Improved Performance of MMSE Multiuser Receivers for Asynchronous CDMA: Preliminary Results

Paolo Banelli, Saverio Cacopardi, Luca Rugini

D.I.E.I. – University of Perugia – Italy – email: {banelli, cacopardi, rugini}@diei.unipg.it

Abstract – **The Minimum Mean Square Error (MMSE) multiuser detector has received great attention in the last years as the optimum linear solution for reducing the Multiple Access Interference (MAI) in DS-CDMA systems. In asynchronous frequency selective channels the covariance matrix estimation errors may introduce BER performance degradation. Aim of this paper is to outline such degradation and to improve the MMSE receiver performance by convenient covariance matrix estimations. Two different approaches are introduced. Simulation results are shown in order to validate the effectiveness of the proposed techniques.**

Keywords – **DS-CDMA, Multiuser Detection, MMSE, Spectral Decomposition, Multiple Access Interference.**

I. INTRODUCTION

The presence of MAI in DS-CDMA systems makes attracting the use of multiuser receivers. The MAI can be exploited to improve the performance by using banks of symbol matched filters as proposed in [1]. Such multiuser detectors are characterised by very cumbersome front-ends, because the number of matched filters (or RAKEs in multipath channels) depends on the active users number. An alternative architecture, that uses only a single chip matched filter and a sampler at the chip rate, is not only characterised by a greater flexibility, but it also allows channel estimation by blind techniques. The main benefit of the blind methods is the bandwidth saving due to the absence of training sequences, and it becomes significant in time-varying channels. Since the blind multiuser detection algorithms are characterised by a very high computational complexity, great attention is dedicated to the class of linear receivers. It is well known that the linear MMSE receiver can be obtained multiplying the inverse of the covariance matrix by the desired user total channel (comprehensive of multipath and spreading) [2]. The estimation errors of the covariance matrix of the signal received through the frequency selective fading channels may introduce BER performance degradation. This problem worsens for time varying channels, because the estimate can be done by averaging on few bits. Moreover, the estimation errors are amplified by the inverse operation, particularly at high SNR, producing bit error rates remarkably higher than those obtained in the ideal situation.

This paper proposes to reduce such errors by two approaches. Both of them rely on the spectral decomposition of the covariance matrix that is also used for the multiuser channel estimation. The first one is based on perturbing the estimated eigenvalues leading to a CMOE type receiver [3], while the other one is based on a successive re-estimation of the covariance matrix by exploiting the estimated channels of all users.

II. SYSTEM MODEL

The multiple-input multiple-output (MIMO) baseband channel model introduced in [4] is herein summarised. The transmitted signal of the *k*th user, in a DS-CDMA system with K users, is expressed by (1)

$$x_k(t) = A_k \sum_{i=0}^{I-1} b_k(i) s_k(t - iT - \tau_k) \qquad (1)$$

where *T* is the symbol duration, A_k and $s_k(t)$ are the amplitude and the spreading waveform of the *k*th user respectively, τ_k is the *k*th user relative delay ($0 \le \tau_k < T$), *I* is the number of transmitted symbols, and $b_k(i)$ is the *i*th symbol of the *k*th user. It is assumed that $b_k(i)$ belongs to a set of indipendent equiprobable $\{\pm 1\}$ random variables. The spreading waveform $s_k(t)$ can be expressed by (2)

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} c_k(j) \psi(t - jT_c) \qquad 0 \le t < T \qquad (2)$$

where *N* is the processing gain, $T_c = T / N$ is the chip duration, $\psi(t)$ is the normalised rectangular chip waveform of duration T_c and $c_k(j)$ is the $\{\pm 1\}$ *j*th value of the *k*th user binary code sequence. We suppose to deal with slowly time varying channels such that they can be considered constant during the transmission of *P* symbols $(P \le I)$. The *k*th user channel is denoted by (3)

$$g_k(t) = \sum_{q=1}^{Q} \alpha_{q,k} \delta(t - \tau_{q,k}) \qquad (3)$$

where *Q* is the number of paths, $\tau_{q,k}$ and $\alpha_{q,k}$ are respectively the delay and the complex amplitude of the *q*th path, $\Delta_k = \max_{q1,q2} \left| \tau_{q1,k} - \tau_{q2,k} \right|$ is the maximum delay spread, $\delta(t)$ is the Dirac function. When the channels are supposed to be constant, the received signal component of the *k*th user is

$$y_k(t) = x_k(t) * g_k(t) = \sum_{q=1}^{Q} \alpha_{q,k} x_k(t - \tau_{q,k}) . \qquad (4)$$

The total received signal is the superposition of the *K* signals $y_k(t)$ and a complex zero-mean white Gaussian noise $v(t)$ with power spectral density σ^2. The received signal $r(t)$, expressed by (5), is first filtered by a chip-matched filter and then sampled at the chip rate $1 / T_c$, obtaining (6)

$$r(t) = \sum_{k=1}^{K} y_k(t) + v(t) = y(t) + v(t) . \qquad (5)$$

$$r_n(l) = y_n(l) + v_n(l) = \int_{lT+nT_c}^{lT+(n+1)T_c} r(t)\psi(t - lT - nT_c)dt \,. \quad (6)$$

The sampling instant is selected arbitrarily, leading to a mean power loss of 1.76 dB with respect to a parallel symbol matched filter architecture characterised by perfect time synchronisation [5]. The discrete-time signal component due to the *k*th user is expressed by (7)

$$\begin{aligned} y_{nk}(l) &= \int_{lT+nT_c}^{lT+(n+1)T_c} y_k(t)\psi(t - lT - nT_c)dt \\ &= A_k \sum_{i=0}^{L_k-1} b_k(l-i) \sum_{j=0}^{N-1} c_k(j) \sum_{q=1}^{Q} \alpha_{q,k} R_\psi(iT + (n-j)T_c - \tau_k - \tau_{q,k}) \end{aligned} \quad (7)$$

where $R_\psi(t)$ is the autocorrelation function of $\psi(t)$. By the following expressions

$$h_k(w) = A_k \sum_{j=0}^{N-1} c_k(j) \sum_{q=1}^{Q} \alpha_{q,k} R_\psi[(w-j)T_c - \tau_k - \tau_{q,k}] \quad ,0 \le w \le L_k N$$

$$h_{nk}(i) = h_k(iN + n) \quad ,0 \le i \le L_k - 1 \qquad 0 \le n \le N-1$$

it is possible to obtain (8), where L_k is the duration in symbol intervals of the *k*th channel taking into account the τ_k delay

$$y_n(l) = \sum_{k=1}^{K} y_{nk}(l) = \sum_{k=1}^{K} b_k(l) * h_{nk}(l) = \sum_{k=1}^{K} \sum_{i=0}^{L_k-1} b_k(l-i) h_{nk}(i) \,. \quad (8)$$

If the discrete-time channel coefficients $h_{nk}(i)$ are grouped in matrices $\underline{H}(l)$, the received samples $r_n(l)$ in $[N \times 1]$ vectors $\underline{r}(l)$, and the symbols $b_k(l)$ in $[K \times 1]$ vectors $\underline{b}(l)$, by the notation in [4], the following MIMO relation (9) is obtained

$$\underline{r}(l) = \underline{y}(l) + \underline{v}(l) = \underline{H}(l) * \underline{b}(l) + \underline{v}(l) \,. \quad (9)$$

If *m* successive vectors $\underline{r}(l)$ are stacked to form a $[Nm \times 1]$ $\mathbf{r}_m(l)$ vector, the expression (10) is obtained

$$\mathbf{r}_m(l) = H_m \mathbf{b}_m(l) + \mathbf{v}_m(l), \quad (10)$$

where H_m is the generalised block Sylvester matrix of dimension $[Nm \times K(m + L - 1)]$ and $L = \max\{L_k\}$ (see [4] for more details). The received signal autocorrelation matrix is defined by (11)

$$\mathbf{C}_r = E\left\{\mathbf{r}_m(l)\mathbf{r}_m(l)^H\right\}. \quad (11)$$

Finally, it is useful to define $\mathbf{h}_k = [h_k(0) \ \cdots \ h_k(LN-1)]^T$ and

$$\underline{h}_k = \begin{cases} [h_k(0) \cdots h_k(LN-1) \ 0 \ \cdots \ 0]^T / \|\mathbf{h}_k\| & ,m > L \\ [h_k(0) \cdots\cdots h_k(mN-1)]^T / \|\mathbf{h}_k\| & ,m \le L. \end{cases} \quad (12)$$

III. MMSE Multiuser Receivers

A. Ideal MMSE and estimated MMSE receivers

If the transmitted data are BPSK mapped, the receiver decision rule is expressed by (13)

$$\hat{b}_k(l) = \mathrm{sgn}\left[\mathrm{Re}(\mathbf{m}_k^H \mathbf{r}_m(l))\right], \quad (13)$$

where the $\mathbf{m}_k$ vector represents the detector. The MMSE receiver is obtained by minimising $E\left[| \, b_k(l) - \mathbf{m}_k^H \mathbf{r}_m(l) \, |^2\right]$, which leads to expression (14)

$$\mathbf{m}_k = \mathbf{C}_r^{-1} \underline{h}_k \,. \quad (14)$$

However, in practical situations, neither $\underline{h}_k$ nor $\mathbf{C}_r$ are known. The channel $\underline{h}_k$ can be estimated either by the aid of training sequences or by blind techniques [4] [6]. In the present paper the blind method proposed in [4] is considered, because the receiver expression $\mathbf{m}_k$ can be obtained at almost no extra computational cost if the channel has been estimated by the same approach. The subspace decomposition of the $\mathbf{C}_r$ matrix is shown in the following expression (15)

$$\begin{aligned} \mathbf{C}_r = \mathbf{U} \, \Lambda \, \mathbf{U}^H &= \begin{bmatrix} \mathbf{U}_S & \mathbf{U}_N \end{bmatrix} \begin{bmatrix} \Lambda_S & \mathbf{0} \\ \mathbf{0} & \Lambda_N \end{bmatrix} \begin{bmatrix} \mathbf{U}_S^H \\ \mathbf{U}_N^H \end{bmatrix} = \\ &= \mathbf{U}_S \Lambda_S \mathbf{U}_S^H + \mathbf{U}_N \Lambda_N \mathbf{U}_N^H = \mathbf{C}_{r,S} + \mathbf{C}_{r,N} \ , \end{aligned} \quad (15)$$

with

$$\Lambda_S = diag\left\{\lambda_1, ..., \lambda_d\right\} \ , \qquad \mathbf{U}_S = \begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_d \end{bmatrix} \ ,$$

$$\Lambda_N = \sigma^2 \mathbf{I}_{Nm-d} \ , \qquad \mathbf{U}_N = \begin{bmatrix} \mathbf{u}_{d+1} & \cdots & \mathbf{u}_{Nm} \end{bmatrix} \ ,$$

$$\lambda_i > \sigma^2 \qquad , \forall i = 1, ..., d \ ,$$

and $d = K(m + L - 1)$ is the signal subspace dimension.

Since the channel $\underline{h}_k$ lies in the signal subspace, a scaled version of $\underline{h}_k$ can be recovered solving equation (16) in the least square sense [4] when only an estimated version of $\mathbf{U}_N^H$ is known (for example when $\mathbf{C}_r$ is estimated by (19))

$$\mathbf{U}_N^H H_m = \mathbf{0} \,. \quad (16)$$

The inverse of $\mathbf{C}_r$ can be easily obtained by the spectral decomposition (15) as expressed by (17)

$$\mathbf{C}_r^{-1} = \mathbf{U} \, \Lambda^{-1} \mathbf{U}^H \,. \quad (17)$$

Taking (16) into account, it is clear that only the signal subspace is effective to express (14) that becomes (18)

$$\mathbf{m}_k = \mathbf{C}_r^{-1} \underline{h}_k = \mathbf{C}_{r,S}^{-1} \underline{h}_k = \mathbf{U}_S \Lambda_S^{-1} \mathbf{U}_S^H \underline{h}_k = \sum_{i=1}^{d} \frac{\mathbf{u}_i^H \underline{h}_k}{\lambda_i} \mathbf{u}_i \,. \quad (18)$$

Nevertheless, when $\mathbf{C}_r$ is replaced by its estimated version[1] (19)

$$\hat{\mathbf{C}}_r = \frac{1}{P - m + 1} \sum_{l=0}^{P-m} \mathbf{r}_m(l) \mathbf{r}_m(l)^H \ , \quad (19)$$

the estimation errors will not be negligible if the P averaging factor is low. It is noteworthy to remind that, in practical applications, it is not possible to increase P as we like in order to reduce the estimation errors, because the channels cannot be considered constant due to the mobility of the users.

The estimation errors effect becomes larger when the *eigenvalue spread* of $\mathbf{C}_r$ increases [7], as it happens if many users are transmitting at high SNR. Therefore the estimated MMSE receiver has to be expressed as function of the true quantities and of the estimation errors, as in equation (20):

[1] The subspace decomposition of the estimated covariance matrix (19) gives the maximum likelihood estimate of the eigenvalues and eigenvectors of $\mathbf{C}_r$ [5].

$$\begin{aligned}
\hat{\mathbf{m}}_k &= \sum_{i=1}^{d} \frac{\hat{\mathbf{u}}_i^H \hat{\underline{h}}_k}{\hat{\lambda}_i} \hat{\mathbf{u}}_i = \sum_{i=1}^{d} \frac{\left(\mathbf{u}_i + \Delta\mathbf{u}_i\right)^H \left(\underline{h}_k + \Delta\underline{h}_k\right)}{\lambda_i + \Delta\lambda_i} \left(\mathbf{u}_i + \Delta\mathbf{u}_i\right) \\
&= \sum_{i=1}^{d} \frac{\mathbf{u}_i^H \underline{h}_k}{\lambda_i} \mathbf{u}_i - \sum_{i=1}^{d} \frac{\Delta\lambda_i}{\lambda_i + \Delta\lambda_i} \frac{\mathbf{u}_i^H \underline{h}_k}{\lambda_i} \mathbf{u}_i + \\
&\quad + \sum_{i=1}^{d} \frac{\Delta\mathbf{u}_i^H \hat{\underline{h}}_k + \mathbf{u}_i^H \Delta\underline{h}_k}{\lambda_i + \Delta\lambda_i} \mathbf{u}_i + \sum_{i=1}^{d} \frac{\hat{\mathbf{u}}_i^H \hat{\underline{h}}_k}{\lambda_i + \Delta\lambda_i} \Delta\mathbf{u}_i \\
&= \mathbf{m}_k + \mathbf{e}_{k,0} + \mathbf{e}_{k,1} + \mathbf{e}_{k,2} \quad .
\end{aligned} \tag{20}$$

B. CMOE receiver

An approach for reducing the $\| \mathbf{e}_{k,0} + \mathbf{e}_{k,1} + \mathbf{e}_{k,2} \|$ norm is to add a positive number ν to the signal subspace eigenvalues. Thus, the new receiver $\mathbf{c}_k$ is expressed by (21)

$$\begin{aligned}
\mathbf{c}_k &= \sum_{i=1}^{d} \frac{\hat{\mathbf{u}}_i^H \hat{\underline{h}}_k}{\hat{\lambda}_i + \nu} \hat{\mathbf{u}}_i = \sum_{i=1}^{d} \frac{\mathbf{u}_i^H \underline{h}_k}{\lambda_i} \mathbf{u}_i - \sum_{i=1}^{d} \frac{\Delta\lambda_i + \nu}{\lambda_i + \Delta\lambda_i + \nu} \frac{\mathbf{u}_i^H \underline{h}_k}{\lambda_i} \mathbf{u}_i \\
&\quad + \sum_{i=1}^{d} \frac{\Delta\mathbf{u}_i^H \hat{\underline{h}}_k + \mathbf{u}_i^H \Delta\underline{h}_k}{\lambda_i + \Delta\lambda + \nu} \mathbf{u}_i + \sum_{i=1}^{d} \frac{\hat{\mathbf{u}}_i^H \hat{\underline{h}}_k}{\lambda_i + \Delta\lambda_i + \nu} \Delta\mathbf{u}_i \\
&= \mathbf{m}_k + \tilde{\mathbf{e}}_{k,0} + \tilde{\mathbf{e}}_{k,1} + \tilde{\mathbf{e}}_{k,2} \quad .
\end{aligned} \tag{21}$$

The error norm $\| \tilde{\mathbf{e}}_{k,1} + \tilde{\mathbf{e}}_{k,2} \|$ decreases by increasing ν as well as the denominators of (21). However, at the same time $\tilde{\mathbf{e}}_{k,0}$ approaches to $-\mathbf{m}_k$, thus suppressing the receiver useful component. As a consequence, the ν parameter has a negative effect for low errors $|\Delta\lambda_i|$, $\|\Delta\mathbf{u}_i\|$ and $\|\Delta\underline{h}_k\|$, because the suppression of $\mathbf{m}_k$ is its main effect (the norm of $(\mathbf{e}_{k,1} + \mathbf{e}_{k,2})$ already being small). On the contrary, if such errors are considerably high, a satisfactory reduction of $\| \tilde{\mathbf{e}}_{k,1} + \tilde{\mathbf{e}}_{k,2} \|$ can be obtained, avoiding an excessive decrease of $\| \mathbf{m}_k + \tilde{\mathbf{e}}_{k,0} \|$ for ν values not too large. It is noteworthy that:

i) The ν parameter allows the eigenvalue spread reduction from $\lambda_{\max} / \lambda_{\min,S}$ to $(\lambda_{\max} + \nu)/(\lambda_{\min,S} + \nu)$, where $\lambda_{\min,S} = \min\{\lambda_1, ..., \lambda_d\}$, even if the matrix $\hat{\mathbf{C}}_{r,S} + \nu\hat{\mathbf{U}}_S\hat{\mathbf{U}}_S^H$ can be considered a biased estimate of $\mathbf{C}_{r,S}$.

ii) In absence of estimation errors, the receiver $\mathbf{c}_k$ includes a huge class of linear receivers: for $\nu = 0$ it is the MMSE receiver, while for $\nu \to +\infty$ it tends to the RAKE receiver. Indeed, if $\nu >> \max\{\lambda_1, ..., \lambda_d\}$, it results that
$$\mathbf{U}[\Lambda + \nu\mathbf{I}_{Nm}]^{-1}\mathbf{U}^H \underline{h}_k \approx \mathbf{U}[\nu\mathbf{I}_{Nm}]^{-1}\mathbf{U}^H \underline{h}_k = \nu^{-1}\underline{h}_k .$$
If negative ν values are considered, also the decorrelator receiver falls in this class ($\nu = -1$).

iii) The solution obtained by adding ν is the same which results by the constrained minimisation (22)
$$\min_{\mathbf{x}_k} \left\{ E\left[\left| b_k(l) - \mathbf{x}_k^H \mathbf{r}_m(l) \right|^2 \right] + \nu \left\| \mathbf{x}_k \right\|^2 \right\} . \tag{22}$$
The proposed receiver is therefore a CMOE receiver. However, the correlator norm constraint is introduced in [3] and in [8] to take into account the signature waveform mismatch $\Delta\underline{h}_k$, which norm decreases as SNR increases. On the contrary, in this case $\Delta\mathbf{u}_i$ and $\Delta\lambda_i$ are the main errors because the problem originates from the $\hat{\mathbf{C}}_r$ inversion and it exists also when $\| \Delta\underline{h}_k \| \approx 0$ [7].

Nevertheless, like in [8], the main difficulty is the choice of ν, because the optimum value should depend not only on the system ill-conditioning (eigenvalue spread of $\mathbf{C}_r$), but also on the amount of the estimate errors (i.e. on the P parameter and on the algorithm used for channel estimation). Algorithms in order to select the optimum value of ν are still under study.

C. Improved MMSE receiver

An alternative estimate of the $\mathbf{C}_r$ matrix can be obtained substituting (10) in (11) which becomes (23)

$$\begin{aligned}
\mathbf{C}_r &= H_m E\left\{ \mathbf{b}_m(l)\mathbf{b}_m(l)^H \right\} H_m^H + E\left\{ \mathbf{v}_m(l)\mathbf{v}_m(l)^H \right\} \\
&= H_m H_m^H + \sigma^2 \mathbf{I}_{Nm}
\end{aligned} \tag{23}$$

because the data and the noise are uncorrelated, with covariance expressed by (24)

$$E\left\{ \mathbf{b}_m(l)\mathbf{b}_m(l)^H \right\} = \mathbf{I}_d \quad , \quad E\left\{ \mathbf{v}_m(l)\mathbf{v}_m(l)^H \right\} = \sigma^2 \mathbf{I}_{Nm} . \tag{24}$$

Therefore it is possible to obtain an alternative estimate of $\mathbf{C}_r$ by means of (25)

$$\tilde{\mathbf{C}}_r = \hat{H}_m \hat{H}_m^H + \hat{\sigma}^2 \mathbf{I}_{Nm} , \tag{25}$$

with $\hat{H}_m$ obtained by solving (16) in the least square sense. The above technique was suggested by observing that the norm of $\Delta\underline{h}_k$ can be very low and it decreases when the SNR increases. Consequently, $\tilde{\mathbf{C}}_r$ should be a good estimate of $\mathbf{C}_r$ and the new receiver expression becomes (26)

$$\mathbf{i}_k = \tilde{\mathbf{C}}_r^{-1} \hat{\underline{h}}_k . \tag{26}$$

It is noteworthy that:

i) The receiver $\mathbf{i}_k$ needs to know the channels $\hat{H}_m$ of all users and consequently it can be implemented only at the base station. The receivers expressed by (20) and (21) can instead be applied also at the mobile station, because they do not depend on $\hat{\underline{h}}_j$ for $j \neq k$.

ii) The computational complexity increases with respect to receivers (20) and (21). Indeed (26) require the inversion of the new covariance matrix $\tilde{\mathbf{C}}_r$, while receivers (20) and (21) exploit the same spectral decomposition of $\hat{\mathbf{C}}_r$ that is used to recover the channels $\hat{\underline{h}}_j$.

iii) The discrete noise power has to be estimated making use of the noise subspace eigenvalues of $\hat{\mathbf{C}}_r$, as expressed by (27)

$$\hat{\sigma}^2 = \frac{1}{Nm - d} \sum_{i=d+1}^{Nm} \hat{\lambda}_i . \tag{27}$$

IV. Simulations results

Gold sequences of length $N = 31$ have been chosen for the short spreading codes $c_k(j)$. The chip rate has been fixed to $1/T_c = 8.192$ Mcps and consequently the users bit rate is approximately equal to 264 kbps. The channels model is compliant with the *pedestrian B* channel of [9]. The chosen chip rate $1/T_c$ and processing gain N lead to a maximum delay spread Δ_k that is little longer than one symbol duration. The uplink situation is considered with delay τ_k uniformly distributed in $[0,T)$. However, both τ_k and Δ_k of each user are assumed known by the receiver within a chip period, and therefore the maximum channel order *L* is assumed known as in [4]. The window size *m* has been selected considering the identifiability conditions (28) [4]:

$$\begin{gathered} m \geq \lceil K(L-1)/(N-K) \rceil \\ (N-K)m^2 + (L-1)(N-2K)m \geq K(L-1)^2 + NL \end{gathered} \tag{28}$$

Most of the simulated situations lead to $m = 2$ or $m = 3$.

It is well known that all blind estimation techniques can recover the channels up to a complex scalar factor [4] [5]. In this paper, the phase of the maximum magnitude coefficient of the discrete-time channel is supposed to be known. The covariance matrix $\hat{\mathbf{C}}_r$ has been estimated by using *P* = 300 bits with an estimation window $\Delta T = PT \approx 1.1$ ms. For a mobile transmitter with speed $V \leq 30$ Km/h and a carrier frequency $f = 2$ GHz, the classic Clarke channel autocorrelation function is $J_0(2\pi V \Delta T / \lambda) \geq 0.96$ and, therefore, the channels can be considered constant as supposed in section II.

The SNR shown in the figures is the one of the weakest user (user 1) and it is defined as $\mathrm{SNR} = A_1^2 / \sigma^2$. The Normalised Root Mean Square Error (NRMSE) is defined by (29)

$$\mathrm{NRMSE} = \sqrt{\frac{1}{N_{runs}} \sum_{i=1}^{N_{runs}} \left\| \underline{\hat{h}}_1^{(i)} - \underline{h}_1^{(i)} \right\|^2 / \left\| \underline{h}_1^{(i)} \right\|^2} \tag{29}$$

where $N_{runs} = 300$ is the number of independent simulation runs. The simulated NRMSE shown in Fig. 1 outlines that an acceptable channel estimation (i.e. NRMSE < 0.15) is achieved with a SNR greater than 13 dB (18 dB) with 5 users (10 users).

The BER performance of all the MMSE receivers in different scenarios is shown in Figs. 2-4. In each figure it is evident the great SNR penalty of the MMSE receiver (20) with respect to the ideal one (14) (ideal estimations). This fact could have been predicted for low values of the SNR, where the channel estimation is not so accurate (for *P* = 300), but it is a little bit surprising for higher values of the SNR, where the channel NRMSE tends to zero as shown in Fig. 1. This behaviour, consequently, depends on the estimation of the inverse covariance matrix $\mathbf{C}_r^{-1}$ that appears in the multiuser detector expression (14). Such penalty tends to increase for increasing SNR, since the estimation error of $\mathbf{C}_r^{-1}$ is higher at high SNR [7].

The optimal CMOE receiver (i.e. the receiver in which the optimum ν for each SNR value is chosen in order to minimise the BER) is able to recover some of the power lost by the estimation of $\mathbf{C}_r^{-1}$. In all scenarios, the optimal CMOE receiver exhibits a SNR gain with respect to the classic estimated MMSE receiver. This gain increases for high SNR values, even if it also increases the SNR penalty with respect to the ideal MMSE receiver. The first fact is caused by the eigenvalue spread reduction obtained by the ν parameter, while the second one depends on the biasing effect introduced by the ν parameter on the $\mathbf{C}_r$ estimation (see section III.2).

On the contrary, the improved MMSE receiver, which significantly outperforms also the optimal CMOE one, exhibits a BER performance that is closer to the ideal one as the SNR increases. This fact is easy to justify because the improved $\tilde{\mathbf{C}}_r$ is obtained by (25) through the estimated channel matrix $\hat{H}_m$, whose estimation error decreases for increasing SNR, as shown in Fig. 1. As a consequence, even better performance could be achieved for low SNR either using better estimation techniques or increasing the P parameter (up to the maximum value that is allowed by the channel coherence time).

It is noteworthy that, at low SNR, the BER in near-far conditions (MAI = 20 dB) outperforms the one in the power control situation (MAI = 0 dB). This fact is particularly evident for the improved MMSE receiver, that exploits the higher power of the other users for a better multiuser channel estimation.

Moreover, it should be pointed out that the optimal CMOE receiver is not effective in near-far situations (Fig. 3) as in ideal power control situations (Fig. 2). This fact does not depend on the channel estimation (as confirmed by the good performance of the improved MMSE receiver) but on the covariance matrix eigenvalue spread that is higher in near-far situations because the users different power transforms in different signal eigenvalue magnitude. The optimum value of the ν parameter depends on the proposed scenario (users number K, MAI, SNR and users power). The BER performance (for fixed SNR) as function of the parameter (ν / σ^2) is shown in Fig. 5, where it is outlined the existence of an optimum value. The values on the abscissa axis are not uniformly spaced and they are compressed by an arctangent-logarithmic function for graphical convenience. Finally, Fig. 6 shows the optimum (ν / σ^2) parameter as function of the user SNR, outlining an increasing linear dependence for the higher SNR values. This fact is not surprising, because the receiver expression (18) only depends on the signal subspace and, if σ^2 is fixed, an increase of the SNR corresponds to an increase of the signal subspace eigenvalues and consequently the correction should scale in a similar manner. On the contrary, when the SNR is low, the ν parameter has to compensate for the channel estimation errors, too.

V. CONCLUSIONS

The effects of noisy covariance matrix estimation on the BER performance degradation of MMSE multiuser detector have been outlined. The two proposed techniques to counteract for such phenomenon seem to be effective as confirmed by simulations. Further analytical investigations are required in order to obtain an automatic strategy to choose the optimum value for the parameter ν, which mainly depends on the user SNR and on the number of active users.

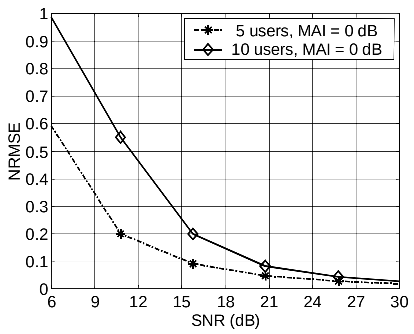

Fig. 1. Channel estimation error (NRMSE)

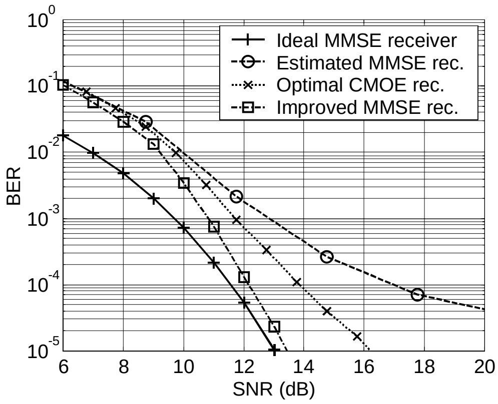

Fig. 2. BER Performance - 5 users - ideal power control

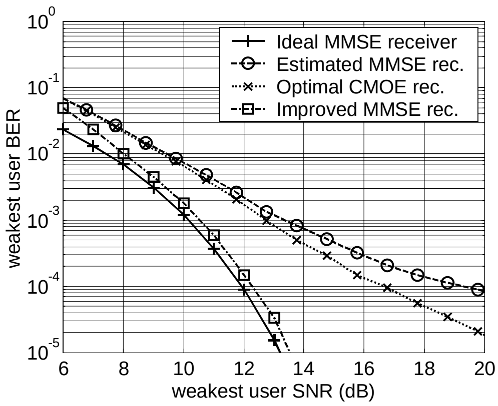

Fig. 3. BER Performance - 5 users - MAI = 20 dB

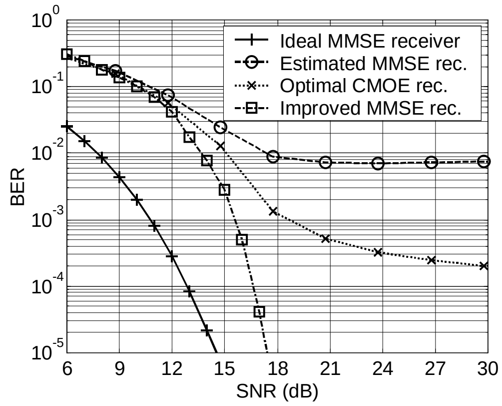

Fig. 4. BER Performance - 10 users - ideal power control

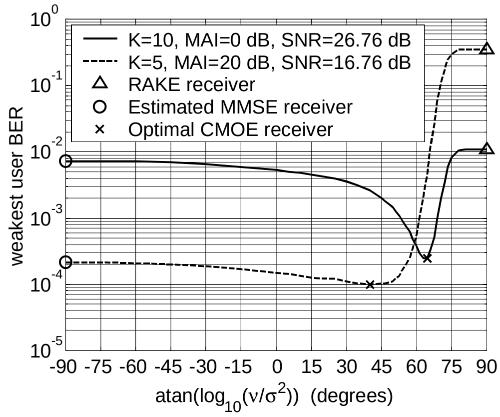

Fig. 5. BER performance as function of the (ν/σ^2) parameter

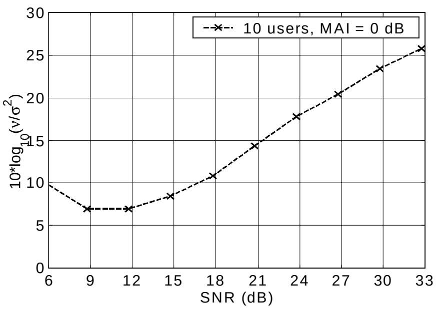

Fig. 6. Optimum (ν/σ^2) as function of the SNR

REFERENCES

[1] S. Verdù, "Multiuser Detection", Cambridge Univ. Press, 1998.

[2] M. Honig and M. Tsatsanis, "Adaptive techniques for multiuser CDMA receivers " IEEE Sig. Proc. Mag., pp. 49-61, May 2000.

[3] M. Honig, U. Madhow and S. Verdù, "Blind adaptive multiuser detection", IEEE Trans. Information Theory, vol. 41, pp. 944-960, July 1995.

[4] X. Wang and H. V. Poor, "Blind equalization and multiuser detection in dispersive CDMA channels", IEEE Trans. Communications, vol. 46, pp. 91-103, Jan. 1998.

[5] S. Bensley and B. Aazhang, "Subspace-based channel estimation for code division multiple access communication systems", IEEE Trans. Communications, vol. 44, pp. 1009-1020, Aug. 1996.

[6] M. Torlak and G. Xu, "Blind multiuser channel estimation in asynchronous CDMA systems", IEEE Trans. Signal Processing, vol. 45, pp. 137-147, Jan. 1997.

[7] P. Banelli, S. Cacopardi and L. Rugini, "Estimation errors sensitivity of MMSE multiuser receivers in DS-CDMA", Proceedings COST 262 Workshop Multiuser Detection in Spread Spectrum Communications, pp. 89-95, Jan. 2001.

[8] U. Madhow, "Blind adaptive interference suppression for the near-far resistant acquisition and demodulation of direct-sequence CDMA signals", IEEE Trans. Signal Processing, vol.45, pp. 124-136, Jan. 1997.

[9] ETSI T.R. 101 112, "Selection procedures for the choice of Radio Transmission Technologies of the UMTS", Apr. 1998.