# WINDOWING TECHNIQUES FOR ICI MITIGATION IN MULTICARRIER SYSTEMS

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#### ABSTRACT

In multicarrier systems, the time variation of the multipath channel generates intercarrier interference (ICI). In this paper, we investigate ICI mitigation techniques that rely on a band approximation of the minimum mean-squared error (MMSE) block linear equalizer (BLE). We show that windowing is an effective method to reduce the ICI at the output of the BLE. We also point up that receiver windowing is more convenient than transmitter windowing. Finally, we show that the windowed MMSE-BLE outperforms an improved MMSE-BLE that takes into account the covariance of the ICI neglected in the band approximation.

### 1. INTRODUCTION

Multicarrier (MC) systems, based on data-block transmissions, gained a lot of popularity thanks to the easy equalization in timeinvariant frequency selective channels, which is enabled by the insertion of a cyclic prefix between successive blocks [1]. However, the increasing request for communication capabilities in highly-mobile environments motivates the design of more advanced equalizers to cope with time-variant frequency selective channels, which destroy the subcarriers orthogonality of MC systems and introduce intercarrier interference (ICI) [2]. In this framework, several equalizers have been recently proposed [4]-[10] to mitigate the BER performance degradation due to the ICI. Among the different alternatives, which also include nonlinear ICI canceling techniques [5][7]-[9], linear receivers maintain an appealing trade-off between performance and complexity. Linear receivers can be generally split in block linear equalizers (BLE), such as those proposed in [5][7][10], and serial linear equalizers (SLE), proposed in [4][6][8][9]. The equalizers proposed in [9] and [10] try to exploit the banded nature of the channel matrix in the Doppler-frequency domain, thus neglecting on each subcarrier the ICI produced by faraway subcarriers. This introduces a modeling error in the equalizer design, which produces a floor in the BER. Windowing techniques [3] have been recognized as a promising approach to reduce the effective band of the channel matrix [9], and thus reduce the BER floor.

In this paper we investigate both design and performance of transmitter windowing (TW) and receiver windowing (RW), opportunely coupled with the MMSE-BLE proposed in [10]. We will show by simulation the BER performance of the proposed windowed MMSE-BLE in OFDM and downlink MC-CDMA systems. Performance comparison with the SLE approaches introduced in [8] and in [9], shows that receiver windowing alone is beneficial to BLE and not to SLE. Thus RW for SLE must be coupled with other techniques such as ICI cancellation, as in [9].

Without resorting to windowing, in order to reduce the BER floor, we also consider banded linear equalizers that explicitly take into account the modeling error. Simulation results show that the windowing approach outperforms the latter for BLEs.

### 2. SYSTEM MODEL

We consider a multicarrier system, such as OFDM, with N subcarriers. Assuming time and frequency synchronization, and employing a cyclic prefix length L greater than the maximum delay spread of the channel, the input-output relation for the *i*th symbol can be expressed by [4]-[10]

$$\underline{\mathbf{z}}[i] = \underline{\mathbf{\Lambda}}[i]\underline{\mathbf{a}}[i] + \underline{\mathbf{n}}[i], \qquad (1)$$

where  $\underline{\mathbf{z}}[i]$  is the  $N \times 1$  received vector,  $\underline{\Lambda}[i] = \mathbf{F} \underline{\mathbf{H}}[i]\mathbf{F}^{H}$  is the  $N \times N$  Doppler-frequency domain channel matrix,  $\underline{\mathbf{H}}[i]$  is the  $N \times N$  time-domain channel matrix,  $\mathbf{F}$  is the  $N \times N$  unitary FFT matrix,  $\underline{\mathbf{a}}[i]$  is the  $N \times 1$  OFDM symbol that contains the frequency-domain data, and  $\underline{\mathbf{n}}[i] = \mathbf{F} \underline{\mathbf{v}}[i]$  is the  $N \times 1$  additive noise vector in the frequency domain. Assuming that  $N_A$  subcarriers are active and  $N_V = N - N_A$  are used as frequency guard bands, we can write  $\underline{\mathbf{a}}[i]^T = [\mathbf{0}_{1 \times N_V/2} \ \mathbf{a}[i]^T \ \mathbf{0}_{1 \times N_V/2}]$ , where  $\mathbf{a}[i]$  is the  $N_A \times 1$  data vector. For MC-CDMA downlink systems, we can use the same model, but in this case the data contained in the  $N_A \times 1$  vector  $\mathbf{a}[i]$  are obtained by multiplexing the symbols of K users, as expressed by  $\mathbf{a}[i] = \mathbf{Cs}[i]$  where  $\mathbf{s}[i]$  is the  $K \times 1$  vector that contains the data symbols of the K users, and  $\mathbf{C}$  is the  $N_A \times K$  matrix whose kth column  $\mathbf{c}_k$  contains the unit-norm spreading code of the kth user.

Assuming that the equalizer does not make use of the data received on the  $N_v$  virtual subcarriers, which contain little signal power, and dropping the block index *i* for the sake of simplicity, (1) becomes

$$\mathbf{z} = \mathbf{\Lambda} \, \mathbf{a} + \mathbf{n} \,, \tag{2}$$

where  $\mathbf{z}$  and  $\mathbf{n}$  are  $N_A \times 1$  vectors obtained by selecting the middle part of  $\underline{\mathbf{z}}[i]$  and  $\underline{\mathbf{n}}[i]$ , respectively, and  $\Lambda$  is the  $N_A \times N_A$  matrix obtained by selecting the central block of  $\underline{\Lambda}[i]$ . For downlink MC-CDMA, we assume that the receiver of the user *k* performs equalization and detection separately. Indeed, by using the separate approach, the receiver does not require the spreading codes of the other users. In addition, we assume that  $\underline{\Lambda}[i]$  is known to the receiver. The topic of TV channel estimation, though important, is not considered herein and can be found elsewhere (see, e.g., [5][11]).

In order to recover  $\mathbf{a}$ , several options are possible [7]. We focus on the banded MMSE-BLE [10], expressed by

$$\tilde{\mathbf{a}} = \mathbf{B}^H (\mathbf{B}\mathbf{B}^H + \gamma^{-1}\mathbf{I}_{N_*})^{-1}\mathbf{z}, \qquad (3)$$

where  $\mathbf{B} = \mathbf{\Lambda} \circ \mathbf{T}^{(Q)}$ ,  $\circ$  denotes the Hadamard (element-wise) product,  $\mathbf{T}^{(Q)}$  is a matrix with lower and upper bandwidth Q [12]

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and all ones within its band, and  $\gamma = \sigma_a^2 / \sigma_n^2$  is the signal-to-noise ratio (SNR), assumed known to the receiver. For MC-CDMA systems, we can recover the transmitted symbol  $s_k$  contained in the *k* th position of **s** by a simple despreading operation, as expressed by  $\tilde{s}_k = \mathbf{c}_k^H \tilde{\mathbf{a}}$ .

By exploiting a band LDL<sup>H</sup> factorization of the band matrix  $\mathbf{M} = \mathbf{B}\mathbf{B}^{H} + \gamma^{-1}\mathbf{I}_{N_{A}}$  the MMSE-BLE (3) requires approximately  $(8Q^{2} + 22Q + 4)N_{A}$  complex operations [10]. The bandwidth parameter Q can be chosen to trade off performance for complexity. Taking into account the rule of thumb  $Q \ge \int f_{D}/\Delta_{f} + 1$  in [9], where  $f_{D}$  is the maximum Doppler frequency and  $\Delta_{f} = 1/T$  is the subcarrier spacing, reasonable choices lie between Q = 1 and Q = 5. Since  $Q \ll N_{A}$ , the computational complexity of the banded MMSE-BLE (3) is significantly smaller than for other linear MMSE equalizers previously proposed [7][8]. In addition, as shown in [10], the complexity of the banded MMSE-BLE (3) is lower than for the banded MMSE-SLE used in [9] to initialize an iterative ICI cancellation technique.

We will now consider how to improve the performance of the banded MMSE-BLE by means of windowing. As shown in [9], windowing is capable to reduce the bandwidth of the channel matrix. In such a way, the parameter Q can be reduced without affecting the quality of the banded approximation.

#### 3. RECEIVER WINDOWING

Let us revisit the system model of (1). By applying an  $N \times 1$  timedomain window  $\mathbf{w}_{R}$  at the receiver before the FFT, the received vector can be expressed by [9]

$$\underline{\mathbf{z}}_{\mathrm{R}} = \underline{\mathbf{\Lambda}}_{\mathrm{R}} \, \underline{\mathbf{a}} + \underline{\mathbf{n}}_{\mathrm{R}} = \underline{\mathbf{C}}_{\mathrm{R}} \underline{\mathbf{\Lambda}} \, \underline{\mathbf{a}} + \underline{\mathbf{C}}_{\mathrm{R}} \underline{\mathbf{n}} \tag{4}$$

where  $\underline{\Lambda}_{R} = \mathbf{F} \underline{\Lambda}_{R} \underline{\mathbf{H}} \mathbf{F}^{H}$  is the frequency-domain windowed channel matrix, with  $\underline{\Lambda}_{R} = \text{diag}(\mathbf{w}_{R})$ ,  $\underline{\mathbf{n}} = \mathbf{F} \underline{\Lambda}_{R} \underline{\mathbf{v}}$  is the windowed noise, and  $\underline{\mathbf{C}}_{R} = \mathbf{F} \underline{\Lambda}_{R} \mathbf{F}^{H}$  is the circulant matrix that represents the windowing operation in the frequency domain. By neglecting the data received on the guard bands, we have

$$\mathbf{z}_{\mathrm{R}} = \boldsymbol{\Lambda}_{\mathrm{R}} \mathbf{a} + \mathbf{C}_{\mathrm{R}} \underline{\mathbf{n}} , \qquad (5)$$

where  $\mathbf{z}_{R}$ ,  $\mathbf{\Lambda}_{R}$ , and  $\mathbf{C}_{R}$  are the middle blocks of  $\underline{\mathbf{z}}_{R}$ ,  $\underline{\mathbf{\Lambda}}_{R}$ , and  $\underline{\mathbf{C}}_{R}$ , respectively, with size  $N_{A} \times 1$ ,  $N_{A} \times N_{A}$ , and  $N_{A} \times N$ , respectively. From the comparison between (5) and (2), it is clear that the main difference is the noise coloring produced by the windowing operation. Hence, by the band approximation  $\mathbf{\Lambda}_{R} \approx \mathbf{B}_{R} = \mathbf{\Lambda}_{R} \circ \mathbf{T}^{(Q)}$ , the RW banded MMSE-BLE becomes

$$\tilde{\mathbf{a}}_{\mathrm{R}} = \mathbf{B}_{\mathrm{R}}^{H} (\mathbf{B}_{\mathrm{R}} \mathbf{B}_{\mathrm{R}}^{H} + \gamma^{-1} \mathbf{C}_{\mathrm{R}} \mathbf{C}_{\mathrm{R}}^{H})^{-1} \mathbf{z}_{\mathrm{R}} .$$
(6)

### 3.1 Window Design

Our goal is to design a receive window with two features. First, the approximation  $\Lambda_{R} \approx \mathbf{B}_{R}$  should be as good as possible, and possibly better than the approximation  $\Lambda \approx B$  . This would reduce the residual ICI of the banded MMSE-BLE. Second, the noise covariance matrix  $\mathbf{C}_{\mathbf{R}}\mathbf{C}_{\mathbf{R}}^{H}$  in (6) should be banded, so that the equalization can exploit the band LDL<sup>H</sup> factorization of  $\mathbf{M}_{\mathrm{R}} = \mathbf{B}_{\mathrm{R}}\mathbf{B}_{\mathrm{R}}^{H} + \gamma^{-1}\mathbf{C}_{\mathrm{R}}\mathbf{C}_{\mathrm{R}}^{H}$ , introduced in [10] to reduce the computational complexity. We point out that, without the band approximation, the application of a time-domain window at the receiver does not change the MSE at the MMSE-BLE output. This is why we adopt the minimum band approximation error (MBAE) criterion, which can be mathematically expressed as follows: Choose **w** that minimizes  $E\{\|\mathbf{E}_{R}\|^{2}\}$ , where  $\mathbf{E}_{R} = \mathbf{\Lambda}_{R} - \mathbf{B}_{R}$  and  $\|\cdot\|$  is the Frobenius norm, subject to the energy constraint  $tr(\Delta_R^2) = N$ . (Equivalently,  $E\{\|\mathbf{B}_{R}\|^{2}\}$  can be maximized subject to the same constraint.) Note that this criterion is similar to the max Average-SINR criterion of [9]. Indeed, also in [9] the goal is to make the

channel matrix more banded, in order to facilitate an iterative ICI cancellation receiver. Differently, in our case, we want to exploit the band LDL<sup>H</sup> factorization, and hence we also require the matrix  $\mathbf{C}_{\mathbf{R}}\mathbf{C}_{\mathbf{R}}^{H}$  in (6) to be banded. Since the  $N_{\mathbf{A}} \times N_{\mathbf{A}}$  matrix  $\mathbf{C}_{\mathbf{R}}\mathbf{C}_{\mathbf{R}}^{H}$  is the middle block of the  $N \times N$  matrix  $\underline{\mathbf{C}}_{\mathbf{R}}\mathbf{C}_{\mathbf{R}}^{H} = \mathbf{F}\mathbf{A}_{\mathbf{R}}\mathbf{A}_{\mathbf{R}}^{H}\mathbf{F}^{H}$ , we impose the following sum-of-exponentials (SOE) constraint: the elements of the window  $\mathbf{w}_{\mathbf{R}}$  should satisfy

$$[\mathbf{w}_{\mathrm{R}}]_n = \sum_{q=-Q}^{Q} b_q \exp(j2\pi qn/N) .$$
<sup>(7)</sup>

Indeed, when  $\mathbf{w}_{R}$  is a sum of 2Q+1 complex exponentials, the diagonal of  $\Delta_{R}\Delta_{R}^{H}$  can be expressed as the sum of 4Q+1 exponentials, and consequently, by the properties of the FFT matrix,  $\mathbf{F}\Delta_{R}\Delta_{R}^{H}\mathbf{F}^{H}$  is exactly banded with lower and upper bandwidth 2Q. Obviously, the class of SOE windows includes some common cosine-based windows such as Hamming, Hann, and Blackman [3]. The SOE constraint (7) can also be expressed by

$$\mathbf{w}_{\mathrm{R}} = \mathbf{F}\mathbf{b} , \qquad (8)$$

where  $\tilde{\mathbf{F}} = \sqrt{N}[\mathbf{f}_{N-Q}, ..., \mathbf{f}_{N-1}, \mathbf{f}_0, \mathbf{f}_1, ..., \mathbf{f}_Q]$  is obtained from the columns  $\{\mathbf{f}_i\}$  of the unitary IFFT matrix  $\mathbf{F}^H$ , and  $\mathbf{b} = [b_{-Q} \cdots b_Q]^T$  is a vector of size 2Q + 1 that contains the design parameters. By applying the MBAE criterion, by the appendix of [9], we obtain

$$E\{\|\mathbf{B}_{R}\|^{2}\} = \mathbf{w}_{R}^{H}(\mathbf{P} \circ \mathbf{A})\mathbf{w}_{R}, \qquad (9)$$

where  $\mathbf{P} = E\{\underline{\mathbf{H}}\underline{\mathbf{H}}^H\}$  contains the time-domain autocorrelation function of the channel, while **A** is defined as

$$[\mathbf{A}]_{m,n} = \frac{\sin(\pi (2Q+1)(n-m)/N)}{N\sin(\pi (n-m)/N)} .$$
(10)

By maximizing (9) with the SOE constraint (8), the window parameters in **b** are obtained by the eigenvector that corresponds to the largest eigenvalue of  $\tilde{\mathbf{F}}^{H}(\mathbf{P} \circ \mathbf{A})\tilde{\mathbf{F}}$ . Note that this maximization leads to  $b_q = b_{-q}^*$ , and consequently the MBAE-SOE window is real and symmetric.

It is worth noting that the application of receive windowing produces a minimal increase in terms of computational complexity, because the design can be performed offline. As a result, it can be shown that the complexity increase of the banded MMSE-BLE due to windowing is approximately  $(2Q+1)N_A$  complex operations, for a total of  $(8Q^2 + 24Q + 5)N_A$  complex operations.

## 4. TRANSMITTER WINDOWING

The windowing operation can be performed at the transmitter, as in [13] for DMT, rather than at the receiver. In this case, by applying an  $N \times 1$  time-domain window  $\mathbf{w}_{T}$  at the transmitter after the IFFT, the received vector can be expressed by [13]

$$\underline{\mathbf{z}}_{\mathrm{T}} = \underline{\mathbf{\Lambda}}_{\mathrm{T}} \, \underline{\mathbf{a}} + \underline{\mathbf{n}} = \underline{\mathbf{\Lambda}} \, \underline{\mathbf{C}}_{\mathrm{T}} \underline{\mathbf{a}} + \underline{\mathbf{n}} \tag{11}$$

where  $\underline{\Lambda}_{T} = F\underline{H}\Delta_{T}F^{H}$  is the frequency-domain windowed channel matrix, with  $\Delta_{T} = \text{diag}(\mathbf{w}_{T})$ , and  $\underline{C}_{T} = F\Delta_{T}F^{H}$  is the circulant matrix that represents the windowing operation in the frequency domain. It is clear that in this case there is no noise coloring, because the window does not change the noise term. For this reason, differently from receiver windowing, the MSE at the MMSE-BLE output depends on the window  $\mathbf{w}_{T}$ . Hence, we have at our disposal two alternative criteria.

*MBAE criterion*: similar to receiver windowing,  $\mathbf{w}_{T}$  can be chosen to minimize the error produced by the band approximation. By some computations similar to those of [9], it can be shown that the optimum MBAE window  $\mathbf{w}_{T}$  is the eigenvector that corresponds to the largest eigenvalue of  $\mathbf{P} \circ \mathbf{A}$ . It can be observed that in this case the SOE constraint is not necessary, because the noise is white. However, this constraint can be useful to reduce the num-

ber of parameters to be designed. If we impose the SOE constraint, the solution is exactly the same for receiver windowing.

*MMSE criterion:* We can select  $\mathbf{w}_{T}$  in order to minimize the MSE at the MMSE-BLE output. In this case, it is useful to understand which would be the optimum MMSE window for the non-banded MMSE-BLE. By [14], the cost function to be minimized is

$$\operatorname{tr}(\mathbf{C}_{\varepsilon\varepsilon}) = \operatorname{tr}(E\{(\mathbf{I}_N + \boldsymbol{\sigma}_n^{-2}\underline{\mathbf{C}}_{\mathrm{T}}^H\underline{\boldsymbol{\Lambda}}^H\underline{\boldsymbol{\Lambda}}\underline{\mathbf{C}}_{\mathrm{T}})^{-1}\}), \qquad (12)$$

where  $\mathbf{C}_{ee}$  is the covariance matrix of the data error  $\underline{\mathbf{\varepsilon}} = \underline{\mathbf{a}} - \hat{\underline{\mathbf{a}}}$ . By exploiting  $\underline{\mathbf{\Lambda}} = \mathbf{F}\underline{\mathbf{H}}\mathbf{F}^{H}$  and  $\underline{\mathbf{C}}_{T} = \mathbf{F}\mathbf{\Lambda}_{T}\mathbf{F}^{H}$ , Eq. (12) becomes

$$\mathbf{r}(\mathbf{C}_{\varepsilon\varepsilon}) = \mathrm{tr}(E\{(\mathbf{I}_{N} + \boldsymbol{\sigma}_{n}^{-2}\mathbf{F}\boldsymbol{\Delta}_{\mathrm{T}}^{H}\mathbf{\underline{H}}^{H}\mathbf{\underline{H}}\boldsymbol{\Delta}_{\mathrm{T}}\mathbf{F}^{H})^{-1}\}).$$
(13)

To obtain an approximated solution, we replace  $\underline{\mathbf{H}}^{H}\underline{\mathbf{H}}$  in (13) with its expected value  $E\{\underline{\mathbf{H}}^{H}\underline{\mathbf{H}}\}=\mathbf{I}_{N}$ , thereby obtaining

$$\operatorname{tr}(\mathbf{C}_{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}}) \cong \operatorname{tr}(\mathbf{F}(\mathbf{I}_N + \boldsymbol{\sigma}_n^{-2} \boldsymbol{\Delta}_{\mathrm{T}}^H \boldsymbol{\Delta}_{\mathrm{T}})^{-1} \mathbf{F}^H), \qquad (14)$$

which by tr(AB) = tr(BA) and  $\Delta_T = diag(w_T)$  leads to

$$\operatorname{tr}(\mathbf{C}_{\varepsilon\varepsilon}) \cong \sum_{i=1}^{N} \frac{1}{1 + \sigma_n^{-2} \left| \left[ \mathbf{w}_{\mathrm{T}} \right]_i \right|^2} \,. \tag{15}$$

As a result, by assuming real-valued windows and taking into account the constraint  $\sum_{i=1}^{N} [\mathbf{w}_{T}]_{i}^{2} = N$ , (15) is minimized when  $[\mathbf{w}_{T}]_{i} = 1$ , i.e., for rectangular windowing, as it could be suggested by intuition.

It is evident that the two approaches and the obtained windows are different, since the MBAE criterion minimizes the band approximation error without considering the MSE, whereas the MMSE criterion minimizes the MSE without taking into account the band approximation error. However, as it will be shown later by simulations, none of them minimizes the BER.

We want to point out that transmitter and receiver windowing can also be used together. In this case, the two windows could be designed either jointly or separately. It is interesting to note that a possible approach can firstly use the MMSE criterion to design the transmitter window, since the receiver window does not change the MSE, and secondly the MBAE criterion to design the receiver window. In this case, the joint design is equivalent to receiver windowing only. However, other criteria and approaches are also possible.

## 5. USING COVARIANCE OF THE UNMODELED ICI

The receiver (3) is based on the assumption that the modeling error  $\mathbf{E} = \mathbf{\Lambda} - \mathbf{B}$  is negligible. On the contrary, windowing tries to reduce this modeling error by compressing the ICI within the band of the channel matrix. A different approach, conceptually simpler than windowing, relies on exploiting the covariance matrix of the modeling error. Indeed, by rewriting (2) as

$$\mathbf{z} = \mathbf{B}\mathbf{a} + \mathbf{E}\mathbf{a} + \mathbf{n} = \mathbf{B}\mathbf{a} + \mathbf{e} + \mathbf{n} , \qquad (16)$$

the modeling error can be explicitly taken into account in the MMSE-BLE expression. Thus, by exploiting the independence among  $\mathbf{a}$ ,  $\mathbf{E}$ , and  $\mathbf{n}$ , it is easy to obtain the MMSE estimate of  $\mathbf{a}$  as

$$\tilde{\mathbf{a}}_{\mathrm{E}} = \mathbf{B}^{H} (\mathbf{B}\mathbf{B}^{H} + \mathbf{C}_{\mathrm{E}} + \gamma^{-1}\mathbf{I}_{N_{A}})^{-1}\mathbf{z} , \qquad (17)$$

where  $\mathbf{C}_{\mathrm{E}} = E\{\mathbf{e}\mathbf{e}^{H}\} = \sigma_{a}^{2} E\{\mathbf{E}\mathbf{E}^{H}\}$ . In the following we assume that an accurate estimate of  $\mathbf{C}_{\mathrm{E}}$  can be obtained by a sample mean estimate over  $N_{c}$  channel realizations, as expressed by  $\hat{\mathbf{C}}_{\mathrm{E}} = \sum_{i=1}^{N_{c}} \mathbf{E}[i]\mathbf{E}[i]^{H} / N_{c}$ . Specifically, we employ  $N_{c} = 1000$  in the simulation results. Moreover, to enable band LDL<sup>H</sup> factorization, we require  $\mathbf{C}_{\mathrm{E}}$  in (17) to be banded and positive definite. Hence, we apply triangular tapering [15] to  $\hat{\mathbf{C}}_{\mathrm{E}}$ , expressed by  $\hat{\mathbf{C}}_{\mathrm{E}} \circ \mathbf{T}_{\mathrm{tri}}^{(Q)}$ , where  $[\mathbf{T}_{\mathrm{tri}}^{(Q)}]_{m,n} = |m-n|/(Q+1)$  for  $|m-n| \leq Q$ , and  $[\mathbf{T}_{\mathrm{tri}}^{(Q)}]_{m,n} = 0$  for |m-n| > Q, and we use  $\hat{\mathbf{C}}_{\mathrm{E}} \circ \mathbf{T}_{\mathrm{tri}}^{(Q)}$  instead

of  $C_E$  in (17). We will resort to simulations to compare the performance of this method with that of windowing.

### 6. SIMULATION RESULTS

In this section we evaluate by simulations the BER performance gains obtained by applying windowing and covariance modeling error to the MMSE-BLE proposed in [10]. We consider both an OFDM system with FFT size N = 128,  $N_A = 96$  active subcarriers, cyclic prefix L = 8, and a partially loaded MC-CDMA system with the same parameters, K = 64 users and Walsh Hadamard spreading codes. Both systems employ QPSK constellations. We also assume Rayleigh fading channels with exponential power delay profile, Jakes' Doppler spectrum and  $f_D / \Delta_f = 0.15$ , which represents high Doppler spread scenarios.

Fig. 1 illustrates the BER performance of the RW for the banded MMSE-BLE and MC-CDMA when Q = 1. Similar results holds true also for OFDM. In this case, since Q = 1, there is a single amplitude parameter to be designed in the MBAE-SOE sense, which is expressed by the ratio  $2|b_1|/b_0$ . Fig. 1 clearly shows that RW is beneficial and that the best performance is obtained for the ratio  $2|b_1|/b_0 = 0.844$ . We point out that also other suboptimum SOE windows outperform the rectangular window (i.e., absence of windowing).

Fig. 2 compares the BER of the RW banded MMSE-BLE with those of other linear equalizers when Q = 2. Banded MMSE-BLE with MBAE-SOE window exhibits the best BER performance among the banded MMSE receivers, thus confirming the goodness of our window design. Fig. 2 also shows the BER for SLEs, with and without windowing, derived from [8] and [9]. Surprisingly, and conversely to the BLE, it turns out that windowing alone is detrimental for SLEs. This means that for SLEs windowing must be coupled with other techniques such as iterative ICI cancellation, as in [9].

Notice that the proposed RW banded MMSE-BLE, which is characterized by linear complexity in the number of subcarriers, outperforms the non-banded SLE of [8], which has a quadratic complexity and the lowest BER among the SLE approaches.

It is also interesting to observe that the application of RW allows for a complexity reduction of the banded MMSE-BLE, by simply reducing the parameter Q. Indeed, by comparing Fig. 1 with Fig. 2, it is evident that the BLE with Q=1 and MBAE-SOE windowing outperforms the BLE with Q=2 and rectangular windowing, while the complexity is reduced roughly to 46%.

Fig. 3 shows the BER performance of the TW banded-MMSE-BLE for OFDM with Q = 1, but similar results hold true also for MC-CDMA and different Q's. First, we highlight that RW with optimum design seems to outperform any TW with SOE design (e.g., for any value of  $2|b_1|/b_0$ ). Second, none TW with SOE design outperforms all the others over the entire SNR range, thus suggesting that the design criterion should take into account the noise power at the receiver side. This means that for low SNR, where the thermal noise **n** dominates the channel modeling error **e**, the MMSE criterion suggests to use an almost rectangular window, while in high SNR regime the window design should take into account also the band approximation.

Fig. 4 shows the BER of the banded MMSE equalizers without windowing for OFDM and Q = 2. Clearly, the simpler technique of taking into account the covariance matrix of the modeling error in the MMSE expression gives some performance improvement reducing the BER floor both for BLEs and SLEs. However such a gain is lower than what obtained by windowing, which also tries to reduce the amount of such a modeling error.

## 7. CONCLUSION

We considered the effect of transmitter and receiver windowing for banded MMSE equalizers employed in multicarrier systems to mitigate the ICI induced by time-varying frequency-selective channels. We found by simulation that receiver windowing is more appealing than transmitter windowing, and that receiver windowing is more beneficial for BLE rather than for SLE. Joint transmitter and receiver windowing design for banded MMSE-BLE could be the subject of future work.

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Fig. 1. BER performance of receiver windowing for MC-CDMA. (Q = 1), different SOE windows).



Fig. 2. BER performance of receiver windowing for MC-CDMA (Q = 2).

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Fig. 3. BER performance of transmitter windowing for OFDM (Q = 1, different SOE windows).



Fig. 4. BER performance with error modeling for OFDM (Q = 2).