

# ERROR SENSITIVITY IN ADAPTIVE PREDISTORTION SYSTEMS

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**ABSTRACT** Digital baseband predistortion is a well known technique to compensate for AM/AM and AM/PM amplifier non linearity. The practical implementation of a flexible digital predistorter suggests introducing some adaptation strategy to the predistortion technique in order to take count of the changes in the non linear characteristics due to ageing, temperature or channel switching of the amplifier. The most effective adaptation criterions are generally based on the estimation of the residual non linear distortion at the output of the predistorted amplifier. A simple criterion is to compare the amplifier output with the predistorter input. This comparison requires an estimation of the input-output time delay of the amplification chain. Aim of this work is to analyse the sensitivity of the adaptation algorithm to errors in the time delay estimation and to noise sources like quantization and inter symbol interference.

## I. Introduction

Bandpass non-linearity like the one of high power amplifiers can be compensated by several techniques like negative feedback, LINC, feed-forward amplification and signal predistortion. The great improvement in the last decade in digital technologies has produced a renewed interest for their digital implementation that guarantees a sharper correction capability. In this field digital baseband predistortion is a technique based on the inversion of the AM/AM and AM/PM distortion curves that characterise a bandpass non linear amplifier [1]. In practical situations the AM/AM and AM/PM distortion curves can change depending on the environmental variations like temperature, biasing voltages, ageing, channel switch and so on. By this reason the predistortion curves that invert the ones of the amplifier must contemplate some adaptation strategy. A simple kind of adaptation can be realised by the monitoring of some characteristic parameters that influence the AM/AM and AM/PM curves. In this way it is possible to guarantee a gross adaptation of the system in each situation by choosing the best predistortion curves from a curve set previously defined as function of the monitored parameters [2]. This approach introduced in the past for analogic predistorter, can be easily extended to digital predistorter, but clearly does not guarantee the best predistortion in each situation because of the limited number of parameters and predistortion curves that can be practically considered. The first effective adaptation system was introduced for a data predistorter [3] and it was easily extended to signal

predistortion [4], [5], [6], [7]. All the systems considered in these papers store the values of the predistortion curves in some look up tables. The adaptation consists in the adjusting of each stored value by the estimation of an input-output error in the amplification chain. The referred papers analyse the convergence property of the algorithm in different situation and predistortion architecture, but all of them make the assumption to correctly compare the input with the output of the amplification chain by means of a perfect synchronisation. In this paper the sensitiveness of the adaptation algorithm to input-output synchronisation will be investigated as well as the sensitiveness to signal quantization and filtering. The basics concept of predistortion is outlined in the next session. A possible predistortion architecture is introduced in session II. The adaptation algorithm and its implementation will be discussed in session III. The simulation results are shown in session IV

## II. Baseband Predistortion

Non-linear power amplifiers are usually modelled as bandpass non linearity [1]. If the input signal  $s(t)$  is expressed by complex base-band representation as

$$s(t) = R(t) \cdot e^{j\theta(t)} \quad (1)$$

the non linear distorted output  $s_d(t)$  is represented by

$$s_d(t) = H[R(t)] \cdot e^{j[\theta(t) + \Phi[R(t)]]} \quad (2)$$

where  $H[R]$  and  $\Phi[R]$  are known as the AM/AM and the AM/PM distortion curves respectively. Baseband predistortion compensates the non linear distortions introduced by RF power amplifiers [1][3][4][5] by two curves  $A[R]$  and  $\Psi[R]$  that globally invert  $H[R]$  and  $\Phi[R]$ . The predistorted amplifier is characterised by a completely cancelled AM/PM and by a residual AM/AM that works like a soft-limiter because of the maximum output power of the amplifier. This paper adopts the Saleh model [8] of non linear amplifiers with the AM/AM and AM/PM distortion curves represented in Fig.1 and expressed by (3).

$$\begin{aligned} H[R] &= \frac{2R}{1+R^2} \\ \Phi[R] &= \frac{\pi}{6} \frac{R^2}{1+R^2} \end{aligned} \quad (3)$$

A digital signal predistorter does not operate on the data at

the output of a symbol generator, but on the digital interpolated baseband signal as shown in Fig.2.

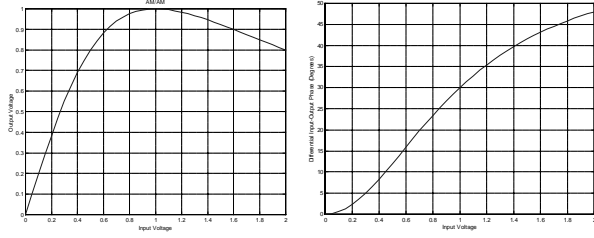


Fig.1 AM/AM and AM/PM distortion curves of the RF power amplifier

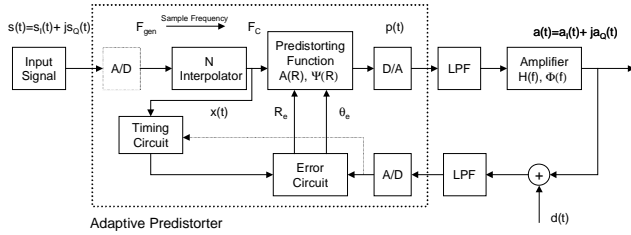


Fig.2 Adaptive Predistorted Amplifier (Baseband model)

The predistorter uses a look-up table to store the predistorting curves  $A[R]$  and  $\Psi[R]$  as shown in Fig.3 where the predistortion action is realised in polar coordinates, by two different implementations. The first one stores the predistorted envelope in the look-up table while the other stores the predistorted gain. Both of them also store the predistorted phase.

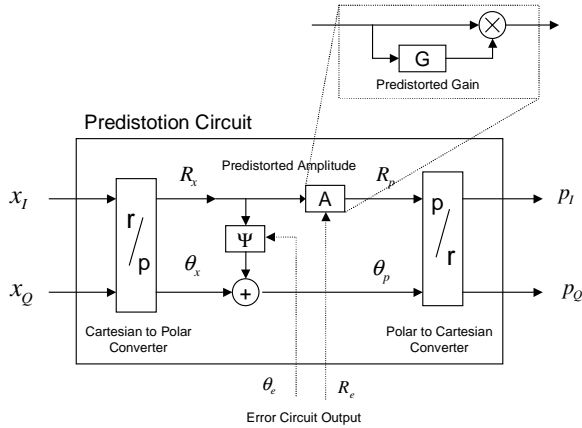


Fig.3 Polar realisation (Envelope or Gain) for the baseband predistortion subsystem

Other systems can realise the same predistortion strategy working in cartesian coordinates and making use of the complex gain representation of the AM/AM and AM/PM expressed by (4) where  $E[R]$  and  $G[R]$  respectively represent the predistorted envelope and the predistorted gain.

$$G_I[R] = \frac{E[R]}{R} \cos \Psi[R] = G[R] \cdot \cos \Psi[R] \quad (4)$$

$$G_Q[R] = \frac{E[R]}{R} \sin \Psi[R] = G[R] \cdot \sin \Psi[R]$$

### III. Adaptive Predistortion

Fig.2 represents the general architecture of a baseband adaptive predistortion scheme, where the Timing Circuit and the Error Circuit are outlined. The Error Circuit has the role to estimate the input-output non-linear error of the predistorted system and provide an error signal to the Predistortion Circuit to update the contents of its lookup tables. The adaptation algorithm can be implemented both in polar or cartesian co-ordinates. The polar realisation is more intuitive because it is directly related to the AM/AM and AM/PM distortions introduced by the HPA. Moreover it was shown [3] that the algorithm convergence is guaranteed in polar co-ordinates while in cartesian is not. The better performance of the polar realisation was also outlined in [7]. As a consequence the polar implementation is considered in the following. The envelope and phase of the  $s(t)$  input are subtracted from the correspondent components of the  $a(t)$  amplifier output in order to estimate the input output error for the predistorted amplifier. The two error signals obtained in this way are weighted by two real coefficients  $\alpha_R$  and  $\alpha_\theta$  to provide the correcting terms for the values that are stored in the predistortion tables of Fig.3. The two error signals are analytically expressed by

$$\begin{aligned} R_e &= \alpha_R \cdot [R_a - K \cdot R_x] \\ \theta_e &= \alpha_\theta \cdot [\theta_a - \vartheta_x] \end{aligned} \quad (5)$$

where  $K$  represents the desired linear gain for the predistorted amplifier.

The  $R_x$  input envelope addresses each value in the Predistortion Tables. The values stored in the table at the  $n$ -th step of the adaptation algorithm are expressed by (6)

$$\begin{aligned} G[R_x]_{n+1} &= G[R_x]_n - \alpha_{R,n} \{ H[G[R_x]_n] - kR_x \} \\ \Psi[R_x]_{n+1} &= \Psi[R_x]_n - \alpha_{\theta,n} \{ \Phi[G[R_x]_n] - \theta_x \} \end{aligned} \quad (6)$$

The general expression of the adaptation algorithm for each value of the AM/AM and AM/PM compensation curve can be expressed by (7)

$$f_{n+1} = f_n - \alpha_n \cdot \{ A[f_n] - k_o \} \quad (7)$$

where

$$A(f_o) - k_o = 0 \quad (8)$$

is the equation to be solved to realise predistortion, and  $f_o = f_o(R_x)$  is the correct value to be stored in the predistortion tables for all the possible  $R_x$  input levels.

The algorithm convergence is assured only if the derivative

of the non linear function  $A[\cdot]$  is different from zero [3]. Under this hypothesis (9) is a general expression for the adaptation coefficients, where  $\alpha_0$  is a real positive constant and  $\eta \in [0,1]$  to guarantee the convergence.

$$\alpha_n = \frac{\alpha_0}{n^\eta} \quad (9)$$

The magnitude  $\alpha_0$  of the adaptation coefficients has to be chosen as a compromise between adaptation speed and noise rejection. If a noise contribution or a measurement error  $d(t)$  affects the non linear output, the recursive equation (7) becomes (10).

$$f_{n+1} = f_n - \alpha_n \cdot \{A[f_n] - k_o + d_n\} \quad (10)$$

$e_n$  represents the error between the real solution of equation (8) and the value assumed at the  $n$ -th step of the recursive algorithm as expressed by (11).

$$e_n = f_n - f_o \quad (11)$$

It was shown [3] that its mean value  $E\{e_n\}$  converges to zero as 'n' approaches infinity with a convergence speed that depends on the 'c' parameter expressed in equation (12)

$$c = \alpha_o A'(f_o) \quad (12)$$

The  $(\sigma_e^2)_{n+1}$  error variance clearly depends on the  $\sigma_d^2$  noise variance but also on the  $\alpha_o$ ,  $\eta$ , and  $A[\cdot]$  parameters [3].

The fixed step size coefficient ( $\eta = 0$ ) gives the best convergence speed, even if is more sensitive to noise contribution in the feedback path [3]. For this particular case the following relation expresses indeed the error convergence

$$\bar{e}_{n+1} = E\{e_{n+1}\} = E\{e_1\} \cdot (1-c)^n < \bar{e}_1 \cdot \exp(-cn) \quad (13)$$

where  $E\{\cdot\}$  is the expected value operator and 'c' must be in the [0,2] range. The variance  $(\sigma_e^2)_{n+1}$  is finite even for high values of the iteration number of the recursive algorithm and it is directly proportional to the noise contribution variance  $\sigma_d^2$  and is expressed by

$$(\sigma_e^2)_{n+1} = E\{(e_{n+1} - \bar{e}_{n+1})^2\} = \frac{c}{2-c} [1 - (1-c)^{2n}] \frac{\sigma_d^2}{[A'(f_o)]^2} \xrightarrow{n \rightarrow \infty} \frac{c}{2-c} \cdot \frac{\sigma_d^2}{[A'(f_o)]^2}$$

This sensitivity to disturbance in the recursive algorithm is clearly exacerbated for points of the non linear function  $A[\cdot]$  with a derivative function that approaches zero. If a non uniform step algorithm is employed the error variance  $(\sigma_e^2)_{n+1}$  converges to zero for large values of  $n$  but with a lower convergence speed [3].

The previous analysis takes count of uncorrelated noise contribution in the demodulation of the amplifier output. However other sources of disturbance to the instantaneous algorithm can be considered. One source is represented by the quantization errors introduced by the A/D and D/A converters in the adaptive digital predistorter. Using a sufficient number of bits in the digital devices can reduce this error source. Another error source is the memory introduced in the system by the filtering stages between the baseband predistorting device and the RF amplifier. These filters produce a sort of inter-symbol interference (ISI) between consecutive values at the Predistorter output and at the Amplifier output. It means that the input to the amplifier at a certain instant 'n', for each value of the predistortion table is not exactly the one that is stored in the predistortion table itself. Indeed, it is corrupted by the interference of the other values previously emitted by the predistorting device and stored in different positions of the predistortion table. In the same way also the amplifier output used to estimate the convergence error is affected by an inter-symbol interference.

Finally the Timing Circuit can introduce a great error source. The Timing Circuit has the role to estimate the loop delay of the adaptive system and compensate it, in order to allow the Error Circuit to correctly compare the Predistortion Circuit Input with the amplifier Output.

The effects of all these contributions to the algorithm convergence will be analysed by simulation in the next session.

#### IV. Simulation Results

The adaptively predistorted amplifier shown in Fig.2 was simulated for several values of the adaptation coefficients  $\alpha_R$  and  $\alpha_\theta$ , with and without perturbation to the recursive non linear algorithms expressed by (6).

The digital to analogue conversion at the output of the predistorting device was taken in count by a signal interpolation that makes the  $F_s$  amplifier simulation frequency,  $N_s$  times greater than the predistorter sample frequency  $F_c$  ( $N_s \gg 1$ ). The  $F_c$  sample frequency was also considered  $N_c$  time ( $N_c \gg 4$ ) greater than the generation frequency  $F_{gen}$  of the input signal in order to correctly represent it in the non linear environment, because of spectral regrowth. The input signal was modelled like a complex random signal with uniform envelope (and phase) distributions so that the convergence speed of each value of the predistortion tables is not influenced by its probability of occurrence. It is possible in this way to concentrate the attention on how the non linear AM/AM and AM/PM shapes influence the convergence speed and precision. If it is not the case like for OFDM signals characterised by a Rayleigh envelope distribution a possible approach is to work with non uniformly addressed look up tables.

The convergence behaviour will be analysed by two parameters. The first, shown in the upper side of Fig.4-5-6, is the Root Mean Square (RMS) Input-Output Error for both envelope and phase, and represent a parameter to estimate the global convergence of the algorithm. The second, shown in the lower side of Fig.4-5-6, is the relative error of the found predistorting solution on respect to the ideal one. It represents a parameter to estimate the punctual convergence of the algorithm for each value of the predistortion tables and is defined as expressed by (14). The input signal bandwidth was imposed to be  $F_{\text{gen}}/2$ .

$$(e_r)_n = \frac{f_n - f_0}{f_0} \quad (14)$$

The results shown in the upper sides of Fig.4 outline how the convergence speed is proportional to the magnitude of the adaptation coefficients  $\alpha_R$  and  $\alpha_\theta$ , as expected from theory. The lower side of the figure shows the relative errors for both AM/AM and AM/PM, when the adaptation time reaches the value corresponding to the last right end point of the upper figures. All the three curves show a good convergence for the intermediate points, for both the AM/AM and AM/PM. Indeed, the derivative function of the non linear distortions introduced by the amplifier does not approach zero in that region. On the contrary the amplifier AM/AM exhibits a quasi zero derivative function for input envelope that approach one, while the AM/PM derivative is very close to zero for input envelope that approaches zero. In this regions the convergence speed is highly lower. However without any kind of disturb (i.e. quantization noise, ISI, input-output synchronisation errors) to the recursive equation the algorithm should converge to a zero RMS error. This is not the case however as shown by the RMS error shape for  $\alpha_R = \alpha_\theta = 0.5$ , that remains constant after about  $1e6$  predistorter samples. The finite dimension of the predistortion tables (i.e. 1024 in this case) imposes this limit to the convergence. Moreover care must be used to compare the three relative errors curves (both for AM/AM and AM/PM). Indeed, the ones for  $\alpha_R = \alpha_\theta = 0.1$ , 0.01 correspond to sample times at which the convergence has not reached the minimum value. This is why the relative errors are higher than for  $\alpha_R = \alpha_\theta = 0.5$  in the regions where the AM/AM and AM/PM derivative functions approaches zero. However in the middle zone of the predistortion curves, where convergence is already obtained, the relative errors are lower than for  $\alpha_R = \alpha_\theta = 0.5$ , as expected from theory. It is useful to point out that the proposed predistorter architecture makes the AM/PM convergence possible if the AM/AM curve has already converged in the same region [5]. This is the reason why the AM/PM predistortion curves exhibits high relative errors also for high values of the input envelope even if the

amplifier AM/PM derivative function does not approach zero in that region. In Fig.5 the same kind of figures helps to analyse the effects of signal quantization in the predistorter when  $\alpha_R = \alpha_\theta = 0.1$ . The figures outlines a low sensitiveness of the AM/AM predistortion tables for a bit number that spans from 12 to 20. On the contrary the AM/PM RMS error suggests using at least 16 bits. The AM/PM relative error is more sensitive to quantization in the lower part of the curve because its quasi zero derivative function amplifies the natural higher sensitiveness of small signals to uniform quantization.

The effects of ISI and time synchronisation errors are outlined in Fig.6 for  $\alpha_R = \alpha_\theta = 0.1$ . Both the upper figures and the lower ones show how the convergence property is seriously affected by ISI between adjacent input values. The figures show the system performance for  $\alpha_R = \alpha_\theta = 0.1$  as function of the fractional delay and the  $N_a$  number of the adaptation samples, and comparing it with the ideal situation. The fractional delay is defined as the ratio between the input-output delay error estimation and the generation time  $T_{\text{gen}} = 1/F_{\text{gen}}$ , while  $N_a$  is defined as the number of consecutive errors that are averaged before updating each predistortion table value. It is possible to see that even without any time delay error estimation the ISI contribution seriously degrades the convergence behaviour that reach a floor level for the RMS envelope error of about  $1.5e-3$ . For a fractional delay equal to  $1/64$  the noise floor grows to  $3e-3$  without a significant difference on respect to the only ISI situation. Greater penalties are introduced for fractional delays equal to  $1/32$  or  $1/16$  that seem to be unacceptable. The relative error for each value of the predistortion table exhibits a stronger sensitiveness for low input envelope. It depends on the fact that the ISI contribution power is the same for all the input envelope and consequently the lower address of the AM/AM predistortion table are characterised by a worst signal to noise ratio. The situation is worst for the AM/PM predistortion table that combines the natural higher sensitiveness of the lower input values with its derivative function close to zero. This fact is clearly outlined by the lower part of figure 6 where the divergence of the algorithm for low input envelope is shown. This divergence is the responsible for the unacceptable RMS phase error that never reaches  $1e-2$ . Moreover figure 6 shows that averaging the magnitude and phase errors on  $N_a=6$  consecutive errors do not give a significant protection from ISI and synchronisation errors

## V. Conclusions

Sensitivity of an adaptive digital predistortion system to quantization, ISI and time delay synchronisation was analysed by simulation. The obtained results outline how the technique is highly sensitive to ISI in the adaptation

loop, which makes the algorithm non instantaneous. The sensitiveness to quantization errors is lower and suggests representing the digital signal by 16 bits. The input-output synchronisation error should not exceed 1/64 of the input generation time in order to maintain an acceptable distance from the ideal predistortion curve. Moreover the dependence of such sensitivity on the AM/AM and AM/PM shapes (i.e. derivative function) was also pointed out. Some benefits on a reduction of such sensitiveness can be obtained by averaging the input-output error of each level of the predistortion tables on a high number of consecutive values. However the way to overcome or reduce the high sensitiveness to ISI interference needs further studies and will be object of future works

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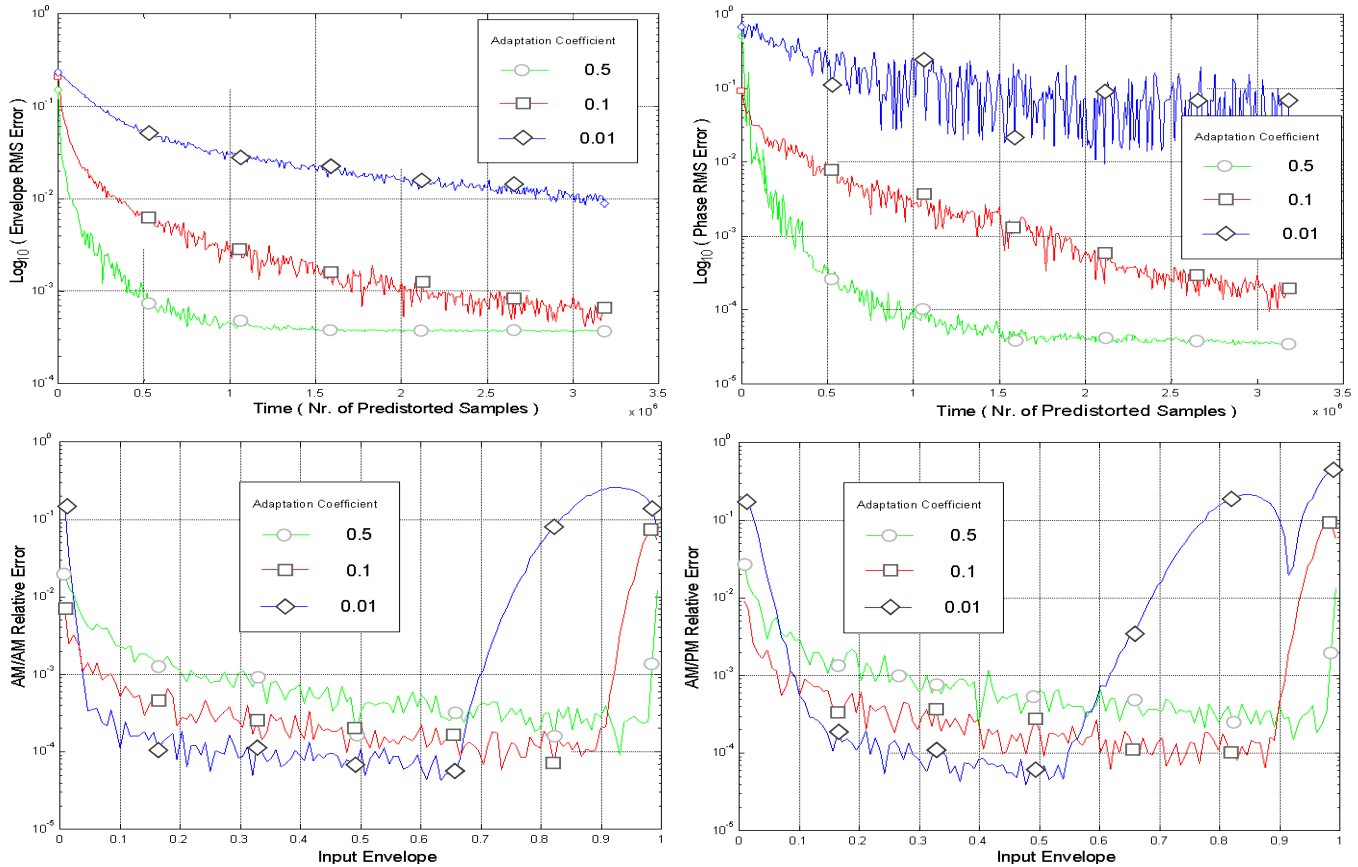


Fig.4 Algorithm Convergence without Quantization, ISI, and Time Synchronisation Errors as function of  $\alpha_R$  and  $\alpha_0$

(Upper) Input-Output Root Mean Square Errors as function of time  
 (Lower) Predistortion Tables Relative Error at the last sample time of Upper Figures

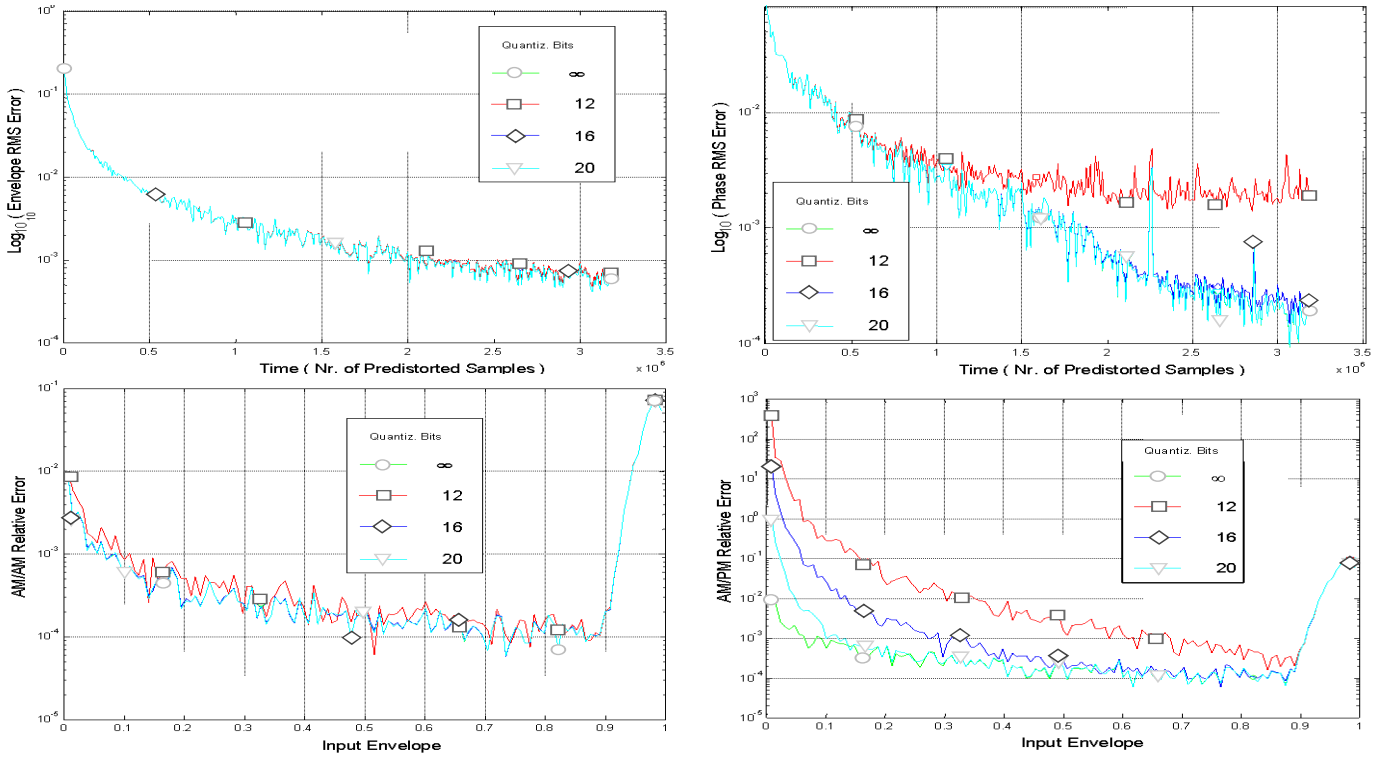


Fig.5 Algorithm Convergence without ISI and Time Synchronisation Errors as function of quantization Bits for  $\alpha_R = \alpha_0 = 0.1$

(Upper) Input-Output Root Mean Square Errors as function of time

(Lower) Predistortion Tables Relative Error at the last sample time of Upper Figures

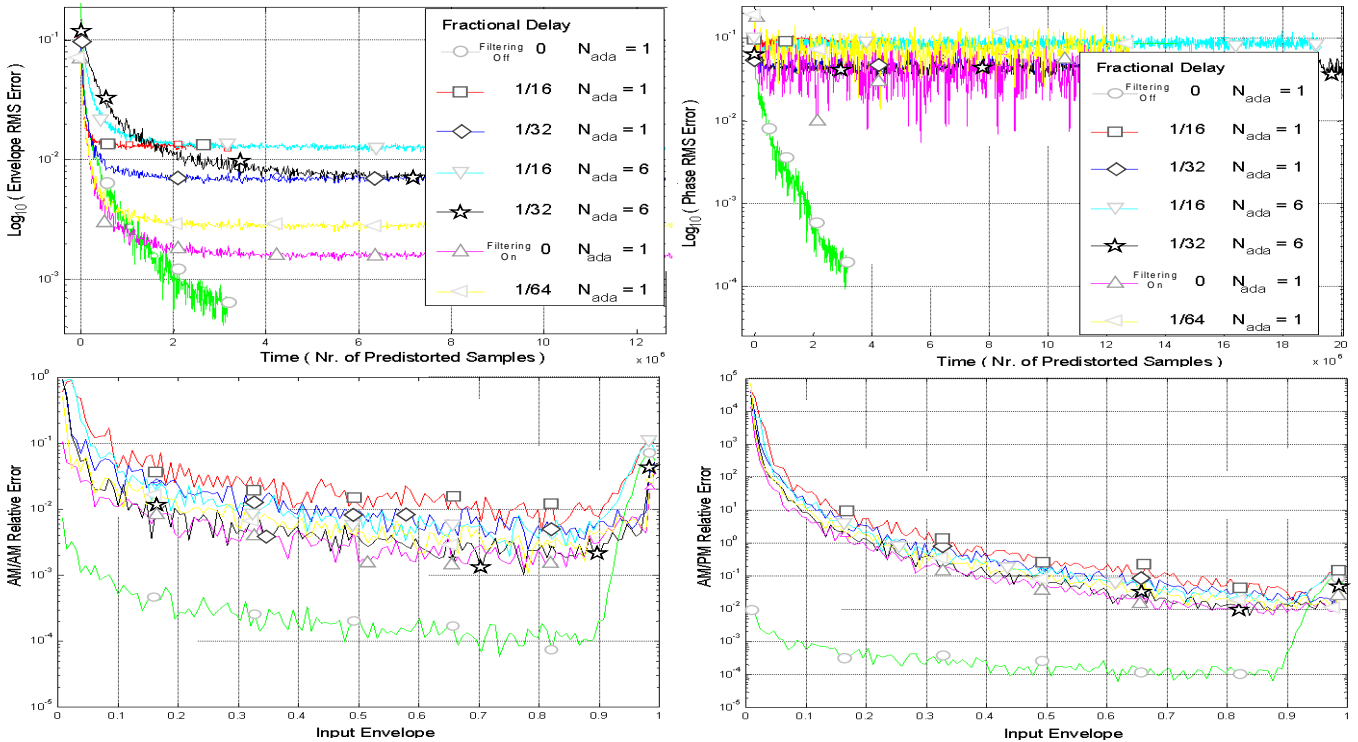


Fig.6 Algorithm Convergence with Quantization and ISI as function of Time Synchronisation Error ( $\% T_{gen}$ ) and Number of Sample Averages

(Upper) Input-Output Root Mean Square Errors as function of time

(Lower) Predistortion Tables Relative Error at the last sample time of Upper Figures