA}},N)}{\sqrt[4]{(1+\gamma)[V_{2,4}(N)]^2}}-\frac{m_{2,4}(N)}{\sqrt{V_{2,4}(N)}}\right), \quad (33)$$

$$B(P_{\mathrm{FA}},N) = \left(\sqrt{V_{2,4}(N)}\; Q^{-1}(P_{\mathrm{FA}})+m_{2,4}(N)+4\right)^{1/2}-2, \quad (34)$$

$$P_{\mathrm{D}} \approx Q\left(\frac{\dfrac{[C(P_{\mathrm{FA}},N)]^3}{3\sqrt{(1+\gamma)}}-\dfrac{[C(P_{\mathrm{FA}},N)]^2}{2\sqrt[3]{(1+\gamma)}}+\dfrac{C(P_{\mathrm{FA}},N)}{\sqrt[6]{(1+\gamma)}}-m_{2,3,6}(N)}{\sqrt{V_{2,3,6}(N)}}\right), \quad (35)$$

$$C(P_{\mathrm{FA}},N) = \frac{1}{2}+\left[\sqrt{[A(1-P_{\mathrm{FA}},N)]^2+(3/4)^3}+A(1-P_{\mathrm{FA}},N)\right]^{1/3} -\left[\sqrt{[A(1-P_{\mathrm{FA}},N)]^2+(3/4)^3}-A(1-P_{\mathrm{FA}},N)\right]^{1/3}, \quad (36)$$

where $m_{2,4}(N)$, $V_{2,4}(N)$, $m_{2,3,6}(N)$, $V_{2,3,6}(N)$ and $A(x,N)$ are expressed by (23), (24) and (32). The required SNR can be approximated as

$$\gamma \approx \left(\frac{B(P_{\mathrm{FA}},N)}{B(P_{\mathrm{D}},N)}\right)^4-1, \qquad \gamma \approx \left(\frac{C(P_{\mathrm{FA}},N)}{C(P_{\mathrm{D}},N)}\right)^6-1. \quad (37)$$

The expressions (33)–(37), despite being slightly longer than the corresponding expressions (15) and (17), only require algebraic operations that can be easily done with low-complexity processing, and hence are suitable for device implementation.

In order to find the required number of samples N, we should analytically invert (33)–(37) with respect to N. For the linear combination in (21), this procedure leads to a quartic equation and hence can be solved analytically, whereas, for the linear combination in (22), since the degree of the resulting equation is larger than four, an analytical solution is not guaranteed due to the Abel-Ruffini Theorem [6]. As a consequence, we propose a low-complexity iterative algorithm that finds the required number of samples N with $O(\log_2(N))$ complexity. Basically, the main idea behind the proposed algorithm is explained in the following. When N is too small, the threshold of the ED is not able to simultaneously ensure both the required probability of detection P_{D} and the required probability of false alarm P_{FA}, for a given SNR γ. Hence, when N is too small, a threshold that ensures the given P_{FA} is necessarily greater than a threshold that ensures the given P_{D}. Therefore, the iterative algorithm looks for the minimum N such that the P_{FA}-based threshold is less than the P_{D}-based threshold: this way, the P_{FA}-based threshold surely guarantees that the probability of detection is equal to (or larger than) P_{D}. The pseudocode of the proposed iterative algorithm is included in the following.

<u>Iterative Algorithm to find the sample-size</u> N

01. Set $N \leftarrow 1$
02. Compute $\lambda_{\mathrm{FA}}(N) = \hat{F}_{2N}^{-1}(1-P_{\mathrm{FA}})$
03. Compute $\lambda_{\mathrm{D}}(N) = (1+\gamma)\hat{F}_{2N}^{-1}(1-P_{\mathrm{D}})$
04. If $\lambda_{\mathrm{FA}}(N) \le \lambda_{\mathrm{D}}(N)$ Then Go-to Line 25
05. Else While $\lambda_{\mathrm{FA}}(N) > \lambda_{\mathrm{D}}(N)$
06.   Set $N \leftarrow 2N$
07.   Compute $\lambda_{\mathrm{FA}}(N) = \hat{F}_{2N}^{-1}(1-P_{\mathrm{FA}})$
08.   Compute $\lambda_{\mathrm{D}}(N) = (1+\gamma)\hat{F}_{2N}^{-1}(1-P_{\mathrm{D}})$
09.   End-of-While
10.   If $N == 2$ Then Go-to Line 25
11.   Else Set $N_{\mathrm{step}} \leftarrow N/2$
12.   Set $s_{\mathrm{sign}} \leftarrow -1$
13.   While $N_{\mathrm{step}} > 1$
14.   Set $N_{\mathrm{step}} \leftarrow N/2$
15.   Set $N \leftarrow N + s_{\mathrm{sign}} N_{\mathrm{step}}$
16.   Compute $\lambda_{\mathrm{FA}}(N) = \hat{F}_{2N}^{-1}(1-P_{\mathrm{FA}})$
17.   Compute $\lambda_{\mathrm{D}}(N) = (1+\gamma)\hat{F}_{2N}^{-1}(1-P_{\mathrm{D}})$
18.   If $\lambda_{\mathrm{FA}}(N) \le \lambda_{\mathrm{D}}(N)$ Then Set $s_{\mathrm{sign}} \leftarrow -1$
19.   Else Set $s_{\mathrm{sign}} \leftarrow 1$
20.   End-of-If
21.   End-of-While
22.   Set $N \leftarrow N + (s_{\mathrm{sign}}+1)/2$
23.   End-of-If
24. End-of-If
25. End-of-Algorithm

The above iterative algorithm finds the required sample size N using two steps. First, the algorithm calculates a power-of-two upper bound on N by means of the "While" loop of Lines 05–09; concurrently, a lower bound on N is obtained as half the upper bound. Second, the algorithm applies a bisection method to refine the value of N between the two bounds, by means of the "While" loop of Lines 13–21. Within the iterative algorithm, $\lambda_{\mathrm{FA}}(N)$ and $\lambda_{\mathrm{D}}(N)$ represent the scaled versions of the P_{FA}-based and P_{D}-based thresholds. The algorithm is valid for both the linear combinations (21) and (22), since the computation of $\hat{F}_{2N}^{-1}(x)$ in the Lines 02, 03, 07, 08, 16 and 17 can be performed using either the approximation (27) or (31). It can be shown that the proposed iterative algorithm evaluates $\hat{F}_{n}^{-1}(x)$ exactly $\max\{4\lceil \log_2(N)\rceil, 2\}$ times, where N is the final solution (we omit the proof for the sake of brevity).

IV. Numerical Comparison

We compare the accuracy of the proposed approximations by means of numerical results. Fig. 1 shows the relative error on the probability of detection P_{D}, as a function of the probability of false alarm P_{FA}, when the SNR is $\gamma = 9$ dB and the sample size is $N = 5$. Among the power transformations, the cube-of-Gaussian approximation of [9] ($r = 3$) yields the lowest error. Also the linear combinations $L_{2,4}$ and $L_{2,3,6}$ give accurate results, which are better than for $r = 3$; however, the relative error for $r = 3$ stays below 10^{-2} for any value of P_{FA}. On the contrary, the approximations with $r = 1$, $r = 2$ and $r = 4$, can be inaccurate, especially for the conventional Gaussian approach ($r = 1$). In case of $r = 2$ and $r = 4$, we have used the means expressed by (13); however, we have verified that using the mean in (12) gives similar results.

The same conclusions of Fig. 1 are confirmed by Fig. 2, which exhibits the probability of detection P_D, as a function of the SNR γ, when $P_{FA} = 0.001$ and $N = 5$. On the other hand, Fig. 3 displays the (signed) error on the estimation of the required sample size N as a function of γ, when $P_D = 0.99$ and $P_{FA} = 0.001$. Again, the three approximations $r = 3$, $L_{2,4}$ and $L_{2,3,6}$ produce very accurate estimates (with minor errors only), while the other approximations overestimate (or underestimate) the required number of samples. Note that results similar to Figs. 1–3 would be obtained for other values of P_{FA} or γ.

V. Conclusions

We have proposed new approximations for energy detection sensors with self-performance computation capabilities. The proposed linear combination approaches, due to their superior accuracy and low complexity, are suitable for device implementation. Future work may include the effect caused by imperfect estimation of the SNR [15].

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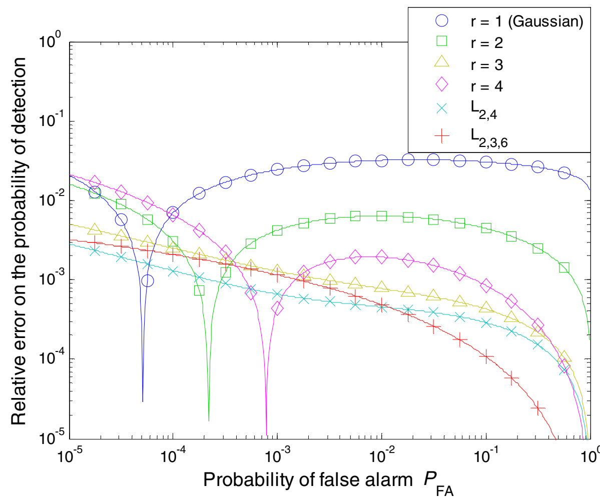

Figure 1. Probability of detection versus the probability of false alarm.

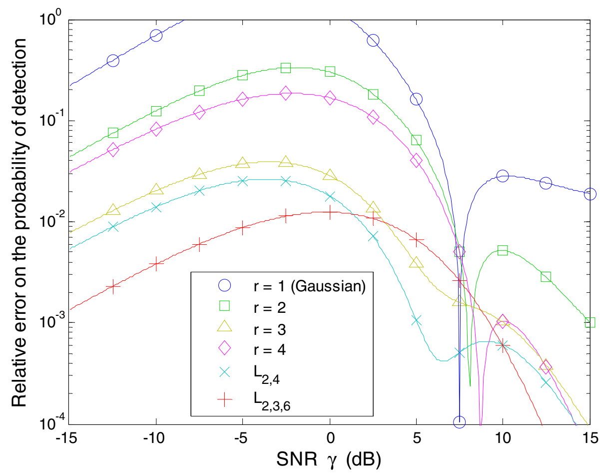

Figure 2. Probability of detection versus the SNR.

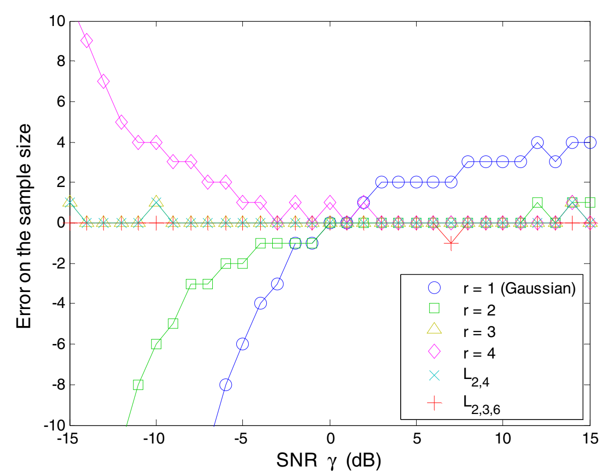

Figure 3. Sample size versus the SNR.