BER OF MC-DS-CDMA SYSTEMS WITH CFO AND NONLINEAR DISTORTIONS

Luca Rugini, Paolo Banelli

DIEI - University of Perugia - Via G. Duranti 93 - 06125 Perugia - Italy

ABSTRACT

Multicarrier systems are highly sensitive to the intercarrier interference (ICI) caused by a carrier frequency offset (CFO) at the receiver and to the intermodulation distortion (IMD) introduced by the nonlinear amplifier (NLA) at the transmitter. We analyze how these impairments degrade the bit-error rate (BER) of multicarrier direct-sequence code-division multiple-access (MC-DS-CDMA) downlink systems in frequency-selective Rayleigh fading channels.

1. INTRODUCTION

The low complexity equalization of cyclic prefixed OFDM systems, and the multiple-access interference (MAI) mitigation capabilities offered by CDMA systems, make multicarrier CDMA techniques attractive for future mobile broadband communications [1].

Differently from single-carrier systems, one of the main problems of multicarrier schemes is the high sensitivity to frequency synchronization errors [2]. Indeed, the CFO, which models the frequency mismatch between the transmitter and receiver oscillators, generates ICI, thereby destroying the frequency-domain orthogonality of the transmitted data. Another critical aspect of multicarrier techniques is the high sensitivity to the NLA at the transmitter, which introduces IMD. Indeed, multicarrier signals are characterized by a high peak-to-average power ratio, and consequently they are significantly distorted when the NLA works close to the saturation, for power efficiency constraints. As a consequence, the degradation introduced by these impairments should be taken into account when evaluating the BER performance of multicarrier systems.

Herein we focus on a system that is commonly identified as MC-DS-CDMA [1], where multiple users are discriminated by spreading in the time domain. Specifically, we consider the BER performance at the mobile end (i.e., downlink), when the receiver is equipped with a low-complexity matched filter (MF) detector. Previous research about this topic considered either the effect of CFO [3] or the effect of the NLA [4]. The purpose of our work is to develop a BER analysis that jointly takes into account CFO, nonlinear distortions, and MAI, in multipath fading channels.

2. SYSTEM MODEL

We consider the downlink of an MC-DS-CDMA system with N subcarriers, spaced by $\Delta_f = 1/T$, and K users. The base station multiplexes N symbols of each user over the N subcarriers, by spreading each symbol in the time domain employing a user-dependent spreading code, which is denoted with $\mathbf{c}_k = [c_{k,0} \cdots c_{k,G-1}]^T$, where G is the processing gain, and $|c_{k,g}| = G^{-1/2}$. The transmitted block, at the input of the NLA, can be expressed by

$$\mathbf{u}_{\mathrm{IN}}[lG+g] = \mathbf{T}_{\mathrm{CP}}\mathbf{F}^H\mathbf{S}[l]\mathbf{c}[g], \tag{1}$$

where $\mathbf{u}_{\mathrm{IN}}[lG+g]$ is a vector of dimension $P = N + L$, L is the cyclic prefix length, $\mathbf{T}_{\mathrm{CP}}$ is the $P \times N$ cyclic prefix insertion matrix [5], $\mathbf{F}$ is the $N \times N$ unitary FFT matrix, $\mathbf{S}[l]$ is the $N \times K$ matrix containing the data symbols, and $\mathbf{c}[g] = [c_{1,g} \cdots c_{K,g}]^T$ is the vector that contains the gth chip of the codes of all the K users. The lth symbol $s_{n,k}[l] = [\mathbf{S}[l]]_{n,k}$ of the kth user on the nth subcarrier, is drawn from the 4-QAM constellation, and all the symbols are assumed to be i.i.d. with power σ_{S}^2.

The NLA is modeled as a memoryless device by means of its AM/AM and AM/PM curves. By using the Bussgang theorem and the index $i = lG + g$, the transmitted block, at the output of the NLA, can be expressed as [6]

$$\mathbf{u}_{\mathrm{OUT}}[i] = \alpha\mathbf{u}_{\mathrm{IN}}[i] + \overline{\mathbf{v}}_{\mathrm{IMD}}[i], \tag{2}$$

where α and the autocorrelation function $\mathbf{R}_{\overline{\mathrm{v}}}(g,g')$ of $\overline{\mathbf{v}}_{\mathrm{IMD}}[i]$ depend on the AM/AM and AM/PM curves of the NLA and on the output back-off (OBO) to the NLA [6]. The validity of (2), based on a Gaussian distribution of $\mathbf{u}_{\mathrm{IN}}[i]$ in (1), is justified by the high number of subcarriers usually employed in multicarrier systems (e.g., $N \geq 32$).

After the parallel-to-serial conversion, the signal stream $u_{\mathrm{OUT}}[iP+p] = [\mathbf{u}_{\mathrm{OUT}}[i]]_{p+1}$, $p = 0,\ldots,P-1$, is transmitted through a multipath FIR channel $h[b]$, characterized by paths with Rayleigh statistics and maximum delay spread smaller than the cyclic prefix duration. We assume that timing information is available at the receiver. In the presence of CFO, the received stream can be expressed as [7]

$$x[a] = e^{j2\pi f_0 aT/N}\sum_{b=0}^{L} h[b]u_{\mathrm{OUT}}[a-b] + x_{\mathrm{WG}}[a], \tag{3}$$

where f_0 is the CFO, and $x_{\mathrm{WG}}[a]$ represents the AWGN, with $a = iP + p$. The P received samples relative to the

gth chip of the lth symbol are grouped to form the vector $\mathbf{x}[i]=\mathbf{x}[lG+g]$, thus obtaining [7]

$$\mathbf{x}[i]=e^{j2\pi\varepsilon iP/N}\tilde{\mathbf{D}}(\mathbf{H}_0\mathbf{u}_{\text{OUT}}[i]+\mathbf{H}_1\mathbf{u}_{\text{OUT}}[i-1])+\mathbf{x}_{\text{WG}}[i], \quad (4)$$

where $[\mathbf{x}[i]]_{p+1}=x[iP+p]$, $\varepsilon=f_0T$ is the normalized CFO, $\tilde{\mathbf{D}}$ is a $P\times P$ diagonal matrix, defined by $[\tilde{\mathbf{D}}]_{p,p}=\exp(j2\pi\varepsilon(p-1)/N)$, and $\mathbf{H}_0$ and $\mathbf{H}_1$ are $P\times P$ Toeplitz matrices defined by $[\mathbf{H}_0]_{m,n}=h[m-n]$ and $[\mathbf{H}_1]_{m,n}=h[m-n+P]$, respectively [5].

If $\mathbf{R}_{\text{CP}}$ denotes the cyclic prefix elimination matrix, by using (1) and (2), the vector $\mathbf{y}[lG+g]=\mathbf{y}[i]=\mathbf{R}_{\text{CP}}\mathbf{x}[i]$ can be expressed as [7]

$$\mathbf{y}[lG+g]=e^{j2\pi\varepsilon((lG+g)P+L)/N}\mathbf{D}\mathbf{H}(\alpha\mathbf{F}^H\mathbf{S}[l]\mathbf{c}[g]+\tilde{\mathbf{v}}_{\text{IMD}}[lG+g])+\mathbf{y}_{\text{WG}}[lG+g], \quad (5)$$

where $\mathbf{D}$ is the $N\times N$ diagonal matrix defined by $[\mathbf{D}]_{n,n}=\exp(j2\pi\varepsilon(n-1)/N)$, $\mathbf{H}=\mathbf{R}_{\text{CP}}\mathbf{H}_0\mathbf{T}_{\text{CP}}$ is the circulant channel matrix, $\tilde{\mathbf{v}}_{\text{IMD}}[lG+g]=\mathbf{R}_{\text{CP}}\bar{\mathbf{v}}_{\text{IMD}}[lG+g]$, and $\mathbf{y}_{\text{WG}}[lG+g]$ stands for the AWGN. Since $\mathbf{H}$ is circulant, it can be expressed as $\mathbf{H}=\mathbf{F}^H\boldsymbol{\Lambda}\mathbf{F}$, where $\boldsymbol{\Lambda}=\text{diag}(\boldsymbol{\lambda})$ is a diagonal matrix representing the frequency-domain channel $\boldsymbol{\lambda}=N^{1/2}\mathbf{F}[h[0]\cdots h[N-1]]^T$.

The recovery of the transmitted data is accomplished by applying the FFT at the receiver, thus obtaining $\mathbf{z}[lG+g]=\mathbf{F}\mathbf{y}[lG+g]$, which by (5) yields

$$\mathbf{z}[lG+g]=e^{j2\pi\varepsilon((lG+g)P+L)/N}\boldsymbol{\Phi}\boldsymbol{\Lambda}(\alpha\mathbf{S}[l]\mathbf{c}[g]+\mathbf{v}_{\text{IMD}}[lG+g])+\mathbf{z}_{\text{WG}}[lG+g], \quad (6)$$

where $\boldsymbol{\Phi}=\mathbf{F}\mathbf{D}\mathbf{F}^H$ is the $N\times N$ circulant matrix that models the ICI due to the CFO, $\mathbf{v}_{\text{IMD}}[lG+g]=\mathbf{F}\tilde{\mathbf{v}}_{\text{IMD}}[lG+g]$ is the IMD after the FFT operation, and $\mathbf{z}_{\text{WG}}[lG+g]$ represents the AWGN.

Due to the spreading in the time domain, in order to decode the N data symbols $\mathbf{s}_k[l]=[s_{1,k}[l]\cdots s_{N,k}[l]]^T$ relative to the lth interval, the receiver of the kth user has to collect G consecutive vectors in (6). From (6), by defining the $N\times G$ matrix $\mathbf{Z}[l]=[\mathbf{z}[lG]\cdots\mathbf{z}[lG+G-1]]$, and $\mathbf{V}_{\text{IMD}}[l]$ and $\mathbf{Z}_{\text{WG}}[l]$ accordingly, we obtain

$$\mathbf{Z}[l]=e^{j2\pi\varepsilon L/N}\boldsymbol{\Phi}\boldsymbol{\Lambda}(\alpha\mathbf{S}[l]\mathbf{C}+\mathbf{V}_{\text{IMD}}[l])\mathbf{E}[l]+\mathbf{Z}_{\text{WG}}[l], \quad (7)$$

where $\mathbf{C}=[\mathbf{c}[0]\cdots\mathbf{c}[G-1]]=[\mathbf{c}_1\ \mathbf{c}_2\ \cdots\ \mathbf{c}_K]^T$ is the row-wise $K\times G$ spreading code matrix, and $\mathbf{E}[l]$ is a $G\times G$ diagonal matrix defined as $[\mathbf{E}[l]]_{g+1,g+1}=e^{j2\pi\varepsilon(lG+g)P/N}$.

We assume that the receiver is able to compensate for the time-invariant phase-shift term $\varphi=e^{j2\pi\varepsilon L/N}e^{j\pi\varepsilon(N-1)/N}$. Regarding the time-variant phase-shift compensation, we consider two alternative hypotheses, H1 and H2.

H1: The receiver is able to obtain an accurate phase-shift estimate in each chip interval, as in [3], with a consequent estimated phase-shift matrix $\hat{\mathbf{E}}[l]=\hat{\mathbf{E}}[l]_{\text{H1}}=\mathbf{E}[l]$.

H2: The receiver is able to estimate only the average phase-shift within a symbol interval. Hence, the estimated phase-shift matrix is diagonal, with elements expressed by $[\hat{\mathbf{E}}[l]_{\text{H2}}]_{g+1,g+1}=\exp\left(j2\pi\varepsilon PN^{-1}\left(lG+(G-1)/2\right)\right)$.

Thus, by assuming perfect channel state information at the receiver side, the equalized data matrix can be constructed as $\mathbf{Z}_{\text{EQ}}[l]=\varphi^*\alpha^{-1}\boldsymbol{\Lambda}^{-1}\mathbf{Z}[l]\hat{\mathbf{E}}[l]^{-1}$. Successively, after despreading, the vector $\mathbf{z}_{\text{DS},k}[l]=\mathbf{Z}_{\text{EQ}}[l]\mathbf{c}_k^*$ contains the N decision variables of the kth user, as expressed by

$$\mathbf{z}_{\text{DS},k}[l]=\boldsymbol{\Lambda}^{-1}\mathbf{M}\boldsymbol{\Lambda}\mathbf{S}[l]\mathbf{C}\boldsymbol{\Theta}\mathbf{c}_k^*+\mathbf{z}_{\text{DS},k,\text{IMD}}[l]+\mathbf{z}_{\text{DS},k,\text{WG}}[l], \quad (8)$$

where $\mathbf{M}=e^{-j\pi\varepsilon(N-1)/N}\boldsymbol{\Phi}$ represents the phase-compensated ICI matrix, $\boldsymbol{\Theta}=\mathbf{E}[l]\hat{\mathbf{E}}[l]^{-1}$ is a diagonal matrix that depends on the hypothesis H1 or H2, $\mathbf{z}_{\text{DS},k,\text{IMD}}[l]=\alpha^{-1}\boldsymbol{\Lambda}^{-1}\mathbf{M}\boldsymbol{\Lambda}\mathbf{V}_{\text{IMD}}[l]\boldsymbol{\Theta}\mathbf{c}_k^*$ represents the IMD after equalization and despreading, and $\mathbf{z}_{\text{DS},k,\text{WG}}[l]=\varphi^*\alpha^{-1}\boldsymbol{\Lambda}^{-1}\mathbf{Z}_{\text{WG}}[l]\hat{\mathbf{E}}[l]^{-1}\mathbf{c}_k^*$ stands for the AWGN.

The K-dimensional vector $\mathbf{C}\boldsymbol{\Theta}\mathbf{c}_k^*$ represents the effect of the CFO-induced time-varying phase-shift. Under H1, employing orthogonal spreading codes, the MAI is eliminated, because the product $\mathbf{S}[l]\mathbf{C}\boldsymbol{\Theta}_{\text{H1}}\mathbf{c}_k^*$ in (8) is equal to $\mathbf{s}_k[l]$. On the contrary, under H2, the MAI cannot be eliminated. Indeed, for $k'\neq k$, $\rho_{k,k',\text{H2}}=[\mathbf{C}\boldsymbol{\Theta}_{\text{H2}}\mathbf{c}_k^*]_{k'}\neq 0$ even for orthogonal sequences.

3. BER ANALYSIS

By denoting with $\lambda_n=[\boldsymbol{\Lambda}]_{n,n}$ the channel gain of the nth subcarrier, the BER can be expressed by

$$(\text{BER})_{n,k}=\int_{\lambda_n}P_{\text{BE},n,k}(\lambda_n)f_{\lambda_n}(\lambda_n)d\lambda_n, \quad (9)$$

$$P_{\text{BE},n,k}(\lambda_n)=\int_{\mathbf{S},\bar{\boldsymbol{\lambda}}_n}P_{\text{BE},n,k}(\mathbf{S},\boldsymbol{\lambda})f_{\bar{\boldsymbol{\lambda}}_n|\lambda_n}(\bar{\boldsymbol{\lambda}}_n\,|\,\lambda_n)f_{\mathbf{S}}(\mathbf{S})d\mathbf{S}d\bar{\boldsymbol{\lambda}}_n, \quad (10)$$

where $f_{\lambda_n}(\lambda_n)$ is the pdf of λ_n, $f_{\bar{\boldsymbol{\lambda}}_n|\lambda_n}(\bar{\boldsymbol{\lambda}}_n\,|\,\lambda_n)$ is pdf of $\bar{\boldsymbol{\lambda}}_n=[\lambda_1\cdots\lambda_{n-1}\ \lambda_{n+1}\cdots\lambda_N]^T$ conditioned on λ_n, and $P_{\text{BE},n,k}(\mathbf{S},\boldsymbol{\lambda})$ is the BER conditioned on $\boldsymbol{\lambda}$ and $\mathbf{S}[l]$. In order to find the BER, we firstly derive an approximated expression for the conditional probability in (10), and successively we average the approximated conditional BER over $f_{\lambda_n}(\lambda_n)$ as in (9). By multiplying (8) with $\lambda_n|\alpha|$, the scaled decision variable $z_{\text{SC},n,k}[l]$ can be expressed by

$$z_{\text{SC},n,k}[l]=|\alpha|\mathbf{m}_n^T\boldsymbol{\Lambda}\mathbf{S}[l]\mathbf{C}\boldsymbol{\Theta}\mathbf{c}_k^*+e^{-j\arg(\alpha)}\mathbf{m}_n^T\boldsymbol{\Lambda}\mathbf{V}_{\text{IMD}}[l]\boldsymbol{\Theta}\mathbf{c}_k^*+\varphi^*e^{-j\arg(\alpha)}\mathbf{z}_{n,\text{WG}}[l]^T\hat{\mathbf{E}}[l]^{-1}\mathbf{c}_k^*, \quad (11)$$

where $\mathbf{z}_{n,\text{WG}}[l]^T$ is the nth row of $\mathbf{Z}_{\text{WG}}[l]$, and $\mathbf{m}_n=[m_{n,1}\cdots m_{n,N}]^T$ is the nth row of $\mathbf{M}$.

By using $\mathbf{S}[l]\mathbf{C}\boldsymbol{\Theta}\mathbf{c}_k^*=\rho\mathbf{s}_k[l]+\sum_{k'=1,k'\neq k}^{K}\rho_{k,k'}\mathbf{s}_{k'}[l]$, and dropping the index l, (11) becomes

$$\begin{aligned}z_{\text{SC},n,k}=&\ |\alpha|\rho m\lambda_n s_{n,k}+|\alpha|\rho\sum_{n'=1,n'\neq n}^{N}m_{n,n'}\lambda_{n'}s_{n',k}\\&+|\alpha|m\lambda_n\sum_{\substack{k'=1\\k'\neq k}}^{K}\rho_{k,k'}s_{n,k'}+|\alpha|\sum_{\substack{n'=1\\n'\neq n}}^{N}m_{n,n'}\lambda_{n'}\sum_{\substack{k'=1\\k'\neq k}}^{K}\rho_{k,k'}s_{n',k'}\\&+\sum_{g=0}^{G-1}e^{-j\arg(\alpha)}\theta_{g+1}c_{k,g}^*\sum_{n'=1}^{N}m_{n,n'}\lambda_{n'}v_{\text{IMD},n',g}+z_{\text{AWGN},n,k},\end{aligned} \quad (12)$$

where $m=m_{n,n}$, $\rho=\rho_{k,k}$, $\theta_{g+1}=[\boldsymbol{\Theta}]_{g+1,g+1}$, $v_{\text{IMD},n,g}$ is the

(n,g)th element of $\mathbf{V}_{\text{IMD}}$, and $z_{\text{AWGN},n,k} = \varphi^* e^{-j\arg(\alpha)} \mathbf{z}_{n,\text{WG}}^T \hat{\mathbf{E}}^{-1} \mathbf{c}_k^*$. By constructing the conditional random variable $t_{\text{SC},n,k} = z_{\text{SC},n,k} \mid \lambda_n$, we can express (12) as a function of the random variables $\{\hat{\lambda}_{n',n}\}$, where $\hat{\lambda}_{n',n} = \lambda_{n'} \mid \lambda_n$ indicates the n'th subcarrier channel gain conditioned on the nth subcarrier channel gain. The $(N-1)\times 1$ vector $\hat{\boldsymbol{\lambda}}_n = [\hat{\lambda}_{1,n} \cdots \hat{\lambda}_{n-1,n}\, \hat{\lambda}_{n+1,n} \cdots \hat{\lambda}_{N,n}]^T$ is still a Gaussian random vector, with mean value $\boldsymbol{\eta}_{\hat{\boldsymbol{\lambda}}_n} = E\{\hat{\boldsymbol{\lambda}}_n\}$ and covariance $\mathbf{R}_{\hat{\boldsymbol{\lambda}}_n} = E\{\hat{\boldsymbol{\lambda}}_n \hat{\boldsymbol{\lambda}}_n^H\}$ expressed by [8]

$$\boldsymbol{\eta}_{\hat{\boldsymbol{\lambda}}_n} = \lambda_n r_{\lambda_n\lambda_n}^{-1} \mathbf{r}_{\bar{\boldsymbol{\lambda}}_n\lambda_n}, \qquad \mathbf{R}_{\hat{\boldsymbol{\lambda}}_n} = \mathbf{R}_{\bar{\boldsymbol{\lambda}}_n} - r_{\lambda_n\lambda_n}^{-1} \mathbf{r}_{\bar{\boldsymbol{\lambda}}_n\lambda_n} \mathbf{r}_{\bar{\boldsymbol{\lambda}}_n\lambda_n}^H, \tag{13}$$

where $r_{\lambda_{n'}\lambda_n} = E\{\lambda_{n'}\lambda_n^*\}$ is the statistical correlation between the channels on the n'th and the nth subcarrier, $\mathbf{r}_{\bar{\boldsymbol{\lambda}}_n\lambda_n} = E\{\bar{\boldsymbol{\lambda}}_n \lambda_n^*\}$ is the crosscorrelation vector between the channel of interest and the other channels, and $\mathbf{R}_{\bar{\boldsymbol{\lambda}}_n} = E\{\bar{\boldsymbol{\lambda}}_n \bar{\boldsymbol{\lambda}}_n^H\}$ is the crosscorrelation matrix of the other channels. Consequently, by defining the $(N-1)\times 1$ zero-mean Gaussian random vector $\boldsymbol{\pi}_n = \hat{\boldsymbol{\lambda}}_n - \boldsymbol{\eta}_{\hat{\boldsymbol{\lambda}}_n} = [\pi_{1,n} \cdots \pi_{n-1,n}\, \pi_{n+1,n} \cdots \pi_{N,n}]^T$, $t_{\text{SC},n,k}$ becomes

$$t_{\text{SC},n,k} = \lambda_n (|\alpha| \rho m s_{n,k} + z_{1,n,k}) + z_{2,n,k}, \tag{14}$$

where

$$z_{1,n,k} = z_{\text{ICI},n,k} + z_{\text{MAI},n,k} + z_{\text{MAICI},n,k} + z_{\text{IMD},n,k}, \tag{15}$$

$$z_{2,n,k} = \tilde{z}_{\text{ICI},n,k} + \tilde{z}_{\text{MAICI},n,k} + \tilde{z}_{\text{IMD},n,k} + z_{\text{AWGN},n,k}, \tag{16}$$

$$z_{\text{ICI},n,k} = |\alpha| \rho r_{\lambda_n\lambda_n}^{-1} \sum_{n'=1, n'\neq n}^{N} m_{n,n'} r_{\lambda_{n'}\lambda_n} s_{n',k}, \tag{17}$$

$$z_{\text{MAI},n,k} = |\alpha| m \sum_{k'=1, k'\neq k}^{K} \rho_{k,k'} s_{n,k'}, \tag{18}$$

$$z_{\text{MAICI},n,k} = |\alpha| r_{\lambda_n\lambda_n}^{-1} \sum_{n'=1, n'\neq n}^{N} m_{n,n'} r_{\lambda_{n'}\lambda_n} \sum_{k'=1, k'\neq k}^{K} \rho_{k,k'} s_{n',k'}, \tag{19}$$

$$z_{\text{IMD},n,k} = e^{-j\arg(\alpha)} r_{\lambda_n\lambda_n}^{-1} \sum_{g=0}^{G-1} \theta_{g+1} c_{k,g}^* \sum_{n'=1}^{N} m_{n,n'} r_{\lambda_{n'}\lambda_n} v_{\text{IMD},n',g}, \tag{20}$$

$$\tilde{z}_{\text{ICI},n,k} = |\alpha| \rho \sum_{n'=1, n'\neq n}^{N} m_{n,n'} \pi_{n',n} s_{n',k}, \tag{21}$$

$$\tilde{z}_{\text{MAICI},n,k} = |\alpha| \sum_{n'=1, n'\neq n}^{N} m_{n,n'} \pi_{n',n} \sum_{k'=1, k'\neq k}^{K} \rho_{k,k'} s_{n',k'}, \tag{22}$$

$$\tilde{z}_{\text{IMD},n,k} = e^{-j\arg(\alpha)} \sum_{g=0}^{G-1} \theta_{g+1} c_{k,g}^* \sum_{n'=1, n'\neq n}^{N} m_{n,n'} \pi_{n',n} v_{\text{IMD},n',g}. \tag{23}$$

The noisy terms in (14) are grouped in two different components. The first one, $\lambda_n z_{1,n,k}$, is proportional to λ_n, and hence it represents the interference that fades *coherently* with the useful signal. The second part $z_{2,n,k}$ represents the interference that fades *independently* of λ_n, because the powers of the terms in (21)-(23) do not depend on the specific channel coefficient λ_n. Consequently, the conditional signal-to-interference plus noise ratio can be expressed as

$$\gamma_{n,k}(\lambda_n) = \frac{|\lambda_n|^2 |\alpha\rho m|^2 \sigma_{\text{S}}^2}{|\lambda_n|^2 \sigma_{1,n,k}^2 + \sigma_{2,n,k}^2}, \tag{24}$$

where

$$\sigma_{1,n,k}^2 = \sigma_{\text{ICI},n,k}^2 + \sigma_{\text{MAI},n,k}^2 + \sigma_{\text{MAICI},n,k}^2 + \sigma_{\text{IMD},n,k}^2, \tag{25}$$

$$\sigma_{2,n,k}^2 = \tilde{\sigma}_{\text{ICI},n,k}^2 + \tilde{\sigma}_{\text{MAICI},n,k}^2 + \tilde{\sigma}_{\text{IMD},n,k}^2 + \sigma_{\text{AWGN}}^2, \tag{26}$$

$$\sigma_{\text{ICI},n,k}^2 = |\alpha\rho r_{\lambda_n\lambda_n}^{-1}|^2 \sum_{n'=1, n'\neq n}^{N} |m_{n,n'} r_{\lambda_{n'}\lambda_n}|^2 \sigma_{\text{S}}^2, \tag{27}$$

$$\sigma_{\text{MAI},n,k}^2 = |\alpha m|^2 \sum_{k'=1, k'\neq k}^{K} |\rho_{k,k'}|^2 \sigma_{\text{S}}^2, \tag{28}$$

$$\sigma_{\text{MAICI},n,k}^2 = |\alpha r_{\lambda_n\lambda_n}^{-1}|^2 \sum_{\substack{n'=1\\ n'\neq n}}^{N} |m_{n,n'} r_{\lambda_{n'}\lambda_n}|^2 \sum_{\substack{k'=1\\ k'\neq k}}^{K} |\rho_{k,k'}|^2 \sigma_{\text{S}}^2, \tag{29}$$

$$\sigma_{\text{IMD},n,k}^2 = r_{\lambda_n\lambda_n}^{-2} \sum_{g=0}^{G-1} \sum_{g'=0}^{G-1} \theta_{g+1} \theta_{g'+1}^* c_{k,g}^* c_{k,g'} \bar{\mathbf{m}}_n^T \mathbf{R}_{\mathbf{v}}(g,g') \bar{\mathbf{m}}_n^*, \tag{30}$$

$$\tilde{\sigma}_{\text{ICI},n,k}^2 = |\alpha\rho|^2 \sum_{\substack{n'=1\\ n'\neq n}}^{N} |m_{n,n'}|^2 (r_{\lambda_{n'}\lambda_n} - r_{\lambda_n\lambda_n}^{-1} |r_{\lambda_{n'}\lambda_n}|^2) \sigma_{\text{S}}^2,$$

$$\tilde{\sigma}_{\text{MAICI},n,k}^2 = |\alpha|^2 \sum_{\substack{n'=1\\ n'\neq n}}^{N} |m_{n,n'}|^2 (r_{\lambda_{n'}\lambda_n} - r_{\lambda_n\lambda_n}^{-1} |r_{\lambda_{n'}\lambda_n}|^2) \sum_{\substack{k'=1\\ k'\neq k}}^{K} |\rho_{k,k'}|^2 \sigma_{\text{S}}^2,$$

$$\tilde{\sigma}_{\text{IMD},n,k}^2 = \sum_{g=0}^{G-1} \sum_{g'=0}^{G-1} \theta_{g+1} \theta_{g'+1}^* c_{k,g}^* c_{k,g'} \cdot \sum_{\substack{n'=1\\ n'\neq n}}^{N} \sum_{\substack{n''=1\\ n''\neq n}}^{N} m_{n,n'} m_{n,n''}^* [\mathbf{J}_n^T \mathbf{R}_{\hat{\boldsymbol{\lambda}}_n} \mathbf{J}_n]_{n',n''} [\mathbf{R}_{\mathbf{v}}(g,g')]_{n',n''}, \tag{31}$$

where $[\bar{\mathbf{m}}_n]_{n'} = m_{n,n'} r_{\lambda_{n'}\lambda_n}$, the $(N-1)\times N$ matrix $\mathbf{J}_n$ is obtained from $\mathbf{I}_N$ by removing the nth row, and $\mathbf{R}_{\mathbf{v}}(g,g')$ is expressed by

$$\mathbf{R}_{\mathbf{v}}(g,g') = E\{\mathbf{v}_{\text{IMD},g} \mathbf{v}_{\text{IMD},g'}^H\} = \mathbf{F}\mathbf{R}_{\text{CP}} \mathbf{R}_{\bar{\mathbf{v}}}(g,g') \mathbf{R}_{\text{CP}}^T \mathbf{F}^H, \tag{32}$$

which can be calculated by exploiting the knowledge of the IMD covariance matrix $\mathbf{R}_{\bar{\mathbf{v}}}(g,g')$.

At this point, we approximate as Gaussian the interference terms in (15) and (16). Such an approximation is reasonable under the hypotheses of a high number of subcarriers and users. Note that for constant-modulus constellations the terms (21) and (22) are exactly Gaussian. Assuming for simplicity 4-QAM with Gray coding, the conditional BER is expressed by

$$P_{\text{BE},n,k}(\lambda_n) = Q\left(\sqrt{\gamma_{n,k}(\lambda_n)}\right), \tag{33}$$

and the resultant BER is obtained by inserting (33), (24), and the Rayleigh pdf of $|\lambda_n|$ in (9), with $\sigma_\lambda^2 = E\{|\lambda_n|^2\}$. The final BER can therefore be obtained as [9]

$$(\text{BER})_{n,k} = \int_0^{+\infty} Q\left(\sqrt{\gamma(\lambda_n)}\right) \frac{2|\lambda_n|}{\sigma_\lambda^2} e^{-\frac{|\lambda_n|^2}{\sigma_\lambda^2}} d|\lambda_n|$$

$$= \frac{1}{2} - \frac{\sqrt{2\mu_{n,k}^2}}{4} e^{-\frac{\mu_{n,k}^2}{2\nu_{n,k}^2}} \sum_{i=0}^{+\infty} \frac{1}{i!} \left(\frac{\mu_{n,k}^2}{2\nu_{n,k}^2}\right)^i {}_2F_0\left(i+\frac{3}{2}, \frac{1}{2};; -\nu_{n,k}^2\right), \tag{34}$$

where ${}_pF_q$ denotes the generalized hypergeometric function, $\mu_{n,k}^2 = |\alpha\rho m|^2 \sigma_{\text{S}}^2 \sigma_\lambda^2 \sigma_{2,n,k}^{-2}$ and $\nu_{n,k}^2 = \sigma_{1,n,k}^2 \sigma_\lambda^2 \sigma_{2,n,k}^{-2}$.

4. SIMULATION RESULTS

We consider a 4-QAM MC-DS-CDMA system with $N = 256$, $\Delta_f = 1/T = 156.25$ kHz, cyclic prefix $L = 64$, a channel characterized by an exponentially decaying power delay profile and rms delay spread of 250 ns, and a perfectly predistorted amplifier [6]. The received E_b / N_0 is defined before the despreading, while the BER is averaged over all the subcarriers and users.

In the first scenario, we assume that the base station employs Walsh-Hadamard (WH) spreading codes of length $G = 16$, and the Hypothesis H1 as true. Fig. 1 shows the joint effect of CFO and NLA on the BER performance when $\varepsilon = 0.02$. The good agreement between theoretical analysis and simulated BER is evident. Moreover, the results highlight that, in the presence of a CFO, increasing the OBO beyond a certain value does not help to reduce the BER, because the IMD reduction is masked by the presence of the ICI.

In the second scenario, we assume that H2 holds true. In this case, since the MAI is present, we consider Gold codes of length $G = 31$. In Fig. 2, it is evident that the BER analysis is quite accurate for $E_b / N_0 \leq 30$ dB, while there is a small mismatch with simulations at high E_b / N_0. This fact is due to the non-perfect Gaussian approximation of the MAI terms. Fig. 2 clearly shows that, if only an average phase estimate is available, the maximum tolerable CFO is rather small, due to the additional MAI. Therefore, the phase compensation step plays a crucial role on the BER performance of realistic MC-DS-CDMA systems.

5. CONCLUSION

In this paper, we have evaluated the BER of 4-QAM MC-DS-CDMA downlink systems subject to both CFO and IMD in frequency-selective Rayleigh fading channels. The BER analysis is quite accurate in several conditions. Particularly, it has been highlighted the importance of an accurate compensation for the CFO-induced phase-shift. The analysis can easily be extended to *M*-QAM or *M*-PSK constellations. Future works may focus on the effects of channel estimation errors and channel coding.

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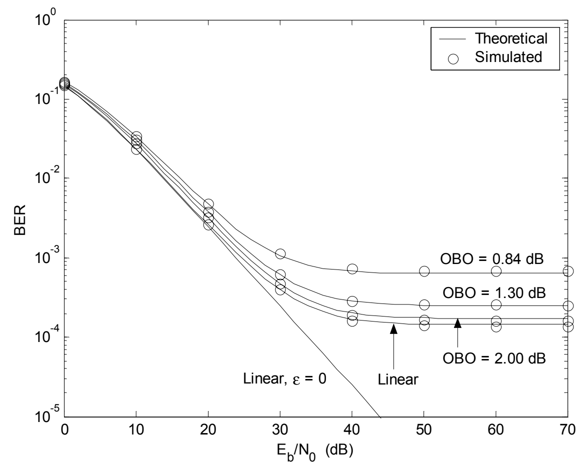

Fig. 1. WH codes, $K = G = 16$, $\varepsilon = 0.02$.

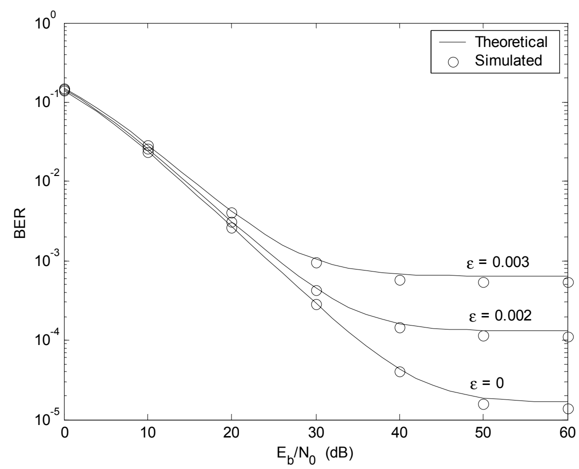

Fig. 2. Gold codes, $G = 31$, $K = 25$, $\text{OBO} = 2.00$ dB.