Performance Analysis of the Decorrelating Multiuser Detector for Nonlinear Amplified DS-CDMA Signals

L. Rugini, P. Banelli, S. Cacopardi
D.I.E.I.- University of Perugia – Via G.Duranti 93/a – 06125 -PERUGIA – Italy

***Abstract*- This paper analyses the effects of the nonlinear distortions introduced by high-power amplifiers (HPAs) on the performance of the linear decorrelating multiuser detector (MUD) in synchronous direct-sequence code-division multiple-access (DS-CDMA) systems. By assuming and motivating the Gaussian distribution of the nonlinear distortion noise, the system bit-error rate (BER) has been derived theoretically in additive white Gaussian noise (AWGN) and frequency-flat Rayleigh fading channels. Simulation results are provided in order to validate the theoretical analysis for ideally predistorted amplifiers.**

I. Introduction

DS-CDMA is a widely employed technique for wireless communications in both satellite and cellular mobile systems. The almost constant envelope exhibited by the single user signal in the uplink scenario is clearly an advantage with respect to other techniques like multicarrier CDMA. Such a characteristic alleviates the system sensitivity to nonlinear distortions introduced mainly at the transmitter side by power amplifiers. Anyway, in the downlink scenario, the overall signal transmitted by a satellite transponder or by a cellular base station is the sum of many independent signals, each one belonging to a different user. Consequently, depending on the fact that each single user signal may add constructively or not to the others, the overall signal amplitude is characterised by a high variability, being exposed to the nonlinear distortions that may be introduced by the common amplifier.

Generally, a predistortion technique [1] is employed at the transmitter side to counteract the nonlinear characteristics of the power amplifier. On the other hand, even if the amplifier is perfectly linearised, a residual clipping (i.e. nonlinear distortion) is not avoidable because of the maximum amplifier output power. Consequently, a soft-limiting characteristic is often employed to model a perfectly predistorted nonlinear amplifier in real scenarios.

Obviously, it is necessary to analyse and quantify the nonlinear distortion effects and the consequent BER performance degradation induced in the system link budget. The *total degradation* (TD) is known in the technical literature as the parameter to optimise the mean output power for a given amplifier and target BER and, consequently, it will be considered herein as the figure of merit to optimise the system link budget.

The same problem has been analysed in [2] for the matched filter detection of DS-CDMA signals in AWGN channels. Anyway, DS-CDMA systems greatly benefit of multiuser detection techniques [3] [4] to improve the BER performance when non-orthogonal spreading waveforms are employed or when frequency-selective fading channels destroy the user orthogonality. MUD techniques become mandatory when the system load (i.e. the ratio between the number of active users and the processing gain) increases, because of their capability in reducing the multiuser interference at the receiver side. The linear MUDs gained great popularity because of their reduced complexity with respect to the nonlinear ones and, among them, the decorrelator is the simplest to implement because it does not require the knowledge of the signal power for each user.

The main object and contribution of this paper is to consider how the BER performance of the decorrelating MUD in DS-CDMA systems is affected by perfectly predistorted nonlinear amplifiers. Analytical BER expressions for binary phase-shift keying (BPSK) modulation in AWGN channels and in flat-fading Rayleigh scenarios have been derived. Simulation results are used in order to verify the analytical findings.

II. System Model

The baseband signal transmitted by a base station to the *k*th user, in the downlink of a CDMA system, is expressed by

$$x_k(t) = A_k \sum_{i=-\infty}^{+\infty} b_k[i] s_k(t-iT)\,, \tag{1}$$

where T is the symbol duration, A_k and $s_k(t)$ are the amplitude and the spreading waveform respectively, and $b_k[i]$ is the *i*th symbol of the *k*th user. The spreading waveform can be expressed as

$$s_k(t) = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} c_k[j] p(t-jT_c)\,, \tag{2}$$

where N is the processing gain, $T_c = T/N$ is the chip duration, $p(t)$ is the impulse response of the chip pulse shaping filter and $c_k[j]$ is the *j*th value of the *k*th user binary spreading code. If BPSK modulation is used, then the symbols $\{b_k[i]\}$ belong to a set of independent and equiprobable $\{\pm 1\}$ random variables, and the overall signal $x_k(t)$ is real.

The base station transmits synchronously the sum of the signals belonging to each user by a HPA that, if supposed to be instantaneous, can be modelled by its AM/AM and AM/PM distortion curves $G(\bullet)$ and $\Phi(\bullet)$ respectively [1]. The above curves can be summarised by a complex nonlinear distortion $F(\bullet)$ as expressed by

$$F(\bullet) = F_R(\bullet) + jF_I(\bullet) = G(\bullet)\exp[j\Phi(\bullet)]\,. \tag{3}$$

The sum $z(t)$ of the K users signals, expressed by

$$z(t) = \sum_{k=1}^{K} x_k(t) = \; |z(t)| \exp(j\arg(z(t)))\,, \tag{4}$$

is transformed by (3) into

$$\begin{aligned} w(t) &= F(|z(t)|)\cdot \exp(j\arg(z(t))) \\ &= [F_R(|z(t)|) + jF_I(|z(t)|)]\cdot \exp(j\arg(z(t)))\,, \end{aligned} \tag{5}$$

which represents the baseband input-output relationship for the nonlinear amplifier. The term $\exp(j\arg(z(t)))$ in (5) can assume values only in the set $\{\pm 1\}$, since the BPSK modulation makes $z(t)$ real. However, as better analysed in the next section, the output signal $w(t)$ can be generally represented as the input signal multiplied by a complex coefficient α, which represents the average linear amplification gain, plus a nonlinear distortion complex noise $n_d(t)$, as expressed by

$$\begin{aligned} w(t) &= \alpha \cdot z(t) + n_d(t) \\ &= |\alpha| \exp(j\arg(\alpha)) \cdot z(t) + n_d(t) \ . \end{aligned} \tag{6}$$

This paper assumes that the signal $w(t)$, which is transmitted by the base station, passes through a frequency-flat channel characterised by an impulse response

$$g(t,\tau) = \beta(t)\exp(j\theta(t)) \cdot \delta(\tau) \ , \tag{7}$$

where $\beta(t)$ and $\theta(t)$ are the gain and the phase-shift of the channel respectively, and $\delta(\tau)$ is the delta Dirac function. Two different situations are considered.

a) Ideal AWGN channel: the amplitude $\beta(t)$ is fixed with time and supposed to be equal to 1. Also the phase-shift $\theta(t) = \theta$ is supposed to be time invariant.
b) Slow fading channel: the amplitude $\beta(t)$ and the phase-shift $\theta(t)$ are supposed to be slowly time varying, so that they can be considered constant during one symbol interval. For a fixed time instant t, the gain $\beta(t)$ is modelled as a Rayleigh random variable, of unit power ($E\{\beta^2\} = 1$) and with probability density function (PDF)

$$f(\beta) = 2\beta \cdot \exp(-\beta^2) \cdot u_{-1}(\beta) \ , \tag{8}$$

where $u_{-1}(\beta)$ is the unitary step function.

At the receiver side, the channel-affected signal is perturbed by a complex zero-mean AWGN $n(t)$, as expressed by

$$\begin{aligned} r(t) &= \int_{-\infty}^{+\infty} g(t,\tau) w(t-\tau) d\tau + n(t) \\ &= \beta(t) |\alpha| \exp(j\theta(t) + j\arg(\alpha)) \cdot z(t) + \\ &\quad + \beta(t)\exp(j\theta(t)) \cdot n_d(t) + n(t) \ . \end{aligned} \tag{9}$$

Firstly, the received signal $r(t)$ is filtered by a chip matched filter and successively sampled at the chip rate $1/T_c$, thus obtaining

$$\begin{aligned} r_n[l] &= \int_{-\infty}^{+\infty} r(t) p^*(t - lT - nT_c) dt \\ &= r_{n,\mathrm{SIG}}[l] + r_{n,\mathrm{NLDN}}[l] + r_{n,\mathrm{AWGN}}[l] \ . \end{aligned} \tag{10}$$

As a consequence of (9), the received sample $r_n[l]$, expressed by (10), is characterised by three additive components: $r_{n,\mathrm{SIG}}[l]$ is the useful part related to $z(t)$, $r_{n,\mathrm{NLDN}}[l]$ is the in-band nonlinear distortion noise due to $n_d(t)$, and $r_{n,\mathrm{AWGN}}[l]$ is the in-band thermal noise. Defining

$$\mathbf{r}[l] = [\ r_0[l] \cdots r_{N-1}[l]\ ]^T \ , \tag{11}$$

$$\mathbf{A} = \mathrm{diag}(A_1, \cdots, A_K) \ , \tag{12}$$

$$\mathbf{b}[l] = [\ b_1[l] \cdots b_K[l]\ ]^T \ , \tag{13}$$

$$\mathbf{C} = \frac{1}{\sqrt{N}} \begin{bmatrix} c_1[0] & \cdots & c_K[0] \\ \vdots & \cdots & \vdots \\ c_1[N-1] & \cdots & c_K[N-1] \end{bmatrix} , \tag{14}$$

we obtain

$$\begin{aligned} \mathbf{r}[l] &= \mathbf{r}_{\mathrm{SIG}}[l] + \mathbf{r}_{\mathrm{NLDN}}[l] + \mathbf{r}_{\mathrm{AWGN}}[l] \\ &= \beta(lT) |\alpha| \exp(j\theta(lT) + j\arg(\alpha)) \cdot \mathbf{CAb}[l] + \\ &\quad + \mathbf{r}_{\mathrm{NLDN}}[l] + \mathbf{r}_{\mathrm{AWGN}}[l] \ , \end{aligned} \tag{15}$$

where the received vectors $\mathbf{r}_{\mathrm{SIG}}[l]$, $\mathbf{r}_{\mathrm{NLDN}}[l]$ and $\mathbf{r}_{\mathrm{AWGN}}[l]$ are obtained as in (11) by collecting the values $r_{n,\mathrm{SIG}}[l]$, $r_{n,\mathrm{NLDN}}[l]$ and $r_{n,\mathrm{AWGN}}[l]$, respectively.

III. Decorrelating Detector BER Analysis

A. *Statistical characterisation of the nonlinear distortion*

The signal $z(t)$, as expressed by (4), is the sum of the signals due to the K users. If the amplitudes $\{A_k\}$ are almost equal and K is sufficiently high, or if K is so high that the users can be grouped in subgroups with a sufficiently high number of users with almost equal amplitudes in each subgroup, then $z(t)$ can be approximated by a real Gaussian random process because of the central limit theorem [2]. However, since K is always a finite value, and since the $\{A_k\}$ are generally equal in satellite communications but not in cellular downlinks, $z(t)$ is not always truly Gaussian, and consequently this approximation has to be carefully checked. Moreover, $E\{z(t)\} = 0$ because the signals $\{x_k(t)\}$ are uncorrelated and zero mean random processes. In such a situation, if the input $z(t)$ is modelled as a zero-mean Gaussian random process, the linear component $\alpha \cdot z(t)$ in (6) and the nonlinear one $n_d(t)$ are mutually uncorrelated, by means of the Bussgang Theorem [5]. Consequently, the output correlation function for a stationary Gaussian input $z(t)$ would be expressed by [6]

$$R_{ww}(\tau) = E\{w^*(t) w(t+\tau)\} = |\alpha|^2 R_{zz}(\tau) + R_{n_d n_d}(\tau) \ , \tag{16a}$$

$$\alpha = R_{zw}(0) / R_{zz}(0) = E\{z^*(t) w(t)\} / E\{z^*(t) z(t)\} \ . \tag{16b}$$

The downlink signal of a synchronous DS-CDMA system is actually not stationary but cyclostationary. Consequently, the autocorrelation functions and the coefficient α in (16) will depend on the time index t. Anyway, such a periodic dependence for the coefficient α is generally negligible with respect to its average in many circumstances (see [2] for deeper details) and consequently relations (16) can be considered valid substituting each quantity with its time average over the symbol period T. Bearing this in mind, the general expression for the correlation of the nonlinearity output driven by Gaussian inputs has the general expression [7]

$$R_{ww}(\tau) = \sum_{i=0}^{+\infty} \gamma_i R_{zz}(\tau)^{2i+1} \ , \tag{17}$$

where $\gamma_0 = |\alpha|^2$. The coefficients γ_i, which depend on the nonlinear function $F(\bullet)$ and on the signal input power, can be calculated by numerical integration or by means of closed form expressions [7] [8]. The knowledge of the coefficients γ_i, together with the statistical properties of the spreading sequences (i.e. the knowledge of the function $R_{zz}(\tau)$), allows the evaluation of the nonlinear noise autocorrelation function by

$$R_{n_d n_d}(\tau) = \sum_{i=1}^{+\infty} \gamma_i R_{zz}(\tau)^{2i+1} . \quad (18)$$

If the spreading codes $\{c_k[j]\}$ are characterised by good autocorrelation properties, such as pseudo-noise or Gold codes, the autocorrelation function $R_{zz}(\tau)$ looks like an impulsive function because $R_{zz}(\tau)/R_{zz}(0) << 1$ when $\tau > T_c$. Therefore, also $R_{ww}(\tau)$ resembles an impulsive function, as well as $R_{n_d n_d}(\tau)$. Consequently $n_d(t)$ is characterised by an approximately flat power spectrum density. The above approximation is reasonably acceptable inside the signal bandwidth, since $R_{zz}(\tau)^{2i+1}$ is represented in the frequency domain by the (2i+1) autoconvolution of the power spectrum $S_{zz}(f)$ [8]. As a consequence, it is possible to approximate the nonlinear distortion noise $n_d(t)$ as a white noise with power

$$\sigma_{n_d}^2 = R_{n_d n_d}(0) = \sum_{i=1}^{+\infty} \gamma_i \sigma_z^{4i+2} . \quad (19)$$

B. *Decorrelator performance in AWGN channels*

Since the transmitted data are BPSK mapped, the receiver decision rule can be expressed as

$$\hat{\mathbf{b}}[l] = \mathrm{sgn}\{\mathrm{Re}(\mathbf{Dr}[l]\varphi)\} \quad (20)$$

where the matrix $\mathbf{D}$ represents the multiuser detector and $\varphi = \exp(-j\theta - j\arg(\alpha))$ is the total phase-shift compensation factor. The channel phase θ and the mean HPA phase-shift $\arg(\alpha)$ are assumed perfectly known to the receiver.

The *linear decorrelating detector* (LDD) is obtained [3] as the Moore-Penrose generalised inverse of the code matrix $\mathbf{C}$:

$$\mathbf{D} = \mathbf{C}^{\dagger} = (\mathbf{C}^T\mathbf{C})^{-1}\mathbf{C}^T = \mathbf{R}^{-1}\mathbf{C}^T , \quad (21)$$

where $\mathbf{R} = \mathbf{C}^T\mathbf{C}$ is the $K \times K$ matrix containing the cross-correlation coefficients of the users code sequences. In (21) it is assumed that the K users codes are linearly independent, from the existence of $\mathbf{R}^{-1}$. It is also assumed that the number of active users K is time-invariant (see [9] for the LDD analysis in a user dynamic environment) and smaller than the processing gain N. Since in AWGN scenarios $\beta(t) = 1$, from (15) and (21) it follows

$$\mathbf{Dr}[l]\varphi = |\alpha|\mathbf{Ab}[l] + \mathbf{R}^{-1}\mathbf{C}^T\mathbf{r}_{\mathrm{NLDN}}[l]\varphi + \mathbf{R}^{-1}\mathbf{C}^T\mathbf{r}_{\mathrm{AWGN}}[l]\varphi . \quad (22)$$

Without loss of generality, we point the attention on the performance of the first user. From (22) we have

$$\begin{aligned} \mathrm{Re}\{\mathbf{D}_{1,:}\mathbf{r}[l]\varphi\} = {} & |\alpha| A_1 b_1[l] + \\ & + \mathbf{R}_{1,:}^{-1}\mathbf{C}^T \mathrm{Re}\{\mathbf{r}_{\mathrm{NLDN}}[l]\varphi\} + \\ & + \mathbf{R}_{1,:}^{-1}\mathbf{C}^T \mathrm{Re}\{\mathbf{r}_{\mathrm{AWGN}}[l]\varphi\} , \end{aligned} \quad (23)$$

where the subscript '1,:' represents the first row of the related matrix. It can be observed that the second term in the right hand side of (23) is the sum of the N elements of $\mathrm{Re}\{\mathbf{r}_{\mathrm{NLDN}}[l]\varphi\}$, weighted by the N elements of $\mathbf{R}_{1,:}^{-1}\mathbf{C}^T$. Therefore, if the processing gain N is high enough, the nonlinear distortion noise $\mathbf{R}_{1,:}^{-1}\mathbf{C}^T \mathrm{Re}\{\mathbf{r}_{\mathrm{NLDN}}[l]\varphi\}$ can be approximated as a Gaussian random variable. As a consequence, since the two noise terms in (23) are uncorrelated, the bit-error probability can be easily evaluated if the powers of the noise terms are known. In the uncorrelated AWGN scenario, $E\{\mathbf{r}_{\mathrm{AWGN}}[l]\mathbf{r}_{\mathrm{AWGN}}[l]^H\} = 2\sigma_{\mathrm{AWGN}}^2\mathbf{I}_N$, and it is easy to prove that

$$\begin{aligned} & \mathbf{R}_{1,:}^{-1}\mathbf{C}^T E\{[\mathrm{Re}\{\mathbf{r}_{\mathrm{AWGN}}[l]\varphi\}][\mathrm{Re}\{\mathbf{r}_{\mathrm{AWGN}}[l]\varphi\}]^T\}\mathbf{C}\left(\mathbf{R}_{1,:}^{-1}\right)^T = \\ & = \mathbf{R}_{1,1}^{-1}\sigma_{\mathrm{AWGN}}^2 , \end{aligned} \quad (24)$$

where the noise enhancement factor $\mathbf{R}_{1,1}^{-1}$ is the 1,1 element of the matrix $\mathbf{R}^{-1}$.

As far as the nonlinear distortion noise is concerned, in the following we will consider the ideal predistortion situation, where the nonlinear function in (5) reduces to a soft-limiter, that is

$$F(|z|) = \begin{cases} |z| & ,|z| < z_{sat} \\ |z_{sat}| & ,|z| > z_{sat} . \end{cases} \quad (25)$$

If the chip-matched filter has a real impulse response, since α is real, from (9) and (10) it follows

$$\mathrm{Re}\{r_{n,\mathrm{NLDN}}[l]\varphi\} = \int_{-\infty}^{+\infty} n_d(t)p(t - lT - nT_c)dt . \quad (26)$$

Therefore the power of the quantity in (26) is equal to

$$\sigma_{\mathrm{NLDN}}^2 = \int_{-\infty}^{+\infty} R_{n_d n_d}(\tau)R_{pp}(-\tau)d\tau , \quad (27)$$

where $R_{pp}(\tau)$ is the autocorrelation of the $p(t)$ waveform and $R_{n_d n_d}(\tau)$ is computable by (18). Since the nonlinear noise has been approximated as a white noise with

$$E\{[\mathrm{Re}\{\mathbf{r}_{\mathrm{NLDN}}[l]\varphi\}][\mathrm{Re}\{\mathbf{r}_{\mathrm{NLDN}}[l]\varphi\}]^T\} \approx \sigma_{\mathrm{NLDN}}^2\mathbf{I}_N , \quad (28)$$

then the nonlinear distortion noise power can be evaluated by $\mathbf{R}_{1,1}^{-1}\sigma_{\mathrm{NLDN}}^2$ analogously to the AWGN noise in (24).

Hence the bit-error probability for the first user is[1]

$$P_{e1} = Q\left(\frac{|\alpha| A_1}{\sqrt{\mathbf{R}_{1,1}^{-1}\sigma_{\mathrm{AWGN}}^2 + \mathbf{R}_{1,1}^{-1}\sigma_{\mathrm{NLDN}}^2}}\right). \quad (29)$$

C. *Decorrelator performance in flat-fading channels*

In a slow flat-fading channel, both the useful signal and the nonlinear distortion noise experience the effect of the random complex gain $\beta\exp(j\theta)$, in which the discrete-time index has been dropped because of the slow time variation. Supposing perfect channel state information and phase compensation, the bit-error probability conditioned to the knowledge of β is expressed by

$$P_{e1}(\beta) = Q\left(\frac{\beta|\alpha| A_1}{\sqrt{\mathbf{R}_{1,1}^{-1}\sigma_{\mathrm{AWGN}}^2 + \beta^2\mathbf{R}_{1,1}^{-1}\sigma_{\mathrm{NLDN}}^2}}\right). \quad (30)$$

The average BER is obtained by integrating (30) over the Rayleigh PDF (8) of the channel gain β, thus obtaining

[1] The Q function is defined as $Q(x) = (2\pi)^{-1/2}\int_x^{+\infty}\exp(-\nu^2/2)d\nu$.

$$P_{e1,\text{FAD}} = \int_{-\infty}^{+\infty} f(\beta) P_{e1}(\beta) d\beta$$
$$= \int_0^{+\infty} 2\beta \exp(-\beta^2) Q\left(\sqrt{\frac{\beta^2 |\alpha|^2 A_1^2}{\mathbf{R}_{1,1}^{-1}(\sigma_{\text{AWGN}}^2 + \beta^2 \sigma_{\text{NLDN}}^2)}}\right) d\beta \quad (31)$$
$$= \int_0^{+\infty} 2\beta \exp(-\beta^2) Q\left(\sqrt{\beta^2 \mu^2 / (1+\beta^2 \lambda^2)}\right) d\beta \ ,$$

where

$$\mu = |\alpha| A_1 \Big/ \left(\sqrt{\mathbf{R}_{1,1}^{-1}} \sigma_{\text{AWGN}}\right) , \qquad \lambda = \sigma_{\text{NLDN}} / \sigma_{\text{AWGN}} \ . \quad (32)$$

The analytical derivation involved with the evaluation of (31) cannot be developed herein because of lack of space and the readers interested in greater details are redirected to [10]. We only report the final expression (33), which is expressed by a series of confluent hypergeometric functions ${}_2F_0$:

$$P_{e1,\text{FAD}} = \frac{1}{2} - \frac{\sqrt{2}}{4} \mu \exp\left(\frac{-\mu^2}{2\lambda^2}\right) \sum_{k=0}^{+\infty} \frac{1}{k!} \left(\frac{\mu^2}{2\lambda^2}\right)^k \cdot {}_2F_0\left(k+\frac{3}{2}, \frac{1}{2};;-\lambda^2\right) . (33)$$

IV. Simulation Results

The situation in which the base station transmits data to $K = 40$ users is considered. The amplitudes $\{A_k\}$ are equal for all users. Gold-like sequences of length $N = 63$ have been chosen for the short spreading codes $\{c_k[j]\}$. The nonlinearity considered is the soft-limiter model (25), which is the envelope input-output characteristic of an ideal predistorted HPA. The user signal-to-noise ratio (SNR) is defined as

$$\text{SNR} = A_1^2 / (2\sigma_{\text{AWGN}}^2) , \quad (34)$$

while the *input back-off* (IBO) and the *output back-off* (OBO) are defined as

$$\text{IBO} = P_{z,\max} / P_z \ , \qquad \text{OBO} = P_{w,\max} / P_w \ , \quad (35)$$

where $P_{z,\max}$ ($P_{w,\max}$) and P_z (P_w) represent the maximum and the average HPA input (output) power respectively.

For a rectangular pulse shaping waveform $p(t)$, the nonlinear distortion noise power σ_{NLDN}^2 in (27) coincides with $R_{n_d n_d}(0)$ in (19) and, by (16a), it obviously simplifies to

$$\sigma_{\text{NLDN}}^2 = P_{w,\max} / \text{OBO} - |\alpha|^2 P_{z,\max} / \text{IBO} \ , \quad (36)$$

where, for soft-limited real Gaussian signals, the coefficient α of (16b) is evaluated as

$$\alpha = \text{erf}\left(\sqrt{\text{IBO}/2}\right) = 2\pi^{-1/2} \int_0^{\sqrt{\text{IBO}/2}} \exp(-\nu^2) d\nu \ . \quad (37)$$

Fig. 1 shows the BER performance of the LDD in AWGN channels for different OBO values. When the OBO values are low, e.g. OBO equal to or smaller than 3.76 dB, the simulated performance does not correspond to the theoretical model because the approximated model of the nonlinear distortion noise is not sufficiently accurate. Indeed, the simulated BER is different from the theoretical one especially when the nonlinear distortion noise is dominant with respect to the AWGN, i.e. at high SNR. Moreover, the good agreement between simulated and theoretical performance at low SNR proves that the signal loss induced by the clipping operation is correctly modelled by the coefficient α in (37). For higher OBO values, the simulated performance exactly matches the analytical model, because the nonlinear distortion noise is smaller than for lower OBOs, and consequently the performance mainly depends on the AWGN.

In order to point out the magnitude of the inaccuracy about the Gaussian assumption of the nonlinear distortion noise (i.e. the second term in the right hand side of (23)), the difference Δ between its estimated cumulative distribution function (CDF) and the ideal Gaussian noise CDF is shown in fig. 2 when OBO = 2.87 dB. Interestingly, in this situation the order of magnitude of Δ is comparable to the error on the BER estimate ($\approx 10^{-3}$). This fact explains the partial mismatch between simulation and analytical results, being the BER performance dependent on the noise CDF by (29).

The BER performance of the LDD in Rayleigh fading channels is shown in fig. 3, where it is confirmed that the theoretical model is more accurate for high OBOs. Anyway, in both AWGN and Rayleigh fading scenarios herein examined, the analytical results can be considered a useful upper bound to the real performance also for low OBO values.

The *total degradation* (TD) is generally considered to establish the most convenient HPA working point in order to optimise the power link budget. The TD to obtain a target BER can be defined as

$$(\text{TD})_{\text{dB}} = [(\text{SNR})_{\text{dB}} - (\text{SNR}_{\text{LIN}})_{\text{dB}}] + (\text{OBO})_{\text{dB}} \ , \quad (38)$$

where SNR and OBO are defined in (34) and (35) respectively, and SNR_{LIN} is the SNR required by the LDD to obtain the target BER in the linear scenario. The term into the square brackets in (38) represents the power penalty with respect to the linear case, while the OBO represents the power penalty with respect to the maximum amplifier output power. Indeed, for a selected target BER, it is desirable to have both low SNR (low transmitted power) and low OBO (high HPA efficiency). These two requirements are in competition with one another because, if the OBO is decreased, the HPA introduces a higher distortion, and consequently the required SNR, for the target BER, must be greater. Therefore, the minimisation of the TD is a fair criterion for the selection of the optimum OBO value if other effects, e.g. the adjacent channel interference, can be neglected or eliminated by linear filtering.

Fig. 4 shows the TD at $\text{BER} = 10^{-3}$ as function of the OBO. The TD is obtained using the approximated analytical model, since little mismatch with the simulation is introduced at this target BER. It is shown that the optimum OBO is close to 5.5 dB for the AWGN channel, while it is approximately 5 dB for the Rayleigh fading environment. It can also be observed that in the Rayleigh fading situation the TD is smaller than in the AWGN scenario for all the OBO values, with a difference of about 0.7 dB between the two optimum values. It can be intuitively explained by considering that, for a fixed OBO, the ratio between the useful signal power and the nonlinear distortion noise power is fixed and that in a Rayleigh fading scenario, where the instantaneous SNR has an exponential PDF, the situation in which the instantaneous SNR is lower than the average SNR is more probable than the opposite. Consequently, the nonlinear distortion noise is partially masked by the thermal noise.

V. Conclusions

An analytical framework to evaluate the BER performance of decorrelating MUDs for DS-CDMA systems subject to amplifier nonlinear distortions in AWGN or Rayleigh fading channels has been introduced. Results for BPSK mapping with rectangular pulse shaping and perfectly predistorted amplifier have been presented, and the OBO that minimises the system TD has been evaluated. Simulation results have shown how the analytical model is appropriate when the number of users and the spreading gain are high enough to justify the Gaussian assumption for the transmitted signal distribution and the nonlinear distortion noise.

The results suggest that the optimum OBO has to be chosen depending on the channel characteristics. Extensions of these results to generally shaped nonlinear amplifier characteristics, as well as to complex constellation mapping (e.g. QPSK, QAM) and to other pulse shaping waveforms will be presented in a future work.

References

[1] A.R. Kaye, D.A. George and M.J. Eric, "Analysis and compensation of bandpass nonlinearities for communications", *IEEE Trans. on Comm.*, vol. COM-20, October 1972, pp. 965-972.

[2] A. Conti, D. Dardari and V. Tralli, "On the performance of CDMA systems with nonlinear amplifier and AWGN", *IEEE ISSSTA 2000*, vol. 1, pp. 197-202.

[3] S.Verdù, *Multiuser Detection*, Cambridge Un.Press, 1998.

[4] M. Honig and M.K. Tsatsanis, "Adaptive techniques for multiuser CDMA receivers" *IEEE Signal Processing Mag.,* May 2000, pp. 49-61.

[5] A. Papoulis, *Probability, Random Variables and Stochastic Processes*, 3ed. McGraw-Hill Inc., 1991.

[6] H.E. Rowe, "Memoryless nonlinearities with Gaussian inputs: elementary results", *Bell Syst. Tech. Journal*, vol. 61, n° 7, September 1982, pp. 1520-1523.

[7] W.B. Davenport, W.L. Root, *An Introduction to the Theory of Random Signals and Noise*, McGraw-Hill, 1958.

[8] P. Banelli and S. Cacopardi, "Theoretical analysis and performance of OFDM signals in nonlinear AWGN channels", *IEEE Trans. on Comm.*, vol. 48, n° 3, March 2000, pp. 430-441.

[9] U. Mitra and H.V. Poor, "Analysis of an adaptive decorrelating detector for synchronous CDMA channels", *IEEE Trans. on Comm.*, vol. 44, Feb. 1996, pp. 257-268.

[10] L. Rugini, P. Banelli and S. Cacopardi, "An analytical upper bound on MMSE performance using approximated MMSE multiuser detector in flat Rayleigh fading channels", *European Wireless 2002,* Florence, Italy.

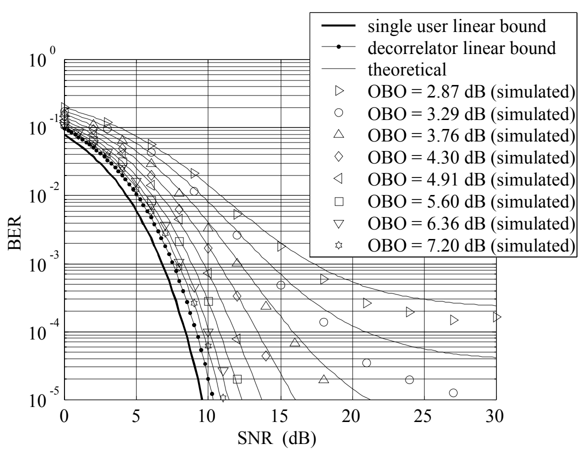

Fig. 1. BER performance in AWGN channels.

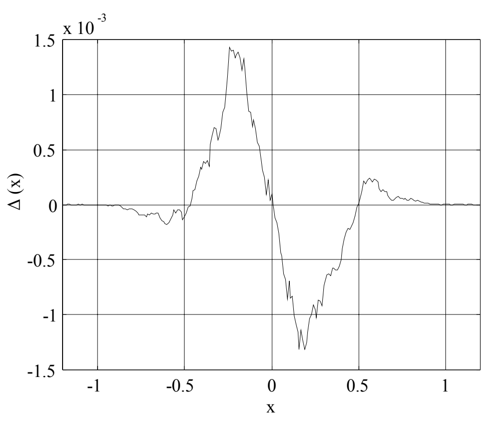

Fig. 2. Difference between the estimated and the ideal CDF.

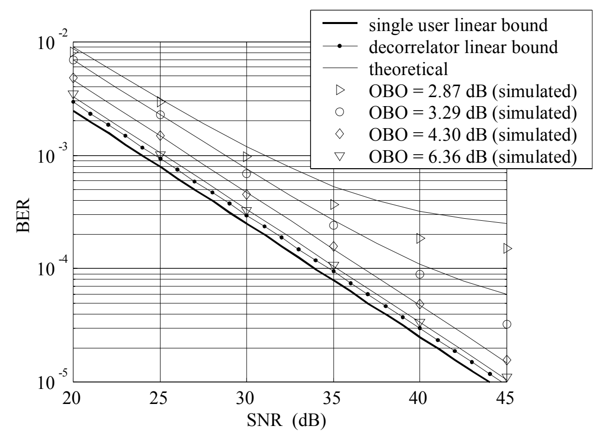

Fig. 3. BER performance in Rayleigh fading channels.

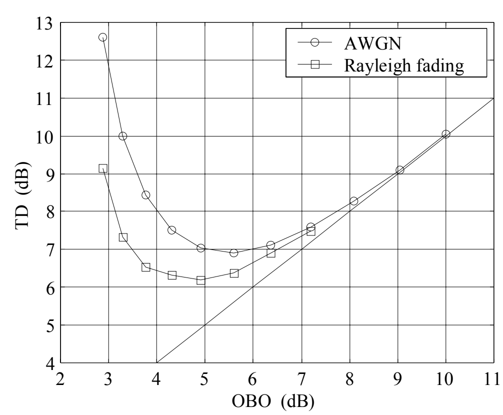

Fig. 4. Total Degradation for $\mathrm{BER} = 10^{-3}$.