BANDED EQUALIZERS FOR MIMO-OFDM IN FAST TIME-VARYING CHANNELS

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ABSTRACT

We propose low-complexity equalizers for multiple-input multiple-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems in frequency-selective time-varying channels, by extending the approach we formerly proposed for single-antenna OFDM systems. Specifically, by neglecting the intercarrier interference (ICI) coming from faraway subcarriers, we design minimum mean-squared error (MMSE) block linear equalizers (BLE) and MMSE block decision-feedback equalizers (BDFE) that employ a band LDL factorization algorithm. The complexity of the proposed banded equalizers is linear in the number of subcarriers, differently from conventional MMSE-BLE and MMSE-BDFE characterized by a cubic complexity. We also consider a receiver window designed to minimize the power of the undesired ICI. Simulation results show that windowing is beneficial in controlling the complexity of the proposed equalizers with acceptable performance loss with respect to the conventional MMSE-BLE and MMSE-BDFE.

1. INTRODUCTION

In the recent past, the use of multiple antennas at both the transmitter and receiver side has received great attention for the high degree of flexibility to increase capacity [1], diversity [2], or both. Multiple-input multiple-output (MIMO) rate-oriented systems in frequency-flat channels can exploit the multiplexing gain offered by multiple transmit antennas by using at least the same number of receive antennas [1]. Classical MIMO techniques developed for frequency-flat channels can be easily extended to frequency-selective channels by resorting to orthogonal frequency-division multiplexing (OFDM) [3] [4]. Indeed, OFDM is particularly attractive because it converts a time-invariant (TI) frequency-selective channel in a set of orthogonal frequency-flat channels [5], thus enabling simple one-tap equalization on each subcarrier.

However, the Doppler spread associated with time-varying (TV) channels, like those experienced in high-mobility environments, destroys the OFDM orthogonality among subcarriers, introducing intercarrier interference (ICI) and seriously degrading the performance of single-tap equalizers [6]. Thus, in single-input single-output (SISO) OFDM systems, more complex equalizers are required to cope with a TV channel, such as those proposed in [7]-[13]. The TV effects are even stronger in MIMO-OFDM systems, because the data received on each subcarrier are affected by the ICI generated by the data transmitted on adjacent subcarriers from all the transmit antennas [14].

The aim of this paper is to extend to MIMO-OFDM the equalization method proposed in [13] for SISO-OFDM, which attains good performance with a very low complexity. Specifically, these nice features are obtained by exploiting the structure of the ICI in the frequency domain, with a philosophy that is common to several papers [7]-[10]. Indeed, by noting that the ICI is mostly introduced by adjacent subcarriers, i.e., the channel matrix in the frequency-domain is nearly banded, we can design a minimum mean squared error (MMSE) block linear equalizer (BLE) and a MMSE block decision feedback equalizers (BDFE) that take advantage of a band LDL factorization algorithm [10]. Moreover, we weapon the equalizers with a receiver windowing, which is known to protect SISO-OFDM systems from Doppler effects. Specifically, this time-domain window can be designed to strengthen the banded assumption of the channel matrix, that is maximizing the ICI power on the very-close subcarriers [9] [13]. This way, the performance of the proposed equalizers can approach those of the conventional non-banded MMSE-BLE and MMSE-BDFE, with a significant reduction in complexity. Simulation results prove that the considered MIMO-OFDM system, equipped with the proposed equalizers, can provide the promised information rate multiplication with respect to SISO-OFDM systems. Moreover, by using more antennas at the receiver than at the transmitter, the proposed equalizers offer improved protection from TV channels, because they are able to collect the diversity gain and reduce the BER floor caused by the ICI.

2. MIMO-OFDM MODEL

We consider a single-user MIMO-OFDM system with M_T transmit antennas, M_R receive antennas, and N subcarriers. We assume that the $M_\mathrm{T}M_\mathrm{R}$ SISO channels are doubly-selective (i.e., both frequency- and time-selective) and characterized by the same fading statistics, with maximum delay spread smaller than the cyclic prefix (CP) length L. We also assume time and frequency synchronization. At the *j*th receive antenna, the received vector, after FFT and CP removal, can be expressed by [14] [15]

$$\underline{\mathbf{z}}_j = \sum_{i=1}^{M_\mathrm{T}} \underline{\mathbf{\Lambda}}_{j,i}\, \underline{\mathbf{a}}_i + \underline{\mathbf{n}}_j \,, \qquad (1)$$

where $\underline{\mathbf{z}}_j$ is the $N\times 1$ received vector, $\underline{\mathbf{\Lambda}}_{j,i}$ is the $N\times N$ frequency-domain channel matrix between the *j*th receive antenna and the *i*th transmit antenna, $\underline{\mathbf{a}}_i$ is the $N\times 1$ OFDM frequency-domain data block transmitted by the *i*th transmit antenna, assumed independent of the data transmitted from the other antennas, and $\underline{\mathbf{n}}_j$ is the $N\times 1$ noise vector of the *j*th receive antenna.

Each frequency-domain channel matrix can be expressed by $\underline{\mathbf{\Lambda}}_{j,i} = \mathbf{F}\mathbf{\Delta}\underline{\mathbf{H}}_{j,i}\mathbf{F}^H$, where $\underline{\mathbf{H}}_{j,i}$ is the corresponding $N\times N$ time-domain channel matrix, $\mathbf{F}$ is the $N\times N$ unitary DFT matrix, and $\mathbf{\Delta} = \mathrm{diag}(\mathbf{w})$, where $\mathbf{w}$ is a time-domain receiver window [9]. For classical (i.e., unwindowed) OFDM, $\mathbf{\Delta} = \mathbf{I}_N$. In time-varying channels, $\underline{\mathbf{H}}_{j,i}$ is not circulant, and hence $\underline{\mathbf{\Lambda}}_{j,i}$ is not diagonal. As a result, a certain amount of ICI is present. However, like in SISO-OFDM systems [9] [12], we can introduce receiver windowing before the FFT, in order to make $\underline{\mathbf{\Lambda}}_{j,i}$ more banded, thus simplifying the equalization step. The price paid is some noise coloring.

Indeed, the windowed noise can be expressed as $\underline{\mathbf{n}}_j = \mathbf{F}\mathbf{\Delta}\underline{\mathbf{v}}_j$, where $\underline{\mathbf{v}}_j$ is the time-domain AWGN vector at the jth receive antenna, with covariance $E\{\underline{\mathbf{v}}_j\underline{\mathbf{v}}_j^H\} = \sigma_v^2\mathbf{I}_N$. For simplicity, we have assumed that all the M_R receive antennas adopt the same window. This is reasonable since all the SISO channels are characterized by the same fading statistics.

By assuming that N_A out of N subcarriers are active, the OFDM data vector $\underline{\mathbf{a}}_i$ transmitted by the ith antenna can be rewritten as

$$\underline{\mathbf{a}}_i = \mathbf{T}_\mathrm{GB}\mathbf{a}_i = [\mathbf{0}^T_{N_\mathrm{V}/2\times 1}\ \mathbf{a}_i^T\ \mathbf{0}^T_{N_\mathrm{V}/2\times 1}]^T , \tag{2}$$

where $\mathbf{T}_\mathrm{GB} = [\mathbf{0}^T_{N_\mathrm{V}/2\times N_\mathrm{A}}\ \mathbf{I}_{N_\mathrm{A}}\ \mathbf{0}^T_{N_\mathrm{V}/2\times N_\mathrm{A}}]^T$ is the $N \times N_\mathrm{A}$ matrix that inserts the N_V frequency guard bands, with $N_\mathrm{V} = N - N_\mathrm{A}$, and $\mathbf{a}_i$ is the $N_\mathrm{A}\times 1$ data vector.

By collecting in a single vector $\underline{\mathbf{z}}$ all the vectors $\{\underline{\mathbf{z}}_j\}_{i=1}^{M_\mathrm{R}}$ received by the M_R antennas, we can write

$$\underline{\mathbf{z}} = \underline{\mathbf{\Lambda}}\,\underline{\mathbf{a}} + \underline{\mathbf{n}} , \tag{3}$$

where $\underline{\mathbf{z}} = [\underline{\mathbf{z}}_1^T \cdots \underline{\mathbf{z}}_{M_\mathrm{R}}^T]^T$, $\underline{\mathbf{a}} = [\underline{\mathbf{a}}_1^T \cdots \underline{\mathbf{a}}_{M_\mathrm{T}}^T]^T$, with $\|\underline{\mathbf{a}}\|_F \le 1$,

$$\underline{\mathbf{\Lambda}} = \begin{bmatrix} \underline{\mathbf{\Lambda}}_{1,1} & \cdots & \underline{\mathbf{\Lambda}}_{1,M_\mathrm{T}} \\ \vdots & & \vdots \\ \underline{\mathbf{\Lambda}}_{M_\mathrm{R},1} & \cdots & \underline{\mathbf{\Lambda}}_{M_\mathrm{R},M_\mathrm{T}} \end{bmatrix} , \tag{4}$$

and $\underline{\mathbf{n}} = [\underline{\mathbf{n}}_1^T \cdots \underline{\mathbf{n}}_{M_\mathrm{R}}^T]^T$, with covariance expressed by $\mathbf{C}_{\underline{\mathbf{nn}}} = \mathbf{I}_{M_\mathrm{R}} \otimes (\sigma_v^2\mathbf{W}\mathbf{W}^H)$, where $\mathbf{W} = \mathbf{F}\mathbf{\Delta}\mathbf{F}^H$ is the circulant matrix that represents the windowing operation in the frequency domain. We now define the permutation matrix $\mathbf{P}_{(M,N)}$ as the $MN \times MN$ matrix that contains 1's in the positions $\{(i+1\ ,\ \lfloor i/M \rfloor + 1 + N\, i_{\mathrm{mod}M})\}_{i=0}^{MN-1}$ and 0's elsewhere. By permuting the received vector in (3), we obtain

$$\underline{\boldsymbol{z}} = \mathbf{P}_{(M_\mathrm{R},N)}\underline{\mathbf{z}} = \underline{\boldsymbol{\Lambda}}\,\underline{\boldsymbol{a}} + \underline{\boldsymbol{n}} , \tag{5}$$

where $\underline{\boldsymbol{\Lambda}} = \mathbf{P}_{(M_\mathrm{R},N)}\underline{\mathbf{\Lambda}}\mathbf{P}^T_{(M_\mathrm{T},N)}$, $\underline{\boldsymbol{a}} = \mathbf{P}_{(M_\mathrm{T},N)}\,\underline{\mathbf{a}}$, and $\underline{\boldsymbol{n}} = \mathbf{P}_{(M_\mathrm{R},N)}\underline{\mathbf{n}}$. Differently from the model expressed by (3), the permuted model of (5), which is equivalent to Eq. 10 in [14], has the property that the data received on the same subcarrier of different antennas are close together. The same property holds for the data transmitted by different antennas. Consequently, also the guard bands relative to data transmitted by different antennas are close together, at the top and at the bottom of $\underline{\boldsymbol{a}}$. By defining $\mathbf{R}_\mathrm{GB} = [\mathbf{0}_{N_\mathrm{A}\times N_\mathrm{V}/2}\ \mathbf{I}_{N_\mathrm{A}}\ \mathbf{0}_{N_\mathrm{A}\times N_\mathrm{V}/2}] = \mathbf{T}_\mathrm{GB}^T$, and $\boldsymbol{R}_\mathrm{GB} = \mathbf{R}_\mathrm{GB} \otimes \mathbf{I}_{M_\mathrm{R}}$ as the $M_\mathrm{R}N_\mathrm{A} \times M_\mathrm{R}N$ guard band removal matrix, the received vector becomes

$$\boldsymbol{z} = \boldsymbol{R}_\mathrm{GB}\underline{\boldsymbol{z}} = \boldsymbol{\Lambda}\boldsymbol{a} + \boldsymbol{n} , \tag{6}$$

where $\boldsymbol{\Lambda} = \boldsymbol{R}_\mathrm{GB}\underline{\boldsymbol{\Lambda}}\boldsymbol{T}_\mathrm{GB}$ is the $M_\mathrm{R}N_\mathrm{A} \times M_\mathrm{T}N_\mathrm{A}$ channel matrix without guard bands, $\boldsymbol{T}_\mathrm{GB} = \mathbf{T}_\mathrm{GB} \otimes \mathbf{I}_{M_\mathrm{T}}$, $\boldsymbol{a} = \boldsymbol{T}_\mathrm{GB}^T\underline{\boldsymbol{a}} = \boldsymbol{R}_\mathrm{GB}\underline{\boldsymbol{a}}$, and $\boldsymbol{n} = \boldsymbol{R}_\mathrm{GB}\underline{\boldsymbol{n}}$. By simple algebra, it is easy to show that

$$\boldsymbol{a} = \boldsymbol{R}_\mathrm{GB}\underline{\boldsymbol{a}} = \boldsymbol{R}_\mathrm{GB}\mathbf{P}_{(M_\mathrm{T},N)}\,\underline{\mathbf{a}} = \boldsymbol{R}_\mathrm{GB}\mathbf{P}_{(M_\mathrm{T},N)}\boldsymbol{T}_\mathrm{GB}\mathbf{a} = \mathbf{P}_{(M_\mathrm{T},N_\mathrm{A})}\mathbf{a} , \tag{7}$$

where $\mathbf{a} = [\mathbf{a}_1^T \cdots \mathbf{a}_{M_\mathrm{T}}^T]^T$ is the aggregate transmitted data vector, which does not contain guard bands. Hence, the estimation of $\boldsymbol{a}$ is equivalent to the estimation of $\mathbf{a}$.

The motivation of using (6) as a basis for the detection procedure lies in the almost block-banded structure of the permuted channel matrix $\boldsymbol{\Lambda}$. Indeed, for SISO-OFDM systems subject to doubly selective channels, it is widely acknowledged that the channel matrix is almost banded [7]-[9], which means that most of the ICI comes from the nearest subcarriers. As a result, the almost-banded structure of $\underline{\mathbf{\Lambda}}_{j,i}$ implies that $\boldsymbol{\Lambda}$ is almost block-banded. This effect can be observed in Fig. 1, which shows the log-magnitude of the elements of $\boldsymbol{\Lambda}$ for a typical channel realization. It is clear that the most significant elements of $\boldsymbol{\Lambda}$ are those around its main block-diagonal.

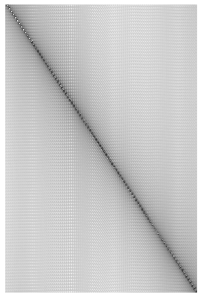

Figure 1 - Log-magnitude of $\boldsymbol{\Lambda}$ ($M_\mathrm{R} = 3$, $M_\mathrm{T} = 2$, $N_\mathrm{A} = 96$).

Hence, we can approximate the channel matrix $\boldsymbol{\Lambda}$ by its banded version, expressed by

$$\boldsymbol{B}_{(Q)} = \boldsymbol{\Lambda} \circ \boldsymbol{\Theta}_{(Q)} , \tag{8}$$

where $\boldsymbol{\Theta}_{(Q)} = \mathbf{\Theta}_{(Q)} \otimes \mathbf{1}_{M_\mathrm{R}\times M_\mathrm{T}}$, and $\mathbf{\Theta}_{(Q)}$ is the $N_\mathrm{A} \times N_\mathrm{A}$ Toeplitz matrix defined as $[\mathbf{\Theta}_{(Q)}]_{m,n} = 1$ for $|m-n| \le Q$ and $[\mathbf{\Theta}_{(Q)}]_{m,n} = 0$ for $|m-n| > Q$. The parameter Q , which controls the width of the block-band, can be chosen as in SISO-OFDM, according to the rule of thumb $Q > \lfloor f_\mathrm{D}/\Delta_f \rfloor$, where f_D is the maximum Doppler frequency and Δ_f is the subcarrier spacing. This leads to very small values of Q , e.g., $1 \le Q \le 5$. We will show that this parameter can be used in the equalizers to trade off performance for complexity.

As a consequence of (8), Eq. (6) can be rewritten as

$$\boldsymbol{z} = \boldsymbol{B}_{(Q)}\boldsymbol{a} + \boldsymbol{E}_{(Q)}\boldsymbol{a} + \boldsymbol{n} , \tag{9}$$

where $\boldsymbol{E}_{(Q)} = \boldsymbol{\Lambda} - \boldsymbol{B}_{(Q)}$. Although $\boldsymbol{E}_{(Q)}\boldsymbol{a}$ could be incorporated into the noise term [12], in this paper we neglect it, and use the approximated model

$$\boldsymbol{z} = \boldsymbol{B}_{(Q)}\boldsymbol{a} + \boldsymbol{n} , \tag{10}$$

which allows the design of low-complexity equalizers herein called *banded equalizers*.

3. BANDED EQUALIZERS

As far as channel state information (CSI) is concerned, we assume that the receiver is aware of the channel matrix. In practice, the channel matrix has to be estimated, e.g., by using the techniques proposed in [14]-[16]. Moreover, we assume that the fading statistics (i.e., Doppler spectrum shape) are known to the receiver. Indeed, these statistics are used to design suitable receiver windows. Equalizers that do not adopt receiver windowing only require a coarse estimate (or an upper bound) of the maximum Doppler frequency f_D in order to guide the choice of the bandwidth parameter Q .

3.1 Linear Equalizers

Linear equalization can be expressed by

$$\tilde{\boldsymbol{a}} = \boldsymbol{G}\boldsymbol{z} , \tag{11}$$

where $\boldsymbol{G}$ is $M_\mathrm{T}N_\mathrm{A} \times M_\mathrm{R}N_\mathrm{A}$. For the banded MMSE-BLE, $\boldsymbol{G}$ can be calculated by

$$\boldsymbol{G} = \boldsymbol{B}_{(Q)}^H(\boldsymbol{B}_{(Q)}\boldsymbol{B}_{(Q)}^H + \mathbf{C}_{\boldsymbol{nn}})^{-1} , \tag{12}$$

or by

$$\boldsymbol{G} = (\mathbf{I}_{M_\mathrm{T}N_\mathrm{A}} + \boldsymbol{B}_{(Q)}^H\mathbf{C}_{\boldsymbol{nn}}^{-1}\boldsymbol{B}_{(Q)})^{-1}\boldsymbol{B}_{(Q)}^H\mathbf{C}_{\boldsymbol{nn}}^{-1} , \tag{13}$$

where

$$\mathbf{C}_{nn} = \boldsymbol{R}_{\text{GB}}\mathbf{P}_{(M_{\text{R}},N)}\mathbf{C}_{\underline{\mathbf{nn}}}\mathbf{P}^{T}_{(M_{\text{R}},N)}\boldsymbol{R}^{T}_{\text{GB}} \quad (14)$$

$$= \boldsymbol{R}_{\text{GB}}\mathbf{P}_{(M_{\text{R}},N)}(\mathbf{I}_{M_{\text{R}}} \otimes (\sigma_v^2\mathbf{W}\mathbf{W}^H))\mathbf{P}^{T}_{(M_{\text{R}},N)}\boldsymbol{R}^{T}_{\text{GB}} \quad (15)$$

$$= \boldsymbol{R}_{\text{GB}}((\sigma_v^2\mathbf{W}\mathbf{W}^H) \otimes \mathbf{I}_{M_{\text{R}}})\boldsymbol{R}^{T}_{\text{GB}} = (\sigma_v^2\boldsymbol{R}_{\text{GB}}\mathbf{W}\mathbf{W}^H\boldsymbol{R}^{T}_{\text{GB}}) \otimes \mathbf{I}_{M_{\text{R}}} . \quad (16)$$

In case of no windowing, $\mathbf{C}_{nn} = \sigma_v^2\mathbf{I}_{M_{\text{R}}N_{\text{A}}}$, and hence, by (11) and (12), we obtain

$$\tilde{\boldsymbol{a}} = \boldsymbol{B}^{H}_{(Q)}(\boldsymbol{B}_{(Q)}\boldsymbol{B}^{H}_{(Q)} + \sigma_v^2\mathbf{I}_{M_{\text{R}}N_{\text{A}}})^{-1}\boldsymbol{z} . \quad (17)$$

Since $\boldsymbol{B}_{(Q)}$ is block-banded, $\boldsymbol{B}_{(Q)}\boldsymbol{B}^{H}_{(Q)}$ is block-banded, and hence is also banded, with bandwidth parameter $Q_1 = (2Q+1)M_{\text{R}} - 1$. As a result, the computation of $\tilde{\boldsymbol{a}}$ in (17) can be performed by means of the band LDL factorization algorithm described in [10]. The computational complexity of this approach is caused mainly by the computation of $\boldsymbol{B}_{(Q)}\boldsymbol{B}^{H}_{(Q)}$, which is $O(Q^2M_{\text{T}}M_{\text{R}}^2N_{\text{A}})$, and by the band LDL factorization algorithm, which is $O(Q_1^2M_{\text{R}}N_{\text{A}})$. To enable linear equalization, $M_{\text{R}} \geq M_{\text{T}}$, and consequently the complexity is $O(Q^2M_{\text{R}}^3N_{\text{A}})$, i.e., linear in the number of subcarriers. For the sake of comparison, the complexity of the full (i.e., non-banded) linear MMSE proposed in [14] is cubic in the number of subcarriers. Hence, the proposed approach permits a significant computational complexity saving, since usually M_{R} and Q are small integers, whereas N_{A} can be very high (even some hundreds or thousands).

The complexity of the banded MMSE-BLE can be reduced by using (13) instead of (12), thus obtaining

$$\tilde{\boldsymbol{a}} = (\sigma_v^2\mathbf{I}_{M_{\text{T}}N_{\text{A}}} + \boldsymbol{B}^{H}_{(Q)}\boldsymbol{B}_{(Q)})^{-1}\boldsymbol{B}^{H}_{(Q)}\boldsymbol{z} . \quad (18)$$

In this second case, the band LDL factorization requires $O(Q^2M_{\text{T}}^3N_{\text{A}})$ flops, and therefore the computational complexity is dominated by the computation of $\boldsymbol{B}^{H}_{(Q)}\boldsymbol{B}_{(Q)}$, which is of order $O(Q^2M_{\text{T}}^2M_{\text{R}}N_{\text{A}})$. Hence, this solution is preferable, because (18) is less complex and mathematically equivalent to (17).

We observe that, since the channel matrix is block-banded, instead of applying a band LDL factorization we could apply a band version of the block LDL factorization algorithm of [14]. This could further reduce complexity. Anyway, for small values of M_{R} and M_{T}, the block size is small, and hence, by considering the matrix to be inverted as banded instead of block-banded, only few additional zeros are included, especially for high Q's.

In case of windowing, the noise covariance $\mathbf{C}_{nn}$ is no longer a scaled identity matrix. Hence, to exploit the low-complexity advantages given by the band LDL factorization, the window $\mathbf{w}$ should be designed in such a way that $\mathbf{C}_{nn}$ is banded. In this view, we consider only sum-of-exponentials (SOE) windows [12], which can be expressed by

$$[\mathbf{w}]_n = \sum_{q=-Q}^{Q} b_q \exp(j2\pi qn/N) . \quad (19)$$

In fact, the use of SOE windows guarantees that the matrix $\mathbf{W}\mathbf{W}^H$ in (16) is perfectly banded. As a result, $\mathbf{C}_{nn}$ is exactly banded, with bandwidth parameter $Q_2 = 2QM_{\text{R}}$. It is worth noting that the class of SOE windows is quite large, and contains many common windows such as Hamming, Blackman and Hann. However, among this class, we are interested in those windows that are able to make the channel matrices $\{\underline{\boldsymbol{\Lambda}}_{j,i}\}$ "more banded." Consequently, we impose the minimum band approximation error (MBAE) criterion [13], where the coefficients $\{b_q\}$ are designed in order to minimize the Frobenius norm of the out-of-band elements of $\underline{\boldsymbol{\Lambda}}_{j,i}$. Since the channel matrices $\{\underline{\boldsymbol{\Lambda}}_{j,i}\}$ are assumed to be uncorrelated, the MBAE-SOE window acts separately on each $\underline{\boldsymbol{\Lambda}}_{j,i}$, and hence $\boldsymbol{\mathit{\Lambda}}$ results "more block-banded" than without windowing. As a consequence, the band approximation error is lower and gives rise to improved performance.

We want to point out that with MBAE-SOE windowing $(\mathbf{W}\mathbf{W}^H)^{-1}$ is not banded, and hence also $\mathbf{C}^{-1}_{nn}$ in (13) is not banded. Consequently, the use of (13) instead of (12) does not reduce complexity. As a result, the complexity of the windowed BLE (W-BLE) is $O(Q^2M_{\text{R}}^3N_{\text{A}})$, that is, higher than the unwindowed BLE.

3.2 Decision-Feedback Equalizers

To design banded BDFEs, we adopt the MMSE approach of [17] and [18]. This approach aims to minimize the quantity $MSE = \text{tr}(\mathbf{R}_{ee})$, where $\mathbf{R}_{\mathbf{xy}} = E\{\mathbf{x}\mathbf{y}^H\}$, $\boldsymbol{e} = \tilde{\boldsymbol{a}} - \boldsymbol{a}$, and $\tilde{\boldsymbol{a}}$ is the soft data estimate at the input of the hard decision device. By denoting with $\boldsymbol{F}_{\text{F}}$ the $M_{\text{T}}N_{\text{A}} \times M_{\text{R}}N_{\text{A}}$ feedforward filter and with $\boldsymbol{F}_{\text{B}}$ the $M_{\text{T}}N_{\text{A}} \times M_{\text{T}}N_{\text{A}}$ feedback filter, the soft data estimate $\tilde{\boldsymbol{a}}$ can be expressed as

$$\tilde{\boldsymbol{a}} = \boldsymbol{F}_{\text{F}}\boldsymbol{z} - \boldsymbol{F}_{\text{B}}\hat{\boldsymbol{a}} , \quad (20)$$

where $\hat{\boldsymbol{a}}$ is the hard data estimate. We make the following standard assumptions:

- The feedback filter $\boldsymbol{F}_{\text{B}}$ is strictly upper triangular, so that the feedback process can be performed by successive cancellation;
- Past decisions are correct, i.e., $\hat{\mathbf{a}} = \mathbf{a}$.

By the orthogonality principle, i.e., $\mathbf{R}_{ez} = \mathbf{0}$, we obtain

$$\boldsymbol{F}_{\text{F}} = (\boldsymbol{F}_{\text{B}} + \mathbf{I}_{M_{\text{T}}N_{\text{A}}})\boldsymbol{G} . \quad (21)$$

This result points out that the feedforward filter is the cascade of the banded BLE, expressed by (12) or (13), and an upper triangular matrix with unit diagonal. To design $\boldsymbol{F}_{\text{B}}$, we use the procedure adopted in [13]. By the matrix inversion lemma

$$\mathbf{R}_{ee} = (\boldsymbol{F}_{\text{B}} + \mathbf{I}_{M_{\text{T}}N_{\text{A}}})\mathbf{M}^{-1}(\boldsymbol{F}_{\text{B}} + \mathbf{I}_{M_{\text{T}}N_{\text{A}}})^H , \quad (22)$$

where

$$\mathbf{M} = \mathbf{I}_{M_{\text{T}}N_{\text{A}}} + \boldsymbol{B}^{H}_{(Q)}\mathbf{C}^{-1}_{nn}\boldsymbol{B}_{(Q)} . \quad (23)$$

If there is no windowing, $\mathbf{M}$ in (23) is block-banded, and hence $\text{tr}(\mathbf{R}_{ee})$ can be minimized by using the band LDL factorization of $\mathbf{M}$, expressed by $\mathbf{M} = \mathbf{L}\mathbf{D}\mathbf{L}^H$, and setting

$$\boldsymbol{F}_{\text{B}} = \mathbf{L}^H - \mathbf{I}_{M_{\text{T}}N_{\text{A}}} . \quad (24)$$

Interestingly, the same band LDL factorization can be used to obtain the feedback filter $\boldsymbol{F}_{\text{B}}$ and the feedforward filter $\boldsymbol{F}_{\text{F}}$, provided that the expression (13) is used for $\boldsymbol{G}$ in (21). As a result, it can be shown that the design of the banded BDFE requires the same complexity as that of the banded BLE (see [13] for details).

In case of windowing, $\mathbf{M}$ in (23) is not banded. However, we can look for reasonable banded approximations, in order to enable band LDL factorization and hence to reduce complexity. Let us consider the matrix $\boldsymbol{K} = \boldsymbol{\mathit{\Lambda}}^H\mathbf{C}^{-1}_{nn}\boldsymbol{\mathit{\Lambda}}$, which can be considered as the non-banded version of $\mathbf{M} - \mathbf{I}_{M_{\text{T}}N_{\text{A}}}$ in (23). The matrix $\boldsymbol{K}$ can be approximated as

$$\boldsymbol{K} = \boldsymbol{\mathit{\Lambda}}^H\mathbf{C}^{-1}_{nn}\boldsymbol{\mathit{\Lambda}} \approx \boldsymbol{R}_{\text{GB}}\underline{\boldsymbol{\mathit{\Lambda}}}^H\mathbf{C}^{-1}_{\underline{nn}}\underline{\boldsymbol{\mathit{\Lambda}}}\boldsymbol{R}^{T}_{\text{GB}} , \quad (25)$$

i.e., the guard bands are removed after multiplication. We have

$$\boldsymbol{R}_{\text{GB}}\underline{\boldsymbol{\mathit{\Lambda}}}^H\mathbf{C}^{-1}_{\underline{nn}}\underline{\boldsymbol{\mathit{\Lambda}}}\boldsymbol{R}^{T}_{\text{GB}} = \boldsymbol{R}_{\text{GB}}\mathbf{P}_{(M_{\text{T}},N)}\underline{\boldsymbol{\Lambda}}^H\mathbf{C}^{-1}_{\underline{\mathbf{nn}}}\underline{\boldsymbol{\Lambda}}\mathbf{P}^{T}_{(M_{\text{T}},N)}\boldsymbol{R}^{T}_{\text{GB}} , \quad (26)$$

where $\underline{\mathbf{K}} = \underline{\boldsymbol{\Lambda}}^H\mathbf{C}^{-1}_{\underline{\mathbf{nn}}}\underline{\boldsymbol{\Lambda}}$ is an $M_{\text{T}} \times M_{\text{T}}$ block-matrix (each block is $N \times N$) whose (*j,i*)th block is expressed by

$$[\underline{\mathbf{K}}]^{(j,i)} = [\underline{\boldsymbol{\Lambda}}^H_{1,j} \cdots \underline{\boldsymbol{\Lambda}}^H_{M_{\text{R}},j}](\mathbf{I}_{M_{\text{R}}} \otimes (\sigma_v^2\mathbf{W}\mathbf{W}^H)^{-1})[\underline{\boldsymbol{\Lambda}}^T_{1,i} \cdots \underline{\boldsymbol{\Lambda}}^T_{M_{\text{R}},i}]^T , \quad (27)$$

which can be rewritten as

$$[\underline{\mathbf{K}}]^{(j,i)} = \sum_{l=1}^{M_{\text{R}}} \underline{\boldsymbol{\Lambda}}^H_{l,j}(\sigma_v^2\mathbf{W}\mathbf{W}^H)^{-1}\underline{\boldsymbol{\Lambda}}_{l,i} . \quad (28)$$

Each term inside the summation in (28) can be expressed by

$$\underline{\mathbf{\Delta}}_{l,j}^{H}(\sigma_v^2 \mathbf{W}\mathbf{W}^H)^{-1}\underline{\mathbf{\Delta}}_{l,i} = \sigma_v^{-2}(\mathbf{F}\underline{\mathbf{H}}_{l,j}^{H}\mathbf{\Delta}^H\mathbf{F}^H)(\mathbf{F}\mathbf{\Delta}\mathbf{\Delta}^H\mathbf{F}^H)^{-1}(\mathbf{F}\mathbf{\Delta}\underline{\mathbf{H}}_{l,i}\mathbf{F}^H) \quad (29)$$

$$= \sigma_v^{-2}\mathbf{F}\underline{\mathbf{H}}_{l,j}^{H}\mathbf{F}^H\mathbf{F}\underline{\mathbf{H}}_{l,i}\mathbf{F}^H = \sigma_v^{-2}\underline{\mathbf{\Gamma}}_{l,j}^{H}\underline{\mathbf{\Gamma}}_{l,i} \quad (30)$$

where $\underline{\mathbf{\Gamma}}_{j,i} = \mathbf{F}\underline{\mathbf{H}}_{j,i}\mathbf{F}^H$ represents the $N \times N$ unwindowed (*j*,*i*)th channel matrix, i.e., $\underline{\mathbf{\Gamma}}_{j,i} = \underline{\mathbf{\Delta}}_{j,i}$ when $\mathbf{\Delta} = \mathbf{I}_N$. Since each $\underline{\mathbf{\Gamma}}_{j,i}$ in (30) has a banded structure, we can approximate $\underline{\mathbf{\Gamma}}_{j,i}$ with its banded version $\underline{\mathbf{\Gamma}}_{j,i} \circ \underline{\mathbf{\Theta}}_{(Q)}$. Consequently, also the matrix expressed by (26) has a banded structure. As a result, we can apply the band LDL factorization also to design the feedback filter. Summarizing, even in the presence of windowing the computational complexity of the BDFE is linear in the number of subcarriers. However, two different LDL factorizations are required, and therefore the complexity of the windowed BDFE (W-BDFE) is roughly doubled with respect to the unwindowed BDFE.

We want to highlight that several different banded BDFEs can be designed depending on the order in which successive cancellation is performed. Indeed, we can change the antenna ordering without destroying the block-banded structure of Λ, provided that the subcarrier ordering is maintained. Since the antenna ordering could be different from subcarrier to subcarrier, the number of possible BDFEs is $2(M_R!)^{N_A}$. However, in this paper we do not consider any ordering scheme.

4. SIMULATION RESULTS

In this section we compare by simulations the BER performance of the proposed techniques. We consider a MIMO-OFDM system with $N = 128$, $N_A = 96$, $L = 8$, and QPSK modulation. We assume that the $M_T M_R$ channels are independent. The power delay profile of the channel follows a truncated exponential distribution with rms delay spread equal to $\sigma = 3$. Each channel path is characterized by Rayleigh fading and has a Jakes' Doppler spectrum.

Fig. 2 compares the BER performance of the banded equalizers with the full MMSE-BLE proposed in [14] and with the full MMSE-BDFE, when $M_T = 2$, $M_R = 3$, and $f_D / \Delta_f = 0.15$. It is clear that the banded BDFE outperforms the banded BLE, although they have the same computational cost. Moreover, MBAE-SOE windowing is more beneficial than decision-feedback in reducing the error floor. In addition, the combination of windowing and BDFE further improves the BER performance, at a double cost with respect to the W-BLE.

Fig. 3 illustrates the performance of the banded W-BLE as a function of the bandwidth parameter Q, when $M_T = 2$, $M_R = 3$, and $f_D / \Delta_f = 0.15$. The performance improvement for higher values of Q, which is due to the lower band approximation error, is accompanied by an increased complexity (quadratic in Q).

Fig. 4 and Fig. 5 show the performance of W-BLE and W-BDFE for different numbers of transmit and receive antennas (M_T, M_R). It is possible to observe that both W-BLE and W-BDFE greatly benefit of the spatial diversity offered by employing more receive than transmit antennas. For equally-balanced transmit and receive antennas, Figs. 4-5 show that the MIMO-OFDM system can achieve the multiplexing gain M_T, while guaranteeing almost the same BER performance of the corresponding SISO-OFDM system. Specifically, when $M_R > M_T$, the proposed equalizers are able to collect the diversity gain and reduce the BER floor caused by the presence of the ICI.

Finally, Fig. 6 exhibits the performance of the W-BLE for different values of the Doppler spread when $M_T = 2$, $M_R = 3$, and $Q = 2$. As expected, higher Doppler spreads give a slight BER improvement at low SNR, due to the higher temporal diversity offered by the channel, and a higher BER floor at high SNR, due to the higher band approximation error.

5. CONCLUSIONS

We have proposed MMSE-BLE and MMSE-BDFE banded equalizers for MIMO-OFDM systems in time-varying frequency-selective channels. Thanks to a band LDL factorization algorithm and a time-domain windowing approach, these equalizers have a small computational complexity (linear in the number of subcarriers) and do not significantly sacrifice performance with respect to more complex equalizers. It is also shown that a MIMO-OFDM system equipped with TV equalizers can benefit from the multiplexing gain also in rapidly TV channels, by using a number of receive antennas equal to the number of transmit antennas. Moreover, just an extra receive antennas can guarantee significant BER performance improvement and resistance to TV scenarios with respect to SISO-OFDM.

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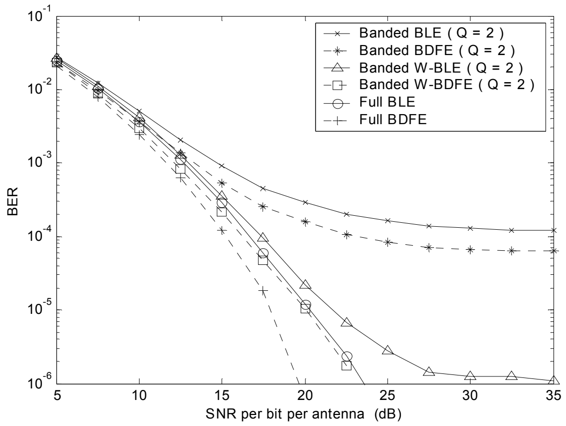

Figure 2 - BER of different equalizers ($M_T = 2$, $M_R = 3$, $f_D / \Delta_f = 0.15$).

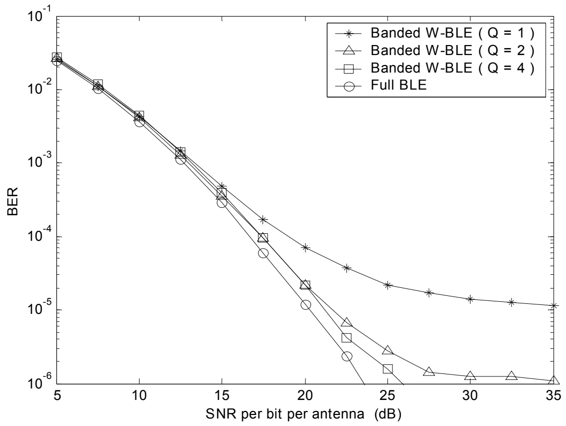

Figure 3 - Effect of the bandwidth parameter Q ($M_T = 2$, $M_R = 3$, $f_D / \Delta_f = 0.15$).

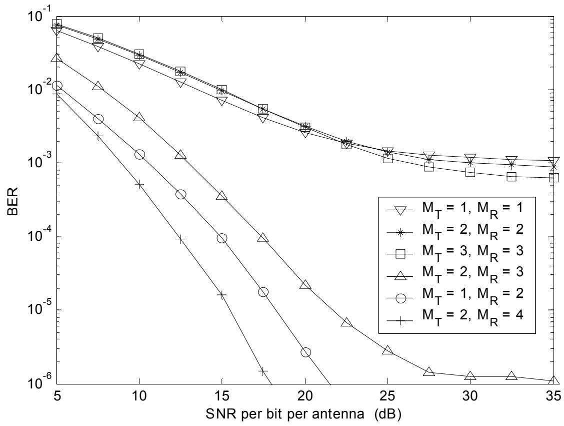

Figure 4 - BER of the banded W-BLE as a function of the number of antennas ($Q = 2$, $f_D / \Delta_f = 0.15$).

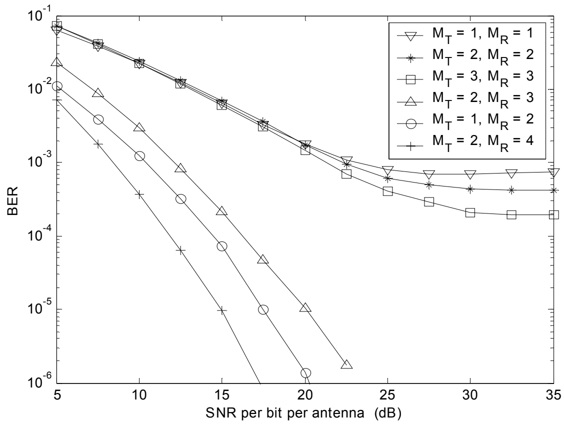

Figure 5 - BER of the banded W-BDFE as a function of the number of antennas ($Q = 2$, $f_D / \Delta_f = 0.15$).

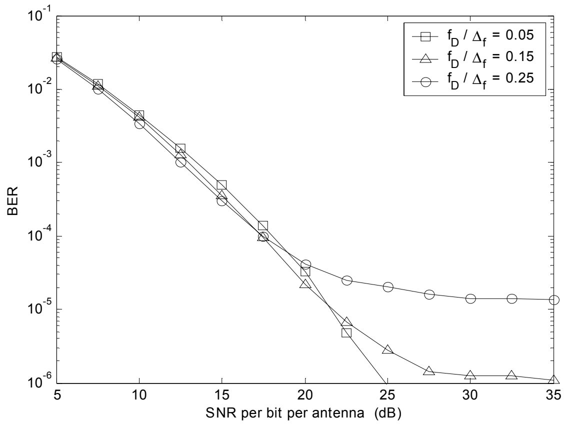

Figure 6 - BER of the banded W-BLE as a function of the Doppler spread ($Q = 2$, $M_T = 2$, $M_R = 3$).