PILOT-AIDED ESTIMATION OF CARRIER FREQUENCY OFFSETS AND CHANNEL IMPULSE RESPONSES FOR OFDM COOPERATIVE COMMUNICATIONS

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ABSTRACT

We consider a cooperative communication system where the destination node receives an orthogonal frequency-division multiplexing (OFDM) signal transmitted from two relay nodes. We propose and compare two pilot-aided algorithms for the estimation of the carrier frequency offsets (CFOs) and the channel impulse responses (CIRs) associated with the two relay-destination links. The first algorithm uses a basis expansion model (BEM) to estimate a time-varying (TV) multiple-input single-output (MISO) channel that incorporates both CFOs and CIRs effects. The CFOs are then extracted from the estimated TV-MISO channel by applying estimation of signal parameters via rotational invariance techniques (ESPRIT). The second algorithm exploits the structure of a specific pilot sequence and estimates the CFOs by directly applying ESPRIT on the observed signal. Simulation results show the effectiveness of the proposed algorithms.

Index Terms—Carrier frequency offset (CFO), channel impulse response (CIR), cooperative communications, ESPRIT, OFDM, pilot-aided estimation, relay.

1. INTRODUCTION

Cooperative transmitting strategies for wireless networks equipped with relay nodes have recently attracted several research efforts [1], [2], and are expected to play an important role in 4G wireless systems. Relays grant wider coverage, reduced power consumption, and potentially transmit diversity by exploiting distributed-array approaches [2]. The simplest relaying strategy consists in amplify-and-forward where the signal is just amplified and retransmitted to the destination. Alternatively, decode-and-forward (DF) relays completely regenerate the signal towards the destination, perhaps introducing a (different) carrier frequency offset (CFO) for each relay. Thus, in fixed and nomadic DF-relay networks, the destination node receives several replicas of the transmitted signal, each one impaired by a different multipath fading channel and CFO. A similar situation occurs in single-frequency broadcasting networks with multiple transmitters in the same area.

Orthogonal frequency-division multiplexing (OFDM) is a common choice to deal with the frequency selectivity induced by multipath; however, (multiple) CFO destroys the subcarrier orthogonality of OFDM, introducing intercarrier interference, which severely degrades the performance. While a single CFO can be fairly estimated and compensated at the receiver side, multiple CFOs in multiple fading channels request both sophisticated estimation and compensation techniques [3], [4], [5].

When coordination among nodes is possible, as in the uplink of orthogonal frequency-division multiple-access (OFDMA) systems, the destination node (e.g., a base station) may be interested in estimating all the CFOs, in order to feed them back to each user and enable CFO precompensation [3]. This way, the destination node has only to compensate for the time-invariant (TI) multiple-input single-output (MISO) channel impulse response (CIR) that conveys the data. But in uncoordinated scenarios, where the different CFOs are not precompensated, the CFOs convert the TI-MISO CIR into a time-varying (TV) MISO CIR, which requests complex TV equalization schemes. In this context, rather than estimating the different CFOs and the TI-MISO CIR, a possible approach is to estimate the overall TV-MISO CIR, especially if the equalizer at the destination node employs the TV-MISO CIR.

Since estimation of CFOs is a classical multiple frequency estimation problem, several approaches are available in the literature, based either on maximum likelihood (ML) methods [6], or on multiple signal classification (MUSIC) [5], [7], or on estimation of signal parameters via rotational invariance techniques (ESPRIT) [8], [9], [10]. These methods can also be exploited for the estimation of the TI-MISO CIR, separately or jointly with the CFOs. Alternatively, the estimation of the TV-MISO CIR, which does not explicitly provides the CFOs and the TI-MISO CIR, can rely on a basis expansion model (BEM) [11], as investigated in [12], [13].

This paper focuses on a two-relay DF-OFDM system, where a training (pilot) sequence is dedicated to CFOs and MISO CIR estimation. The system under consideration does not exploit orthogonal (subband-based) training among the relays [3], [4], which would simplify the estimation problem but would also request higher network coordination. Actually, the two relays exploit the same training, which is simply received by the source node. Additionally, it is assumed coarse time synchronization, because of the OFDM cyclic prefix (CP). In this framework, to avoid the complexity of an ML estimate [3], [6], this paper proposes two ESPRIT-based algorithms for separate estimation of CFOs and TI-MISO CIR. The first algorithm suggests a BEM estimation of the TV-MISO CIR, and applies ESPRIT on the estimated TV-MISO CIR to extract the CFOs. The second algorithm takes advantage of a time-domain Kronecker-delta (TDKD) training, which is mean-squared error (MSE) optimal [14], enabling a direct application of ESPRIT on the observed data. In both algorithms, the TI-MISO CIR is reconstructed using a least-squares (LS) approach. The performance of the two algorithms, expressed in terms of MSE for both CFO and TV-MISO CIR estimates, is compared by means of simulations, which highlight the effectiveness of the proposed algorithms, as well as the effects of different system parameters.

2. TWO-RELAY SYSTEM MODEL

Let us consider an OFDM-based cooperative communication with a source node, two relay nodes, and a destination node. In the

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broadcasting phase, the source node sends a pilot sequence to two relays. In the relaying phase, the relays forward the pilot sequence to the destination node. We assume that there are no errors on the broadcasting phase, and we focus on the relaying phase.

We assume OFDM with N subcarriers and CP length L_{CP}. Let us denote with $\mathbf{x}$ the time-domain pilot sequence of size N transmitted by the source node. Therefore, the $R=2$ relays forward the same pilot sequence $\mathbf{x}$ to the destination. We assume that the asynchronism delay between the two relays, which is negligible with respect to the CP duration, can be incorporated into the CIRs. By denoting with ε_r the CFO (normalized to the subcarrier spacing) between the r th relay and the destination, the time-domain received signal after CP removal can be expressed by

$$\mathbf{y}[k]=\varphi_1[k]\mathbf{D}(\varepsilon_1)\mathbf{H}_1\mathbf{x}+\varphi_2[k]\mathbf{D}(\varepsilon_2)\mathbf{H}_2\mathbf{x}+\mathbf{w}[k], \quad (1)$$

where $\varphi_r[k]=\exp(j2\pi\varepsilon_r(L_{\mathrm{CP}}+k(N+L_{\mathrm{CP}}))/N)$ is the phase offset induced by the CFO of relay r in the k th OFDM block, $\mathbf{D}(\varepsilon_r)=\mathrm{Diag}(1,\exp(j2\pi\varepsilon_r/N),...,\exp(j2\pi\varepsilon_r(N-1)/N))$ is the diagonal matrix that expresses the CFO of relay r, $\mathbf{H}_r$ is the circulant $N\times N$ matrix with $[\mathbf{h}_r^T,0,...,0]^T$ as its first column, $\mathbf{h}_r=[h_{r,0},...,h_{r,L_r-1}]^T$ is the TI CIR of relay r, whose length $L_r\le L_{\mathrm{CP}}+1$ includes the asynchronism delay, and $\mathbf{w}[k]$ is the additive white Gaussian noise (AWGN) vector. In (1) we have assumed that the pilot sequence $\mathbf{x}$ is the same for $k=0,...,K-1$, where K is the number of OFDM blocks. Our aim is to estimate the CFOs ε_1 and ε_2 and the TI CIRs $\mathbf{h}_1$ and $\mathbf{h}_2$.

3. CFO AND CIR ESTIMATION

3.1. TV-MISO CIR Estimation Using BEM

The first algorithm uses the BEM [11] to estimate a TV-MISO CIR that includes both effects of CFOs and TI CIRs. We recast (1) as

$$\mathbf{y}[k]=\mathbf{H}[k]\mathbf{x}+\mathbf{w}[k], \quad (2)$$

where $\mathbf{H}[k]=\varphi_1[k]\mathbf{D}(\varepsilon_1)\mathbf{H}_1+\varphi_2[k]\mathbf{D}(\varepsilon_2)\mathbf{H}_2$ is the time-domain matrix of the TV channel to be estimated. Indeed, by defining $(a)_{\mathrm{mod}\,N}$ as the nonnegative remainder of the integer division of a by N, the element $(n,(n-l)_{\mathrm{mod}\,N})$ of $\mathbf{H}[k]$ is

$$H_{n,(n-l)_{\mathrm{mod}\,N}}[k]=\varphi_1[k]e^{j2\pi\varepsilon_1 n/N}h_{1,l}+\varphi_2[k]e^{j2\pi\varepsilon_2 n/N}h_{2,l}, \quad (3)$$

which represents the l th tap (at time n) of the aggregate TV-MISO CIR in the k th OFDM block. Hence, an estimate of $\mathbf{H}[k]$ implicitly contains both the effects of CFOs ε_1 and ε_2 and TI CIRs $\mathbf{h}_1$ and $\mathbf{h}_2$. Using the BEM, $\mathbf{H}[k]$ is modeled as [12], [13]

$$\mathbf{H}[k]=\sum_{l=0}^{L-1}\sum_{q=0}^{Q-1}c_{l,q}[k]\mathrm{Diag}(\mathbf{b}_q)\mathbf{J}_l, \quad (4)$$

where $L=\max\{L_1,L_2\}$ is the multipath channel order, Q is the number of BEM basis functions, $\{c_{l,q}[k]\}$ are the unknown BEM coefficients, $\mathbf{b}_q$ is the q th known basis function, and $\mathbf{J}_l$ is the $N\times N$ circulant cyclic-shift matrix, with ones on the l th lower diagonal and zeros elsewhere. By defining the $N\times NLQ$ matrix

$$\mathbf{\Gamma}=[\mathrm{Diag}(\mathbf{b}_0)\mathbf{J}_0,...,\mathrm{Diag}(\mathbf{b}_0)\mathbf{J}_{L-1},...,\mathrm{Diag}(\mathbf{b}_{Q-1})\mathbf{J}_0,...,\mathrm{Diag}(\mathbf{b}_{Q-1})\mathbf{J}_{L-1}]$$

and $\mathbf{c}[k]=[c_{0,0}[k],...,c_{L-1,0}[k],...,c_{0,Q-1}[k],...,c_{L-1,Q-1}[k]]^T$, it can be shown that $\mathbf{H}[k]\mathbf{x}=\mathbf{\Gamma}(\mathbf{c}[k]\otimes\mathbf{I}_N)\mathbf{x}=\mathbf{\Gamma}(\mathbf{I}_{LQ}\otimes\mathbf{x})\mathbf{c}[k]$, where $\otimes$ denotes the Kronecker product, leading to

$$\mathbf{y}[k]=\mathbf{P}\mathbf{c}[k]+\mathbf{w}[k], \quad (5)$$

where $\mathbf{P}=\mathbf{\Gamma}(\mathbf{I}_{LQ}\otimes\mathbf{x})$ is known. As a result, the unknown BEM coefficients in $\mathbf{c}[k]$ can be estimated by linear minimum MSE (LMMSE) methods or LS approaches, such as $\hat{\mathbf{c}}[k]=\mathbf{P}^{+}\mathbf{y}[k]$, where $^{+}$ denotes Moore-Penrose pseudo-inversion. The aggregate TV channel matrix of block k can then be estimated by

$$\hat{\mathbf{H}}[k]=\mathbf{\Gamma}(\hat{\mathbf{c}}[k]\otimes\mathbf{I}_N). \quad (6)$$

3.2. CFO Estimation Using BEM-Based ESPRIT

By (3), it is evident that the knowledge of $H_{n,(n-l)_{\mathrm{mod}\,N}}[k]$ would permit to estimate ε_1 and ε_2 by classical frequency estimation algorithms, such as MUSIC and ESPRIT [3]. Thus, we propose a CFO estimation algorithm that applies ESPRIT on the TV CIR taps estimated by the BEM in (6). By collecting N values into the vector $\mathbf{t}_l[k]=[H_{0,(-l)_{\mathrm{mod}\,N}}[k],...,H_{N-1,(N-1-l)_{\mathrm{mod}\,N}}[k]]^T$, we obtain

$$\mathbf{t}_l[k]=\mathbf{E}\mathbf{v}_l[k], \quad (7)$$

$$\mathbf{E}=\begin{bmatrix}1 & 1\\ e^{j2\pi\varepsilon_1/N} & e^{j2\pi\varepsilon_2/N}\\ \vdots & \vdots\\ e^{j2\pi\varepsilon_1(N-1)/N} & e^{j2\pi\varepsilon_2(N-1)/N}\end{bmatrix}, \quad (8)$$

where $\mathbf{v}_l[k]=[\varphi_1[k]h_{1,l}\ ,\ \varphi_2[k]h_{2,l}]^T$. The structure of (8) allows for the estimation of the CFOs ε_1 and ε_2 using ESPRIT. We define the downsampling factor D and, assuming that N/D is integer, the $N/D\times N$ matrix $\mathbf{R}=\mathbf{I}_{N/D}\otimes\mathbf{i}_D$, where $\mathbf{i}_D=[1,0,...,0]$ is the first row of $\mathbf{I}_D$. To reduce the computational complexity, we define the reduced vector $\mathbf{z}_l[k]$, with size N/D, as

$$\mathbf{z}_l[k]=\mathbf{R}\mathbf{t}_l[k]. \quad (9)$$

This way, ESPRIT can be applied to the reduced equation (9) rather than to (7). To simplify the notation, we denote the elements of $\mathbf{z}_l[k]=[H_{0,(-l)_{\mathrm{mod}\,N}}[k],H_{D,(D-l)_{\mathrm{mod}\,N}}[k],...,H_{N-D,(N-D-l)_{\mathrm{mod}\,N}}[k]]^T$ in (9) as $\mathbf{z}_l[k]=[z_{0,l}[k],z_{1,l}[k],...,z_{N/D-1,l}[k]]^T$, and define the Hankel matrix $\mathbf{Z}_l[k]$ as

$$\mathbf{Z}_l[k]=\begin{bmatrix}z_{0,l}[k] & z_{1,l}[k] & \cdots & z_{N/D-M,l}[k]\\ z_{1,l}[k] & z_{2,l}[k] & \cdots & z_{N/D-M+1,l}[k]\\ \vdots & \vdots & & \vdots\\ z_{M-1,l}[k] & z_{M,l}[k] & \cdots & z_{N/D-1,l}[k]\end{bmatrix}_{M\times N/D-M+1}, \quad (10)$$

where M is a design parameter, with $2<M<N/D$. By constructing the $M\times(N/D-M+1)LK$ matrix $\mathbf{Z}=[\mathbf{Z}_0[0],...,\mathbf{Z}_{L-1}[0],...,\mathbf{Z}_0[K-1],...,\mathbf{Z}_{L-1}[K-1]]$, we obtain $\mathbf{Z}=\mathbf{A}\mathbf{\Phi}\mathbf{S}$, where

$$\mathbf{A}=\begin{bmatrix}1 & 1\\ e^{j2\pi\varepsilon_1 D/N} & e^{j2\pi\varepsilon_2 D/N}\\ \vdots & \vdots\\ e^{j2\pi\varepsilon_1(M-1)D/N} & e^{j2\pi\varepsilon_2(M-1)D/N}\end{bmatrix}_{M\times 2}, \quad (11)$$

$$\mathbf{\Phi}=[\mathbf{\Phi}_{0,0},...,\mathbf{\Phi}_{L-1,0},...,\mathbf{\Phi}_{0,K-1},...,\mathbf{\Phi}_{L-1,K-1}]_{2\times 2LK}, \quad (12)$$

$$\mathbf{\Phi}_{l,k}=\begin{bmatrix}h_{1,l}e^{j2\pi\varepsilon_1(L_{\mathrm{CP}}+k(N+L_{\mathrm{CP}}))/N} & 0\\ 0 & h_{2,l}e^{j2\pi\varepsilon_2(L_{\mathrm{CP}}+k(N+L_{\mathrm{CP}}))/N}\end{bmatrix}, \quad (13)$$

$$\mathbf{S} = \mathbf{I}_{LK} \otimes \tilde{\mathbf{S}}, \tag{14}$$

$$\tilde{\mathbf{S}} = \begin{bmatrix} 1 & e^{j2\pi\varepsilon_1 D/N} & \cdots & e^{j2\pi\varepsilon_1 (N/D-M)D/N} \\ 1 & e^{j2\pi\varepsilon_2 D/N} & \cdots & e^{j2\pi\varepsilon_2 (N/D-M)D/N} \end{bmatrix}_{2\times N/D-M+1}. \tag{15}$$

Because of (11), $\text{rank}(\mathbf{Z}) = 2$ when $\varepsilon_1 \neq \varepsilon_2$. Hence, we estimate the CFOs using a version of ESPRIT based on the singular value decomposition (SVD) [8]. By the SVD $\mathbf{Z} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$, $\mathbf{U}$ has the same column space of $\mathbf{A}$. Then, we denote with $\mathbf{U}_\text{x}$ and $\mathbf{U}_\text{y}$ the $M-1\times 2$ matrices obtained from $\mathbf{U}$ by deleting its last row and its first row, respectively. From the eigenvalue decomposition $\mathbf{U}_\text{x}^+\mathbf{U}_\text{y} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^H$, we obtain

$$\mathbf{\Lambda} = \text{Diag}(\lambda_1, \lambda_2), \tag{16}$$

where $\lambda_r = e^{j2\pi\varepsilon_r D/N}$, for $r = 1,2$. In practice, only the estimated vector $\hat{\mathbf{z}}_l[k] = [\hat{H}_{0,(-l)_{\text{mod}\,N}}[k], \hat{H}_{D,(D-l)_{\text{mod}\,N}}[k], ..., \hat{H}_{N-D,(N-D-l)_{\text{mod}\,N}}[k]]^T$ is available, and the SVD is performed on the estimated matrix $\hat{\mathbf{Z}} = \hat{\mathbf{U}}\hat{\mathbf{\Sigma}}\hat{\mathbf{V}}^H$. Due to the BEM modeling error, and to the AWGN-induced estimation errors, in general $\text{rank}(\hat{\mathbf{Z}}) > 2$. Consequently, the proposed algorithm constructs $\hat{\mathbf{U}}_\text{x}$ and $\hat{\mathbf{U}}_\text{y}$ using only the first two columns of $\hat{\mathbf{U}}$. Then, the CFOs are estimated as

$$\hat{\varepsilon}_r = \frac{N}{2\pi D}\text{angle}(\hat{\lambda}_r), \tag{17}$$

where $\hat{\lambda}_r$, for $r = 1,2$, is an eigenvalue of $\hat{\mathbf{U}}_\text{x}^+\hat{\mathbf{U}}_\text{y}$.

3.3. Estimation of TI-MISO CIR

To estimate the two TI CIRs associated with the two relay-destination links, we resort to a frequency-domain approach. For simplicity, $L_1 = L_2 = L$ is assumed in the following. From (1), we compute $\mathbf{y}_\text{F}[k] = \mathbf{F}\mathbf{y}[k]$, where $\mathbf{F}$ is the $N\times N$ unitary FFT matrix, and collect the observation data into $\underline{\mathbf{y}}_\text{F} = [\mathbf{y}_\text{F}[0]^T, ..., \mathbf{y}_\text{F}[K-1]^T]^T$. The frequency-domain vector $\underline{\mathbf{y}}_\text{F}$ can be expressed as

$$\underline{\mathbf{y}}_\text{F} = \mathbf{\Xi}_{\varepsilon_1,\varepsilon_2}\mathbf{h} + \underline{\mathbf{w}}_\text{F}, \tag{18}$$

$$\mathbf{\Xi}_{\varepsilon_1,\varepsilon_2} = (\mathbf{\Psi}_{\varepsilon_1,\varepsilon_2} \otimes \mathbf{I}_N)\mathbf{G}_{\varepsilon_1,\varepsilon_2}(\sqrt{N}\mathbf{I}_2 \otimes \text{Diag}(\mathbf{x}_\text{F})\mathbf{F}_L), \tag{19}$$

$$\mathbf{\Psi}_{\varepsilon_1,\varepsilon_2} = \begin{bmatrix} \varphi_1[0] & \varphi_2[0] \\ \vdots & \vdots \\ \varphi_1[K-1] & \varphi_2[K-1] \end{bmatrix}, \tag{20}$$

$$\mathbf{G}_{\varepsilon_1,\varepsilon_2} = \begin{bmatrix} \mathbf{F}\mathbf{D}(\varepsilon_1)\mathbf{F}^H & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \mathbf{F}\mathbf{D}(\varepsilon_2)\mathbf{F}^H \end{bmatrix}, \tag{21}$$

where $\mathbf{x}_\text{F} = \mathbf{F}\mathbf{x}$, $\mathbf{F}_L$ is the $N\times L$ matrix that contains the first L columns of $\mathbf{F}$, $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_2^T]^T$ is the TI-MISO CIR vector, $\underline{\mathbf{w}}_\text{F} = [\mathbf{w}_\text{F}[0]^T, ..., \mathbf{w}_\text{F}[K-1]^T]^T$, and $\mathbf{w}_\text{F}[k] = \mathbf{F}\mathbf{w}[k]$. Hence, the TI-MISO CIR can be estimated by LMMSE or LS approaches, like

$$\hat{\mathbf{h}} = \mathbf{\Xi}_{\hat{\varepsilon}_1,\hat{\varepsilon}_2}^+ \underline{\mathbf{y}}_\text{F}. \tag{22}$$

3.4. CFO Estimation Using Observation-Based ESPRIT

For the estimation of CFOs ε_1 and ε_2, and TI CIRs $\mathbf{h}_1$ and $\mathbf{h}_2$, we propose a second algorithm that exploits a TDKD pilot structure specially designed for TV channels [14], expressed by

$$\mathbf{x} = \mathbf{1}_{N/L\times 1} \otimes \mathbf{x}_L, \tag{23}$$

where $\mathbf{1}_{N/L\times 1}$ is the all-ones column vector with size N/L (assumed integer), and $\mathbf{x}_L = [x, 0, ..., 0]^T$ has size L. Indeed, the TDKD training is MSE-optimal for the estimation of TV multipath channels [14]. Note that the destination node must know the maximum length L of the TI-MISO CIR (including a possible asynchronism), or alternatively should use L_CP instead of L.

Interestingly, the TDKD pilot sequence enables the estimation of the CFOs by directly applying ESPRIT on the received signal: indeed, the TDKD pilot directly produces repeated noisy estimates of the TV-MISO CIR into the received vector $\mathbf{y}[k]$ in (1) [14]. By defining $y_n[k]$ as the n th element of $\mathbf{y}[k]$, we have

$$y_n[k] = \varphi_1[k]e^{j2\pi\varepsilon_1 n/N}h_{1,(n)_{\text{mod}\,L}}x + \varphi_2[k]e^{j2\pi\varepsilon_2 n/N}h_{2,(n)_{\text{mod}\,L}}x + w_n[k], \tag{24}$$

where the AWGN $w_n[k]$ is the n th element of $\mathbf{w}[k]$. Note that (24) has the same form of (3), apart for the pilot x and the AWGN $w_n[k]$, and hence can represent the basis for CFO estimation using ESPRIT. Assuming $2 < M < N/L$, we define the Hankel matrix

$$\mathbf{Y}_l[k] = \begin{bmatrix} y_l[k] & y_{l+L}[k] & \cdots & y_{l+(N/L-M)L}[k] \\ y_{l+L}[k] & y_{l+2L}[k] & \cdots & y_{l+(N/L-M+1)L}[k] \\ \vdots & \vdots & & \vdots \\ y_{l+(M-1)L}[k] & y_{l+ML}[k] & \cdots & y_{l+(N/L-1)L}[k] \end{bmatrix}_{M\times N/L-M+1}, \tag{25}$$

where $0 \le l \le L-1$, and the $M\times(N/L-M+1)LK$ matrix $\mathbf{Y} = [\mathbf{Y}_0[0], ..., \mathbf{Y}_{L-1}[0], ..., \mathbf{Y}_0[K-1], ..., \mathbf{Y}_{L-1}[K-1]]$. Similarly to (25), we define $\mathbf{W}_l[k]$ starting from the AWGN samples $\{w_n[k]\}$, and $\mathbf{W} = [\mathbf{W}_0[0], ..., \mathbf{W}_{L-1}[0], ..., \mathbf{W}_0[K-1], ..., \mathbf{W}_{L-1}[K-1]]$. It can be shown that $\mathbf{Y} = \bar{\mathbf{Z}} + \mathbf{W}$, where $\bar{\mathbf{Z}} = \bar{\mathbf{A}}\bar{\mathbf{\Phi}}\bar{\mathbf{S}}$,

$$\bar{\mathbf{A}} = \begin{bmatrix} 1 & 1 \\ e^{j2\pi\varepsilon_1 L/N} & e^{j2\pi\varepsilon_2 L/N} \\ \vdots & \vdots \\ e^{j2\pi\varepsilon_1 (M-1)L/N} & e^{j2\pi\varepsilon_2 (M-1)L/N} \end{bmatrix}_{M\times 2}, \tag{26}$$

$$\bar{\mathbf{\Phi}} = [\bar{\mathbf{\Phi}}_{0,0}, ..., \bar{\mathbf{\Phi}}_{L-1,0}, ..., \bar{\mathbf{\Phi}}_{0,K-1}, ..., \bar{\mathbf{\Phi}}_{L-1,K-1}]_{2\times 2LK}, \tag{27}$$

$$\bar{\mathbf{\Phi}}_{l,k} = \begin{bmatrix} xh_{1,l}e^{j2\pi\varepsilon_1(L_\text{CP}+l+k(N+L_\text{CP}))/N} & 0 \\ 0 & xh_{2,l}e^{j2\pi\varepsilon_2(L_\text{CP}+l+k(N+L_\text{CP}))/N} \end{bmatrix}, \tag{28}$$

$$\bar{\mathbf{S}} = \mathbf{I}_{LK} \otimes \bar{\tilde{\mathbf{S}}}, \tag{29}$$

$$\bar{\tilde{\mathbf{S}}} = \begin{bmatrix} 1 & e^{j2\pi\varepsilon_1 L/N} & \cdots & e^{j2\pi\varepsilon_1 (N/L-M)L/N} \\ 1 & e^{j2\pi\varepsilon_2 L/N} & \cdots & e^{j2\pi\varepsilon_2 (N/L-M)L/N} \end{bmatrix}_{2\times N/L-M+1}. \tag{30}$$

Clearly, if $\varepsilon_1 \neq \varepsilon_2$, $\text{rank}(\bar{\mathbf{Z}}) = 2$. By the SVD $\mathbf{Y} = \bar{\mathbf{U}}\bar{\mathbf{\Sigma}}\bar{\mathbf{V}}^H$, we construct $\bar{\mathbf{U}}_\text{x}$ and $\bar{\mathbf{U}}_\text{y}$ as the $M-1\times 2$ matrices obtained from the first two columns of $\bar{\mathbf{U}}$ by deleting the last row and the first row, respectively. Then, the two eigenvalues $\bar{\lambda}_r$ of $\bar{\mathbf{U}}_\text{x}^+\bar{\mathbf{U}}_\text{y}$ allow for the estimation of the two CFOs as

$$\hat{\varepsilon}_r = \frac{N}{2\pi L}\text{angle}(\bar{\lambda}_r). \tag{31}$$

Finally, the TI-MISO CIR can be reconstructed using the LS approach expressed by (18)-(22) in Section 3.3.

3.5. Identifiability of CFO and TI-MISO CIR

In the absence of noise and BEM modeling errors, the BEM-based ESPRIT of Section 3.2 performs the SVD of $\mathbf{Z} = \mathbf{A\Phi S}$, where $\mathbf{Z}$ contains the exact taps $\mathbf{t}_l[k]$. If $\varepsilon_1 \neq \varepsilon_2$, (11) yields $\text{rank}(\mathbf{Z}) = 2$, and therefore both CFOs are identifiable when $|\varepsilon_r| < \varepsilon_{\max}$, for $r = 1,2$. From (17), we obtain $\varepsilon_{\max} = 0.5\, N / D$. The presence of BEM modeling errors may reduce the estimation range: using complex exponentials [11], the estimated $\hat{\mathbf{t}}_l[k]$ are projections onto $\mathbf{b}_q = [1,\, e^{j2\pi q/N},\, ...,\, e^{j2\pi q(N-1)/N}]^T$, for $|q| \leq (Q-1)/2$, leading to $\varepsilon_{\max} = 0.5 \min\{Q-1,\, N/D\}$. On the other hand, if $\varepsilon_1 = \varepsilon_2$, then $\text{rank}(\mathbf{Z}) = 1$, and only a single CFO is identifiable, but this estimation problem simplifies to single CFO estimation for a single relay with equivalent TI CIR $\mathbf{h}_1 + \mathbf{h}_2$. A similar analysis applies to the observation-based ESPRIT of Section 3.4: both CFOs are identifiable when $\varepsilon_1 \neq \varepsilon_2$ and $|\varepsilon_r| < \varepsilon_{\max}$, where $\varepsilon_{\max} = 0.5\, N / L$, while a single CFO is identifiable when $\varepsilon_1 = \varepsilon_2$ and $|\varepsilon_r| < \varepsilon_{\max}$.

The necessary and sufficient condition for the identifiability of the TI-MISO CIR $\mathbf{h}$ in (18) is the full column rank condition $\text{rank}(\mathbf{\Xi}_{\varepsilon_1,\varepsilon_2}) = 2L$. From (19), there are five necessary conditions for full column rank: $\varepsilon_1 \neq \varepsilon_2$; $|\varepsilon_r| < \varepsilon_{\max}$; $KN \geq 2L$; $N \geq L$; $\text{rank}(\text{Diag}(\mathbf{x}_\text{F})\mathbf{F}_L) = L$. Specifically, the last condition requires that the frequency-domain pilot $\mathbf{x}_\text{F}$ must have at least L nonzero values. Together, the five conditions constitute a sufficient condition for the full column rank condition $\text{rank}(\mathbf{\Xi}_{\varepsilon_1,\varepsilon_2}) = 2L$.

3.6. Computational Complexity of CFO Estimation

The main computational effort of ESPRIT-based algorithms lies in the SVD. Since the proposed CFO estimation requires only $R = 2$ singular vectors in $\mathbf{U}$ or $\bar{\mathbf{U}}$, the SVD computational complexity is $O(RM^2)$ [15], which could be further reduced by exploiting the Hankel property of (10) and (25) [15]. The BEM-based ESPRIT requires also the TV-MISO CIR estimation of Section 3.1, whose computational complexity is $O(LQN)$ per OFDM block.

4. PERFORMANCE COMPARISON

We compare by simulations the MSE for the proposed BEM-based and observation-based ESPRIT algorithms. We assume OFDM with $N = 128$ and $L_\text{CP} = 4$. We use a TDKD training for both algorithms. We suppose that the two relay-destination links have the same CIR length $L_1 = L_2 = L = 4$, the same power-delay profile, and the same signal-to-noise ratio (SNR). The CFOs have been randomly selected, with $|\varepsilon_r| < 0.5$, which coincides with the CFO estimation range for subband orthogonal training designs [4]. We assume an orthogonal polynomial BEM [13] with $Q = 5$, which grants negligible BEM modeling error. For the BEM-based ESPRIT, we use $D = 4$, which gives the same CFO estimation range and SVD complexity of the observation-based ESPRIT.

Fig. 1 illustrates the MSE of the TV-MISO CIR, defined as

$$\text{MSE}_{\text{TV-MISO CIR}} = \frac{1}{NK} \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} E\{\| \hat{\mathbf{t}}_l[k] - \mathbf{t}_l[k] \|^2\}, \qquad (32)$$

estimated by averaging over 100 random CFO couples and 100 random CIR couples. The SVD row size is $M = 16$. In this scenario, when the number of pilot blocks is $K = 1$, the TV-MISO CIRs estimated using ESPRIT outperform the TV-MISO CIR estimated by the BEM in (6). When $K = 5$, the MSEs for the ESPRIT-based algorithms increase: indeed, when $K = 1$, the CFO and CIR estimates are allowed to vary from block to block. For the same scenario, Fig. 2 shows the MSE of the CFO, defined as

$$\text{MSE}_{\text{CFO}} = \frac{1}{2} \sum_{r=1}^{2} E\{|\hat{\varepsilon}_r - \varepsilon_r|^2\}. \qquad (33)$$

Note that other CFO definitions are possible: for instance, [5] refers to $\nu_r = \varepsilon_r / N$, and hence our MSE values are N^2 times greater than [5]. When $K = 1$, the MSE performance of the observation-based ESPRIT is above the upper bound $b = 1$ obtained for $|\hat{\varepsilon}_r - \varepsilon_r| = 1$ in (33). Indeed, when sporadically $\varepsilon_1 \approx \varepsilon_2$, the observation-based ESPRIT recognizes only one CFO value, and the second estimate is practically random within the CFO range, thus deteriorating the average MSE. To avoid this problem, a CFO range constraint can be inserted: for instance, we can set $\hat{\varepsilon}_r = 0$ whenever ESPRIT produces $|\hat{\varepsilon}_r| \geq 0.5$. This constraint also improves the CFO MSE for the BEM-based ESPRIT.

Fig. 3 displays the MSE of the constrained CFO estimates when the number of pilot blocks is $K \in \{1,5\}$. The training size increase is more beneficial for the observation-based rather than for the BEM-based ESPRIT. Fig. 4 exhibits the constrained CFO MSE of the observation-based ESPRIT, for $K = 1$ and different values of M: the MSE is minimized for $M \approx 0.75\, N / L$.

5. CONCLUSIONS

This paper has presented and compared two ESPRIT algorithms for the estimation of CFOs and CIRs in a two-relay DF-OFDM system that employs a common training signal. Topics for further research and improvements include a theoretical analysis of the MSE, the investigation about the best basis functions for the BEM, and refined ESPRIT algorithms, such as the unitary ESPRIT proposed in [16], and the total LS approaches suggested in [8].

6. ACKNOWLEDGMENT

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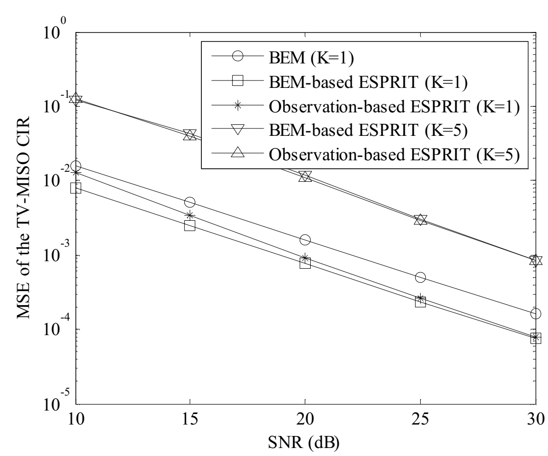

Fig. 1. MSE of the TV-MISO CIR ($M = 16$).

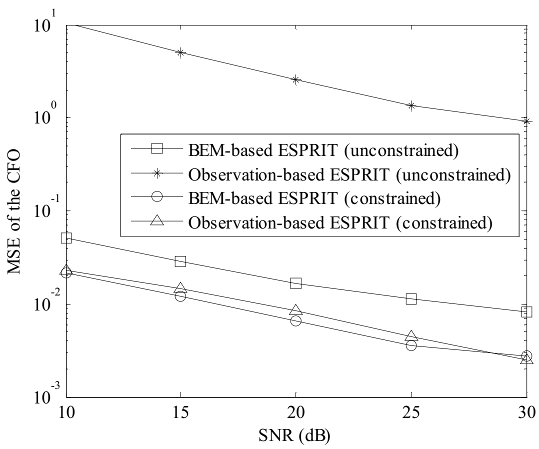

Fig. 2. MSE of the CFO ($K = 1$, $M = 16$).

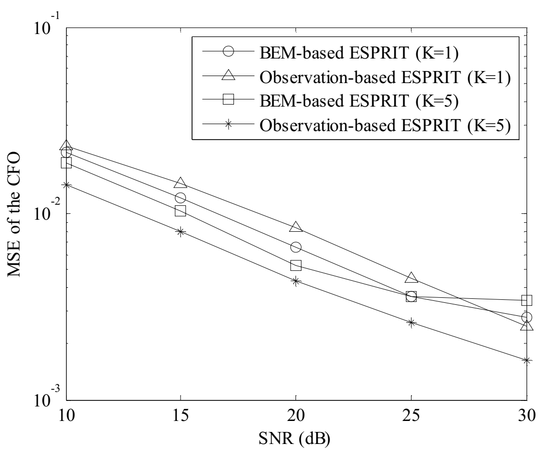

Fig. 3. MSE of the CFO ($M = 16$).

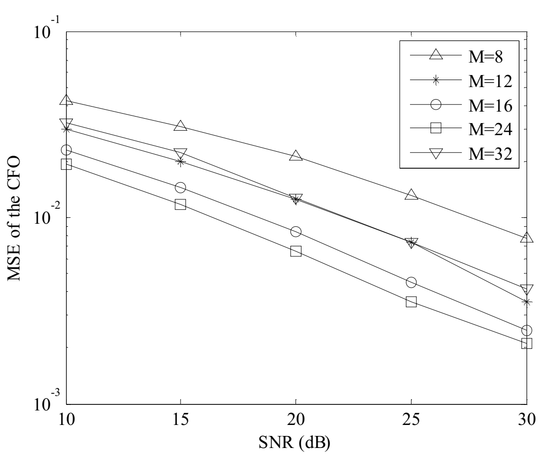

Fig. 4. MSE of the CFO (Observation-based ESPRIT, $K = 1$).

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