Huffman Sequence Design for Coded Excitation in Medical Ultrasound

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Abstract

This paper deals with the design of coded-excitation signal for medical ultrasound imaging. In order to design a code sequence that generates an ultrasound signal with good detail resolution and signal-to-noise ratio, both in frequency-dependent and frequency-independent attenuating media, we propose to use a linear Huffman code obtained by an efficiency-driven optimizing procedure. By resorting to computer simulations, we show that this approach is particularly effective and it outperforms other linear coding schemes commonly used in coded-excitation ultrasound imaging.

1. Introduction

Ultrasound coding excitation (CE) in medical imaging is a transmission technique that allows to increase the transmitted energy without increasing the pulse amplitude [1]–[5], by exciting the ultrasound transducer with a long modulated pulse characterized by a Time-Bandwidth-Product greater than one. In the CE framework, some authors proposed single [3] and double transmissions [4] with binary-coded modulations, which are characterized by extremely low complexity. For instance, complementary Golay codes [4], thanks to the side-lobes cancellation granted by double transmissions, provide ideal performance in frequency flat environments, although they highly suffer the presence of frequency-dependent attenuations [2]. On the contrary, linear-FM codes [2] (i.e. chirp) are robust against frequency-dependent attenuations, but suffer of a non-ideal autocorrelation function and, thus, of a worse contrast resolution. In order to have a CE technique characterized by quasi-ideal performance, both in the absence and in the presence of frequency-dependent attenuations, we propose to code the signal by resorting to Huffman sequences, which combine amplitude with phase (and thus frequency) modulation.

To this end, Section 2 provides details of a typical CE architecture, while Section 3 briefly summarizes the Huffman coding theory. Section 4 tests the proposed Huffman coding approach by means of typical ultrasound performance indexes, also employing the Field II [6] simulator, and it makes comparisons with double-transmission Golay CE [4] and with a single-transmission binary CE that employs inverse filtering [3].

2. System Architecture and Ultrasound Coding Excitation

Fig. 1 describes the TX/RX block diagram of a CE ultrasound system that employs a phased-array probe with Q piezoelectric elements.

![Figure 1. Block diagram of a coded-excitation ultrasound system.](image)

The code generator produces the discrete-time baseband digital signal \( \hat{s}[n] \) that is modulated to obtain the RF signal \( s[n] \) expressed by

\[
s[n] = \Re\{\hat{s}[n]e^{j2\pi f_0 T_s n}\}, \tag{1}
\]

where \( f_0 \) is the ultrasound center frequency and \( T_s = 1/f_s \) is the sampling generation time. After digital-to-analog conversion (DAC) and proper amplification, the signal \( s(t) \) excites the ultrasound piezoelectric elements. At the receiver side, in the case of a single scatterer and ignoring noise and signal attenuation, the analog-to-digital converter (ADC) output \( r[n] \) can be approximated by the time- and frequency-shifted version of the transmitted signal \( s[n] \), as expressed by

\[
r[n] \approx \Re\{\hat{s}[n-k_0]e^{j2\pi [-f_d(n-k_0)T_s]}\}, \tag{2}
\]

where \( k_0 \) is the digital round trip delay, \( f_d \) is the frequency shift induced by the frequency-dependent attenuation [1], and \( T_s \) is the sampling interval that, for simplicity, is
assumed equal to that one used in the generation process. The frequency shift is typically approximated by [7]

$$f_d = \beta B_r^2 f_0^2 z,$$

(3)

where $\beta$ is the frequency dependent attenuation coefficient, $B_r$ is the relative bandwidth of the transmitted pulse and $z$ is the depth of the reflecting scatterer.

The discrete-time RF signal $r[n]$ is successively processed to compress (decode) the effective impulse response, and consequently restore the spatial resolution. More precisely, the output of the pulse compressor is obtained by cross-correlating the received waveform $r[n]$ with the pulse compression waveform $\psi[n]$, as expressed by

$$R_{r\psi}[k] = \sum_{m=-\infty}^{+\infty} r[m] \psi[k + m],$$

(4)

which is summarized by its baseband complex counterpart $\tilde{R}_{r\psi}[k]$ expressed by

$$\tilde{R}_{r\psi}[k] = \sum_{m=-\infty}^{+\infty} \tilde{r}[m] \tilde{\psi}[k + m],$$

(5)

and where, by means of (1), $\tilde{r}[n]$ and $\tilde{\psi}[n]$ are the complex envelope associated to the RF received signal $r[n]$ and the compression waveform $\psi[n]$, respectively. While in the absence of frequency-dependent attenuation the pulse compression output is $\tilde{R}_{r\psi}[k] \approx \tilde{R}_{r\psi}[k + k_0]$, when $f_d \neq 0$ (5) becomes $\tilde{R}_{r\psi}[k] \approx \tilde{\chi}_{\tilde{s}\tilde{r}}(k + k_0, f_d)$, where the ambiguity function

$$\tilde{\chi}_{\tilde{s}\tilde{r}}(k, f_d) = \sum_{m=-\infty}^{+\infty} s^*[m] \tilde{\psi}[m + k] e^{-j2\pi f_d m T_s},$$

(6)

shows how the cross-correlation function changes with a frequency variation $f_d$. Our aim is to design a system characterized by a ridge [1] ambiguity function, in order to guarantee, even with a frequency-dependent attenuation, both a good detail and a good contrast resolution, which depend, respectively, on the width of the main lobe and on the ratio main lobe and the side lobes of $\psi[n]$. The discrete-time RF signal $r[n]$ is successively processed to compress (decode) the effective impulse response, and consequently restore the spatial resolution. More precisely, the output of the pulse compressor is obtained by cross-correlating the received waveform $r[n]$ with the pulse compression waveform $\psi[n]$, as expressed by

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$$\tilde{s}[n] = \sum_{i=0}^{N} c_i p[n - iM],$$

(7)

where $p[n]$ is the pulse shaping waveform, $c_i$ are the codes of length $N$, and $M$ is an opportune upsampling factor. We will compare different (L)-CE approaches with respect to the detail and the contrast resolution, and the signal-to-noise ratio gain $GSNR$, which is defined by

$$GSNR = \frac{SNR_c}{SNR_0} = \frac{|R_{r\psi}[0]|^2}{(|R_{r\psi}[0]|^2 + |R_{ss}[0]|^2)^2},$$

(8)

where $\sigma^2$ is the system noise power, $SNR_c$ is the signal-to-noise ratio guaranteed by the CE technique and a pulse compression waveform $\psi[n]$, and $SNR_0$ is the $SNR$ at the reception of $r_0[n]$ when a single pulse (without CE) $s_0[n] = p[n] \sin[2\pi f_0 n]$ is transmitted.

3. Huffman coding

The design of good linear CE sequences aims at obtaining a coded waveform $s[n]$ characterized by an autocorrelation function $\tilde{R}_{ss}[k]$ that is similar to that of a single strong pulse, and, in the presence of frequency shifts, a ridge ambiguity function. Thus, by means of eq. (7) our goal is to find a sequence $\{c_i\}$ whose discrete autocorrelation function is similar to a Kronecker delta $(R_{ss}[k]) = \sum_{i=0}^{N} c_i c_{i+k} = \delta[k]$ such that $\tilde{R}_{ss}[k] = R_{pp}[k]$, and successively to design $p[n]$ in order to meet our requirements.

In 1962 Huffman [8] found out a family of complex discrete sequences $\{c_{H}\}$ with autocorrelation functions $R_{c_{H}c_{H}}[k]$ expressed by

$$R_{c_{H}c_{H}}[k] = \begin{cases} 
\sum_{i=0}^{N} |c_{H,i}|^2, & k = 0 \\
R_{c_{H}c_{H}}[0] X^{-N}, & 0 < k < N \\
1 - X^{-2N}, & k = N
\end{cases},$$

(9)

where $X$ is a design parameter, and that, by means of (9), are close to our desired target, except for $k = N$. Huffman demonstrated that a sequence $\{c_i\}$ has the autocorrelation function expressed by (9) if its Z-transform $C_H(z)$

$$C_H(z) = c_{H,0} + c_{H,1} z^{-1} + \ldots + c_{H,N} z^{-N},$$

$$C_H(z) = c_{H,0} \prod_{i=1}^{N} (1 - z^{-2} z_i),$$

(10)

has all the zeros $z_i$ that are spaced at equal angular intervals in the $z$-plane and lie in one of two origin-centered circles, with radius $X$ and $1/X$, as expressed by

$$z_i = \begin{cases} 
X \cos(2\pi i / N), & \text{if the } i\text{th zero has radius } X \\
X^{-1} \cos(2\pi i / N), & \text{if the } i\text{th zero has radius } 1/X.
\end{cases}$$

(11)

Further properties of this kind of sequences can be summarized as

$$MSR = R_{c_{H}c_{H}}[0] = X^N + X^{-N},$$

$$\eta = \frac{MSR}{\max_{n} |c_{H,n}|^2},$$

(12)

where the $MSR$ is equal to the code energy, and $\eta$ represents the efficiency of the sequence, which influences the $GSNR$ achievable with the specific code.
Once the two parameters $N$ and $X$ are selected (e.g. by choosing the maximum sequence length and the $MSR$ in (12)), according to (10), there are $2^N$ different sequences with the same autocorrelation function expressed by (9), each one characterized by its own efficiency $\eta$ and ambiguity function $\tilde{\gamma}_{ss}(k, f_a)$. We suggest to apply the synthesizing method described by Ackroyd in [9] in order to choose the Huffman sequence $\{c_{Hi}\}$. Indeed, although this procedure consists on the search of the Huffman zero pattern that maximize the code efficiency $\eta$ in (12), it also provides a sequence with a very ridge ambiguity function.

4. Coding Performance

In this section we consider the performance of the proposed Huffman sequence design with two other linear coding approaches described in [3] and [4]. First we evaluate the matched and mismatched filter output for the different coding methods. Successively, by exploiting the Field II simulator [6], we compare the scan-lines amplitudes obtained with a B-mode imaging approach and a specific beamforming scheme, taking into account also the tissue attenuation (e.g. the ambiguity function impact).

We consider a linear code length $N = 26$ and a pulse shaping waveform $p[n]$ designed as a 120 taps FIR filter with $B = 2.6$ MHz that implements the Gaussian ($\alpha = 3.5$) window described in [10], whose pulse compression performance are better evaluated in [11]. The up-sampling factor has been set to $M = \lceil f_s/B \rceil = 38$, which corresponds to a signal duration $T \approx 10\mu s$ at $f_s = 100$ MHz.

The Huffman sequence has been generated according to the Ackroyd approach [9], by setting in (12) the parameter $X$ in order to guarantee $MSR = 100$ dB.

For a first comparison we use the binary inverse filtering (BIF) code sequence found in [3] with $N = 26$. This is the "near optimal sequence" and a FIR least square inverse filter is employed as pulse compression mismatched filtering. We use a filter code length $N_v = 3N$, as suggested in [3].

An alternative linear CE is the Golay coding approach described in [4], which provides ideal impulse-like auto-correlation performance at the price of a double transmission and, consequently, of a frame-rate reduction in B-mode images. Additionally, motion artifacts are expected to degrade the side-lobe cancellation.

The performance are evaluated when the received signal is altered by the presence of a transducer impulse response. More precisely we consider a 4 MHz transducer with 65% of fractional bandwidth, modeled as a linear band-pass filter implemented as a two stage 101 taps FIR filter that employs Hamming windowing.

Fig. 2 compares the matched filter output $R_{m}^{THU} [k]$ of the Huffman sequence, versus the mismatched filter output of a binary sequence with inverse filtering designed as [3] and versus a double transmission Golay matched filter as in [4], in the absence of frequency-dependent attenuation. It is clear that, in this scenario, the Golay approach provides ideal $MSR$ performance, the inverse filtering method guarantees $MSR \approx 45$ dB, while the Huffman coding is designed in order to have $MSR = 100$ dB. The main lobe, and thus the axial resolution, of all the three methods is identical. Indeed it can be easily demonstrated that for linear coding the main-lobe amplitude depends only on the pulse shaping function $p[n]$ that is used (see [11] for further details).

Table 1 shows, as concern the $GSNR$ performance, the Huffman code largely outperforms the single transmission (BIF) described in [3], while the Golay code [4], also thanks to its double transmission, provides significantly higher values of $GSNR$.

![Figure 2. Pulse compression performance comparison in absence of frequency-dependent attenuation.](image)

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<tr>
<td>$GSNR$ (dB)</td>
<td>9.3</td>
<td>8.9</td>
<td>13.6</td>
</tr>
<tr>
<td>$MSR$ (dB)</td>
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<td>$\infty$</td>
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<tr>
<td>Axial Resolution (mm)</td>
<td>1.7</td>
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other, at absolute distances ranging from 40 to 200 mm from the transducer, which is modeled as a 32 elements phased array probe. We also used a fixed focus transmission beamforming [12] at 100 mm distance and a dynamic receive beamforming [12]. As we did for the results of Fig. 2, each piezoelectric element is modeled by a filter with center nominal frequency $f_0 = 4$ MHz and with 65% fractional bandwidth.

![Figure 3](image_url)

**Figure 3.** Compressed central rf-line in frequency-independent (left) and frequency-dependent attenuating medium (right) for a) Huffman coding (top) - b) BIF coding (middle) - c) Golay coding (bottom).

Fig. 3 (a)-(c) show the compressed RF-line in a frequency-independent and in frequency-dependent mediums ($\beta = 0.7$ dB/(MHz cm)) for Huffman, BIF and Golay coding, respectively. It is now evident that, in the presence of attenuation, both the BIF and the Golay coding suffer a pronounced degrade of MSR, especially for far scatterers. On the contrary, thanks to the quasi-ridge ambiguity function of the design we propose, Huffman coding provides very good performance also in the presence of frequency-dependent attenuation, thus outperforming the other linear CE approaches in practical scenarios.

### 5. Conclusions

This paper has presented a new approach for linear coding excitation in medical ultrasound systems, based on Huffman coding theory. The transmitted Huffman sequence is designed by a technique that, for a fixed code length, optimizes the code efficiency and thus the final GSNR and that also provides a very good ambiguity function to ensure robustness of the system against frequency-dependent tissue attenuations. Pulse compression is performed by a matched filtering approach and provides a very good contrast resolution if compared with single [3] and double transmission [4] coding, especially in attenuating tissues.

### References


