SER Performance of Linear Multiuser Detectors for DS-CDMA Downlink With Transmitter Nonlinear Distortions

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***Abstract*—The aim of this paper is to examine the effects of the nonlinear distortions introduced by the high-power amplifier in the downlink of direct-sequence code-division multiple-access systems. By modeling the nonlinear distortion effects as a signal power loss and a superimposed colored Gaussian noise, we derive the symbol-error rate (SER) of the minimum mean-squared error (mmse) detector in additive white Gaussian noise channels. Successively, we make use of a semianalytical approach to obtain the SER performance of linear multiuser detectors, such as the RAKE, decorrelator, and mmse receiver, in frequency-selective fading channels. Simulation results confirm the effectiveness of the theoretical approach.**

***Index Terms*—Direct-sequence code-division multiple-access (DS-CDMA), downlink, multiuser detectors, nonlinear distortions.**

I. Introduction

AMONG THE techniques employed for satellite and cellular mobile communications, direct-sequence code-division multiple-access (DS-CDMA) is quite robust to many types of interferences, such as narrow-band interferences, and, thanks to the combination of many chip values, it is able to exploit the multipath diversity offered by the channel [1]. Another well-known feature of the DS-CDMA technique is the small sensitivity to nonlinear distortions with respect to multicarrier systems. Indeed, when phase-shift keying (PSK) modulations are used, the single-user DS-CDMA signal is characterized by an almost-constant envelope; therefore, the transmitting high-power amplifier (HPA) can work efficiently (i.e., close to the saturation) without introducing significant distortions.

However, the constant envelope characteristic is lost when many independent DS-CDMA signals have to be transmitted, because the constructive and destructive superposition of these signals gives rise to an aggregate signal characterized by a high peak-to-average power ratio (PAR). This effect is evident in the downlink scenario, where the base station has to serve all the active users of a specific cell. Moreover, if a mobile user is allowed to occupy multiple spreading codes, the same problem can also occur in the uplink [2].

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In the technical literature, few papers consider the presence of nonlinear distortions in DS-CDMA systems. Lee and Miller [3] and Lau [4] analyzed the PAR behavior and its reduction in downlink scenarios. In [5], Conti *et al.* presented a bit-error rate (BER) analysis of the matched-filter (MF) receiver in additive white Gaussian noise (AWGN), for binary PSK (BPSK) and quaternary PSK (QPSK) modulations. In [2], Guo and Milstein analyzed the BER performance of the MF for multicode transmissions in AWGN and examined some methods to reduce the nonlinear distortions.

Although the AWGN condition can be appropriate under some circumstances, e.g., in satellite channels with a strong line-of-sight (LoS) component, in many cases DS-CDMA systems have to face a frequency-selective fading channel due to the multipath delay spread. This effect is especially emphasized in CDMA cellular systems characterized by a wide frequency band, e.g., IMT-2000 [1]. Since frequency-selective channels destroy the user orthogonality, high performance improvements can be obtained making use of multiuser detection techniques [6], because of their capability in reducing the multiple-access interference (MAI) at the receiver side. However, the sensitivity of CDMA systems to nonlinear distortions is higher when the receiver employs multiuser detection techniques rather than conventional matched filtering. Indeed, multiuser detectors tolerate a higher MAI (i.e., more users) than MF, leading to a HPA input signal with higher PAR. Therefore, if we want to exploit the high capacity of DS-CDMA systems by means of multiuser detectors, the performance degradation introduced by the nonlinear HPA should be taken into account carefully. Unfortunately, the majority of works dealing with performance analysis of multiuser detectors neglect the presence of nonlinear amplifiers (just to consider few examples, see [6]–[10]).

In this paper, we evaluate the symbol-error rate (SER) of linear multiuser detectors in downlink channels, taking into account the presence of a nonlinear HPA at the transmitter. We emphasize that this paper is intended to outline the performance degradation induced by the HPA when multiuser detectors designed for linear scenarios are employed. More sophisticated detectors that are specially designed for nonlinear environments [11], [12] are not considered here. First, we extend to the minimum mean-squared error (mmse) detector some of the results obtained in [5] for the MF and in [13] for the linear decorrelating detector (LDD) in AWGN channels. Although in downlink AWGN channels the need for multiuser receivers can be circumvented by adopting orthogonal codes, our aim is to develop a theoretical background to be exploited

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in frequency-selective channels, where the users' signals are nonorthogonal independently of the spreading codes. Second, we analyze the degradation induced by the nonlinear amplifier in frequency-selective channels, obtaining a semianalytic SER expression for QPSK modulations. Simulation results, which validate the analytical approach, are presented for RAKE, decorrelating, and mmse detectors.

This paper is organized as follows. Section II introduces the CDMA system model and Section III is dedicated to the statistical characterization of the nonlinear distortions. The SER analysis of linear multiuser detectors in AWGN channels, contained in Section IV, will be extended to frequency-selective scenarios in Section V. Section VI compares analytical and simulated results and Section VII concludes the paper.

II. System Model

First, we introduce some basic notations. We use lower (upper) bold face letters to denote vectors (matrices); superscripts *, ^{T}, ^{H}, and † to represent complex conjugate, transpose, Hermitian, and Moore–Penrose pseudoinverse operators, respectively; $E\{\cdot\}$ to represent the statistical expectation; and $\lceil x \rceil$ and $\mathrm{csgn}(x)$ to denote the smallest integer greater than x and the complex signum of x, respectively. The Q-function is defined as $Q(x) = (2\pi)^{-1/2}\int_x^{+\infty}\exp(-\nu^2/2)d\nu$. The symbols $*$ and $\otimes$ denote convolution and Kronecker matrix product, respectively, $\mathbf{0}_{M\times N}$ is the $M \times N$ all-zero matrix, $\mathbf{I}_N$ is the $N \times N$ identity matrix, and $\mathrm{diag}(a_1,\ldots,a_N)$ is the $N \times N$ diagonal matrix with a_n as its (n,n)th entry. We define $(\mathbf{A})_{m,n}$ as the (m,n)th entry of the matrix $\mathbf{A}$, $(\mathbf{A})_{:,n}$ as its nth column vector $\big((\mathbf{A})_{m,:}$ as the mth row vector$\big)$, and $(\mathbf{a})_{m,1}$ as the mth entry of the column vector $\mathbf{a}$ $\big((\mathbf{a})_{1,n}$ as the nth entry of the row vector $\mathbf{a}\big)$. Similarly, we denote with $(\mathbf{A})_{:,m:n}$ the submatrix of $\mathbf{A}$, obtained by selecting the columns from the mth to the nth $\big((\mathbf{A})_{m:n,:}$ obtained by selecting the rows from the mth to the nth$\big)$ and $(\mathbf{A})_{m:n,p:q}$ the submatrix of $\mathbf{A}$ having $(\mathbf{A})_{m,p}$ as its first element and $(\mathbf{A})_{n,q}$ as its last element.

We consider the downlink of a DS-CDMA system. Since the base station attends all the users that are active in a given cell, the signal to be transmitted is expressed by

$$x(t) = \sum_{k=1}^{K} x_k(t) \tag{1}$$

where $x_k(t)$ represents the signal component relative to the kth user and K is the number of active users. Each signal $x_k(t)$ can be expressed by

$$x_k(t) = A_k \sum_{i=-\infty}^{+\infty} b_k[i] s_k(t - iT) \tag{2}$$

where T is the symbol interval and A_k, $s_k(t)$, and $b_k[i]$ are the amplitude, spreading waveform, and ith symbol, respectively, of the user k. The spreading waveform is expressed by $s_k(t) = N^{-1/2}\sum_{m=0}^{N-1} c_k[m]p(t - mT_c)$, where N is the processing gain, $T_c = T/N$ is the chip duration, $p(t)$ is the chip pulse-shaping waveform with unit energy, and $c_k[m]$ is the mth value of the kth user spreading code, with $|c_k[m]| = 1$. It is assumed that the data symbols $\{b_k[i]\}$ belong to a set of independent and equiprobable random variables, drawn from the set $B = \{\pm(\sqrt{2}/2) \pm j(\sqrt{2}/2)\}$.

If the transmitting HPA is supposed to be instantaneous, it can be modeled by its AM/AM and AM/PM distortion curves $G(\cdot)$ and $\Phi(\cdot)$, respectively, [14] or, equivalently, by a complex nonlinear distorting function $F(\cdot) = G(\cdot)\exp[j\Phi(\cdot)]$. Hence, the signal $x(t)$ in (1) is transformed by the HPA into

$$w(t) = F(|x(t)|)e^{j\arg(x(t))} \tag{3}$$

which represents the base-band-equivalent input–output relationship for the nonlinear amplifier. As motivated in Section III, the HPA output signal in (3) can be alternatively expressed as

$$w(t) = \alpha_0 x(t) + n_d(t) \tag{4}$$

where α_0 represents the average linear amplification gain and $n_d(t)$ is the nonlinear distortion noise.

This paper assumes that the signal $w(t)$, which is transmitted by the base station, passes through a slowly varying multipath channel, characterized by an impulse response

$$g(\tau) = \sum_{q=1}^{Q} \beta_q e^{j\theta_q}\delta(\tau - \tau_q) \tag{5}$$

where Q is the number of paths of the channel; β_q, θ_q, and τ_q are the gain, phase shift, and propagation delay, respectively, of the qth path; and $\delta(\cdot)$ is the Dirac delta function. By exploiting (4), the signal $r(t)$ at the input of the receiver can be expressed by

$$\begin{aligned} r(t) &= \int_{-\infty}^{+\infty} g(\tau)w(t-\tau)d\tau + n(t) \\ &= r_{\mathrm{SIG}}(t) + r_{\mathrm{NL}}(t) + n(t) \end{aligned} \tag{6}$$

where $r_{\mathrm{SIG}}(t) = \sum_{q=1}^{Q}\beta_q|\alpha_0|e^{j\theta_q + j\arg(\alpha_0)}x(t-\tau_q)$ represents the useful received signal, $r_{\mathrm{NL}}(t) = \sum_{q=1}^{Q}\beta_q e^{j\theta_q}n_d(t-\tau_q)$ denotes the nonlinear distortion noise, and $n(t)$ stands for the thermal noise.

At the receiver side, assuming perfect synchronization and channel-state information, $r(t)$ is filtered by a chip-matched filter and successively sampled at the chip rate, thus obtaining

$$\begin{aligned} r_n[l] &= \int_{-\infty}^{+\infty} r(t)p^*(t - lT - nT_c)dt \\ &= r_{n,\mathrm{SIG}}[l] + r_{n,\mathrm{NL}}[l] + r_{n,\mathrm{AWGN}}[l]. \end{aligned} \tag{7}$$

The received chip $r_n[l]$, expressed by (7), is characterized by three additive parts: $r_{n,\mathrm{SIG}}[l]$ is the useful component related to $x(t)$, $r_{n,\mathrm{NL}}[l]$ is the inband nonlinear distortion noise introduced by the quantity $n_d(t)$, and $r_{n,\mathrm{AWGN}}[l]$ is the inband thermal noise with power $\sigma^2_{\mathrm{AWGN}} = E\{|r_{n,\mathrm{AWGN}}[l]|^2\}$.

Assuming that the maximum delay spread of the channel is smaller than the symbol interval (i.e., $\max\{\tau_Q\} < T$), the channel spreads the information related to a certain symbol over two symbol intervals and, consequently, a receiver window of $2N$ consecutive chips contains all the energy related to the symbol of interest. Hence, we will consider a receiving window of two symbol intervals. However, the extension to longer windows is straightforward.

The discrete-time equivalent impulse response of the channel is expressed by $g[l] = \left[\int_{-\infty}^{+\infty} g(\tau)R_{pp}(t-\tau)d\tau\right]_{t=lT_c}$, where $R_{pp}(\tau)$ is the autocorrelation function of the pulse waveform $p(t)$. Defining the channel order as $L = \lceil \max\{\tau_Q\}/T_c \rceil$, it holds true that $g[l] = 0$ when $l > L$ and when $l < 0$. Moreover, we define the column vectors $\mathbf{r}[l] = [r_0[l] \cdots r_{N-1}[l]]^T$, $\mathbf{r}_{\mathrm{NL}}[l] = [r_{0,\mathrm{NL}}[l] \cdots r_{N-1,\mathrm{NL}}[l]]^T$, $\mathbf{r}_{\mathrm{AWGN}}[l] = [r_{0,\mathrm{AWGN}}[l] \cdots r_{N-1,\mathrm{AWGN}}[l]]^T$, $\mathbf{c}_k = N^{-1/2}[c_k[0] \ldots c_k[N-1]]^T$, $\mathbf{b}[l] = [b_1[l] \cdots b_K[l]]^T$, $\underline{\mathbf{r}}[l] = [\mathbf{r}[l]^T \mathbf{r}[l+1]^T]^T$, $\underline{\mathbf{r}}_{\mathrm{NL}}[l] = [\mathbf{r}_{\mathrm{NL}}[l]^T \mathbf{r}_{\mathrm{NL}}[l+1]^T]^T$, $\underline{\mathbf{r}}_{\mathrm{AWGN}}[l] = [\mathbf{r}_{\mathrm{AWGN}}[l]^T \mathbf{r}_{\mathrm{AWGN}}[l+1]^T]^T$, and $\underline{\mathbf{b}}[l] = [\mathbf{b}[l-1]^T \mathbf{b}[l]^T \mathbf{b}[l+1]^T]^T$ and the matrices $\mathbf{C} = [\mathbf{c}_1 \ldots \mathbf{c}_K]$, $\tilde{\mathbf{C}} = (\mathbf{C})_{N-L+1:N,:}$, $\mathbf{A} = \mathrm{diag}(A_1, \ldots, A_K)$, $\underline{\mathbf{A}} = \mathbf{I}_3 \otimes \mathbf{A}$, and

$$\underline{\mathbf{G}} = \begin{bmatrix} g[L] & \ldots & g[0] & 0 & \ldots & 0 \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & g[L] & \ldots & g[0] \end{bmatrix}_{2N \times 2N+L} \quad (8)$$

$$\underline{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{C}} & \mathbf{0}_{L\times K} & \mathbf{0}_{L\times K} \\ \mathbf{0}_{N\times K} & \mathbf{C} & \mathbf{0}_{N\times K} \\ \mathbf{0}_{N\times K} & \mathbf{0}_{N\times K} & \mathbf{C} \end{bmatrix}_{2N+L \times 3K} \quad (9)$$

thus obtaining the received vector $\underline{\mathbf{r}}[l]$, which is expressed by

$$\underline{\mathbf{r}}[l] = |\alpha_0| e^{j\arg(\alpha_0)} \underline{\mathbf{G}}\underline{\mathbf{C}}\underline{\mathbf{A}}\underline{\mathbf{b}}[l] + \underline{\mathbf{r}}_{\mathrm{NL}}[l] + \underline{\mathbf{r}}_{\mathrm{AWGN}}[l]. \quad (10)$$

For any linear detector, the decision variable is obtained by a linear combination of the elements of the received vector $\underline{\mathbf{r}}[l]$ in (10). Focusing on the user k and assuming QPSK modulation, the estimate $\hat{b}_k[l]$ of the transmitted symbol $b_k[l]$ is expressed by

$$\hat{b}_k[l] = 2^{-1/2} \mathrm{csgn}(\underline{\mathbf{d}}_k \underline{\mathbf{r}}[l]) \quad (11)$$

where the $2N$-row vector $\underline{\mathbf{d}}_k$ represents a linear receiver chosen according to the detection criterion (e.g., MF, LDD, mmse, etc.). The extension of such a model to other constellations or longer delay spreads is straightforward.

III. Characterization of the Nonlinear Distortions

The aim of this section is to characterize the statistical nature of the signal at the input and at the output of the transmitter HPA. The input signal $x(t)$, as summarized by (1), is the sum of those signals belonging to the K active users. Since we are considering the downlink scenario, where signal and interference experience the same attenuation, we will assume users' signals with roughly equal amplitudes. This assumption, which allows to avoid a power-control strategy, guarantees that the signal-to-interference ratio (SIR) at the detector input is approximately the same for all the mobile users. This choice is reasonable, because a multiuser DS-CDMA system is mainly interference limited when several users are active. Alternatively, a power control strategy could balance the signal-to-interference plus noise ratios (SINRs) instead of the users' SIRs, by exploiting the knowledge of the user distances or the received signal levels. However, this strategy cannot produce significant differences among the users' amplitudes, which would penalize too much the SER of the users with weaker amplitudes. Thus, by assuming that all the amplitudes $\{A_k\}$ are nearly equal with one another and that K is sufficiently high (e.g., $K > 20$), $x(t)$ can be approximated by a Gaussian-random process [5] because of the central limit theorem (CLT) [15]. The Gaussian distribution of $x(t)$ holds true even if the number K of active users is high enough to group the users in subgroups, with each subgroup satisfying the above hypotheses. In practice, since K is always upper bounded, $x(t)$ is not always truly Gaussian and the following approximation has to be checked. However, the hypothesis of a DS-CDMA system with many active users seems to be realistic when multiuser detectors at the receiver side counteract the introduced MAI. Moreover, for multicode transmissions, it is possible to interpret K as the number of physical channels belonging to $\tilde{K}$ users, with $\tilde{K} < K$.

The signal $x(t)$ in (1) is a zero-mean random process, i.e., $E\{x(t)\} = 0$, because the signals $\{x_k(t)\}_{k=1}^K$ are zero-mean random processes. By the previous considerations, the input $x(t)$ can be modeled as a zero-mean Gaussian random process and, consequently, using the complex extension of the Bussgang theorem [5], [15], [16], the HPA output is characterized by two mutually uncorrelated components, $\alpha(t)x(t)$ and $n_d(t)$, as expressed by

$$w(t) = \alpha(t)x(t) + n_d(t) \quad (12)$$

$$E\{x^*(t)n_d(t+\tau)\} = 0, \quad \forall\, t, \forall\, \tau. \quad (13)$$

The linear gain $\alpha(t)$, for Gaussian inputs $x(t)$, is expressed by

$$\alpha(t) = \frac{E\{w(t)x^*(t+\tau)\}}{E\{x(t)x^*(t+\tau)\}} = \frac{E\{w(t)x^*(t)\}}{E\{|x(t)|^2\}} \quad (14)$$

and it depends only on $\sigma_x^2(t) = E\{|x(t)|^2\}$ and on the amplifier nonlinearity $F(\cdot)$ [17].

If the HPA input $x(t)$ is stationary, then the linear gain is time nvariant, i.e., $\alpha(t) = \alpha_0$, because both the numerator and denominator in (14) are independent of the time index t. In such a situation, the relation between the autocorrelation $R_{ww}(\tau)$ of the HPA output signal $w(t)$ and the autocorrelation $R_{xx}(\tau)$ of the HPA input signal $x(t)$ is expressed by [18]

$$R_{ww}(\tau) = \sum_{i=0}^{+\infty} \gamma_i R_{xx}(\tau)^{2i+1} \quad (15)$$

where the coefficients γ_i, which depend on the nonlinear function $F(\cdot)$ and on the signal input power σ_x^2, can be calculated by numerical integration or by means of closed-form expressions [18], [19]. Using (4) and exploiting (13), it is easy to split the output autocorrelation $R_{ww}(\tau)$ in (15) as the sum of the useful part $|\alpha_0|^2 R_{xx}(\tau)$ and of the nonlinear noise part $R_{n_d n_d}(\tau)$, as expressed by

$$R_{ww}(\tau) = |\alpha_0|^2 R_{xx}(\tau) + R_{n_d n_d}(\tau) \quad (16)$$

where

$$R_{n_d n_d}(\tau) = \sum_{i=1}^{+\infty} \gamma_i R_{xx}(\tau)^{2i+1}. \quad (17)$$

However, in the downlink of a DS-CDMA system, the signal $x(t)$ is not stationary. In our scenario, $x(t)$ is cyclostationary with a period equal to the duration T of the short spreading codes. By defining the time-varying cross-correlation function [15] of two random processes $u(t)$ and $v(t)$ as $R_{uv}(t,t+\tau) = E\{u^*(t)v(t+\tau)\}$, a relation similar to (16) can be derived from (12) and (13), which is

$$R_{ww}(t,t+\tau) = \alpha^*(t)\alpha(t+\tau)R_{xx}(t,t+\tau) + R_{n_dn_d}(t,t+\tau). \tag{18}$$

It should be observed that, if $E\{|x(t)|^2\}$ is almost constant with time, the linear gain $\alpha(t)$ in (14) is characterized by a small time variability, which yields $\alpha(t) \approx \alpha(t+\tau) \approx \alpha_0$. This property is realistic in many circumstances, e.g., when the pulse-shaping waveform is rectangular or when it is a raised cosine function with a small rolloff factor [5]. Assuming that such a kind of pulse shaper is used, the time dependence of the linear gain $\alpha(t)$ can be neglected, thereby motivating the use of α_0 in (4). The simplified model obtained by replacing $\alpha(t)$ with α_0 will be used in Sections IV–VI to analyze the performance of linear multiuser detectors.

It is noteworthy that the cyclostationarity of the input $x(t)$ holds true in the strict sense, since $x(t)$ is regarded as Gaussian. The output $w(t)$ is also cyclostationary (in the strict sense) with the same period T, because the amplifier is modeled as memoryless. By defining the time-averaged cross-correlation function [20] of two cyclostationary processes, $u(t)$ and $v(t)$, as $\bar{R}_{uv}(\tau) = (1/T)\int_0^T R_{uv}(t,t+\tau)dt$, the hypothesis of the slow time variability of $\alpha(t) \approx \alpha_0$ allows us to express (18) as

$$\bar{R}_{ww}(\tau) = |\alpha_0|^2\bar{R}_{xx}(\tau) + \bar{R}_{n_dn_d}(\tau). \tag{19}$$

Equivalently, (19) can be interpreted as the output autocorrelation function of the stationary random process $\tilde{w}(t) = w(t+\Theta)$, where the random variable Θ is uniformly distributed between 0 and T. As a consequence, $\bar{R}_{ww}(\tau)$ can be expressed as a power-series expansion of $\bar{R}_{xx}(\tau)$, as in (15), with the same coefficients γ_i. Analogously, $\bar{R}_{n_dn_d}(\tau)$ in (19) can be evaluated by replacing the quantity $R_{xx}(\tau)$ with $\bar{R}_{xx}(\tau)$ in (17), thus obtaining

$$\bar{R}_{n_dn_d}(\tau) = \sum_{i=1}^{+\infty}\gamma_i\bar{R}_{xx}(\tau)^{2i+1}. \tag{20}$$

The knowledge of the quantities $|\alpha_0|^2$ and $\bar{R}_{n_dn_d}(\tau)$ allows a complete characterization of both the effects (e.g., linear gain and nonlinear distortion noise) produced by the nonlinear amplification.

IV. SER Performance in Nonlinear AWGN Channels

In AWGN channels, since $L = 0$, the channel matrix $\underline{\mathbf{G}}$ in (8) becomes $\underline{\mathbf{G}} = g_0\mathbf{I}_{2N}$, where $g_0 = g[0]$ and, hence, a receiving window of one symbol interval (i.e., of N samples) is wide enough to recover the symbol of interest. In this situation, (10) simply reduces to

$$\mathbf{r}[l] = |\alpha_0 g_0|e^{j\varphi_0}\mathbf{CAb}[l] + \mathbf{r}_{\mathrm{NL}}[l] + \mathbf{r}_{\mathrm{AWGN}}[l] \tag{21}$$

where $\varphi_0 = \arg(\alpha_0 g_0)$ is the phase shift due to both the channel and the HPA. Therefore, the receiver of the kth user can be expressed by an N-row vector

$$\mathbf{d}_k = e^{-j\varphi_0}(\mathbf{D})_{k,:} \tag{22}$$

where $\mathbf{D}$ is the $K \times N$ matrix that contains the detectors of all the users. For the MF receiver, the detector acts like a simple despreader. Therefore, $\mathbf{D}$ becomes

$$\mathbf{D}_{\mathrm{MF}} = \mathbf{C}^H \tag{23}$$

and the SER performance can be obtained in closed form by modeling the MAI and the nonlinear distortion noise as Gaussian-random variables. These approximations are reasonably justified by the high number K of active users and by the high processing gain $N \geq K$ [5]. Hence, we obtain that the SER $P_{\mathrm{AWGN},k}$ of the user k is expressed by

$$P_{\mathrm{AWGN},k} = 2Q\left(\sqrt{\frac{\sigma^2_{\mathrm{SIG},k}}{\sigma^2_{\mathrm{MAI},k} + \sigma^2_{\mathrm{NL},k} + \sigma^2_{\mathrm{AWGN},k}}}\right) - Q^2\left(\sqrt{\frac{\sigma^2_{\mathrm{SIG},k}}{\sigma^2_{\mathrm{MAI},k} + \sigma^2_{\mathrm{NL},k} + \sigma^2_{\mathrm{AWGN},k}}}\right) \tag{24}$$

where, by denoting with $\mathbf{R} = \mathbf{C}^H\mathbf{C}$ the matrix that contains the cross-correlation coefficients of the users' spreading codes, $\sigma^2_{\mathrm{SIG},k} = |\alpha_0 g_0|^2 A_k^2$, $\sigma^2_{\mathrm{MAI},k} = |\alpha_0 g_0|^2 \sum_{j=1, j\neq k}^{K} |(\mathbf{R})_{k,j}|^2 A_j^2$, and $\sigma^2_{\mathrm{AWGN},k} = \sigma^2_{\mathrm{AWGN}}$.

In [5], the theoretical evaluation of the nonlinear distortion noise power $\sigma^2_{\mathrm{NL},k}$ has been carried out supposing a rectangular chip waveform $p(t)$, leading to $\sigma^2_{\mathrm{NL},k} = |g_0|^2\bar{R}_{ww}(0) - |\alpha_0 g_0|^2\bar{R}_{xx}(0)$. Anyway, this assumption is not always realistic in band-limited channels. By assuming a more realistic band-limited chip waveform $p(t)$, the nonlinear distortion noise power can be evaluated as $\sigma^2_{\mathrm{NL},k} = |g_0|^2(\mathbf{C}^H\mathbf{\Psi}\mathbf{C})_{k,k}$, where the $N \times N$ matrix $\mathbf{\Psi}$ is expressed by $(\mathbf{\Psi})_{m,n} = \left[\bar{R}_{n_dn_d}(\tau) * R_{pp}(\tau)\right]_{\tau=(n-m)T_c}$, with $\bar{R}_{n_dn_d}(\tau)$ computed using (20).

Assuming that the code matrix $\mathbf{C}$ is full rank, by using the decorrelating detector, expressed by

$$\mathbf{D}_{\mathrm{LDD}} = \mathbf{C}^\dagger \tag{25}$$

the MAI is completely eliminated [6]. The SER performance can be expressed as in (24) [13], with $\sigma^2_{\mathrm{SIG},k} = |\alpha_0 g_0|^2 A_k^2$, $\sigma^2_{\mathrm{MAI},k} = 0$, $\sigma^2_{\mathrm{NL},k} = |g_0|^2(\mathbf{C}^\dagger\mathbf{\Psi}\mathbf{C}^{\dagger H})_{k,k}$, and $\sigma^2_{\mathrm{AWGN},k} = (\mathbf{R}^{-1})_{k,k}\sigma^2_{\mathrm{AWGN}}$, where the quantity $(\mathbf{R}^{-1})_{k,k}$ is the thermal noise amplification factor due to the decorrelating operation. Also in this case, the nonlinear distortion noise $\left(\mathbf{C}^\dagger\mathbf{r}_{\mathrm{NL}}[l]\right)_{k,1}$ can be considered Gaussian because of the high processing gain N [13].

A scaled version of the mmse receiver can be expressed by

$$\mathbf{D}_{\text{mmse}} = \mathbf{M} \tag{26}$$

where [21]

$$\begin{aligned}\mathbf{M} &= (\mathbf{I}_K + |\alpha_0 g_0|^2 \mathbf{A}\mathbf{C}^H \mathbf{W}^{-1}\mathbf{C}\mathbf{A})^{-1}\mathbf{A}\mathbf{C}^H\mathbf{W}^{-1} \\ &= \mathbf{A}\mathbf{C}^H(|\alpha_0 g_0|^2\mathbf{C}\mathbf{A}^2\mathbf{C}^H + \mathbf{W})^{-1}\end{aligned} \tag{27}$$

and $\mathbf{W} = |g_0|^2\mathbf{\Psi} + \sigma^2_{\text{AWGN}}\mathbf{I}_N$ is the covariance matrix of the total colored noise term $\mathbf{r}_{\text{NL}}[l] + \mathbf{r}_{\text{AWGN}}[l]$.

As far as the SER of the mmse detector is concerned, we propose as in [5] and [13] to model the nonlinear distortion noise as Gaussian. Moreover, as shown in [7], even the residual MAI at the mmse output is well approximated by a Gaussian-random variable, leading to a SER that can be expressed by (24) as well, where $\sigma^2_{\text{SIG},k} = |\alpha_0 g_0|^2 A_k^2 (\mathbf{MC})^2_{k,k}$

$$\sigma^2_{\text{MAI},k} = |\alpha_0 g_0|^2 \sum_{\substack{j=1 \\ j\neq k}}^{K} |(\mathbf{MC})_{k,j}|^2 A_j^2$$

$$\sigma^2_{\text{NL},k} = |g_0|^2(\mathbf{M\Psi M}^H)_{k,k}$$

and

$$\sigma^2_{\text{AWGN},k} = (\mathbf{MM}^H)_{k,k}\sigma^2_{\text{AWGN}}.$$

Although the LDD and the mmse detector depend on the spreading codes of the other users, it should be noted that these receivers can also be obtained blindly, i.e., without explicit knowledge of the spreading codes of the other users [22]. As an example, the matrix $\mathbf{P} = |\alpha_0 g_0|^2\mathbf{C}\mathbf{A}^2\mathbf{C}^H + \mathbf{W}$ that appears in the mmse detector expression (27) is the covariance matrix of the received signal $\mathbf{r}[l]$ in (21) and can be estimated by approximating $\mathbf{P} = E\{\mathbf{r}[l]\mathbf{r}[l]^H\}$ with $\hat{\mathbf{P}} = (1/N_{\text{data}})\sum_{l=1}^{N_{\text{data}}}\mathbf{r}[l]\mathbf{r}[l]^H$. Hence, since $\mathbf{A}$ is diagonal, the detector (22) reduces to $\mathbf{d}_{\text{mmse},k} = e^{-j\varphi_0}A_k\mathbf{c}_k^H\hat{\mathbf{P}}$, which requires only the spreading code $\mathbf{c}_k$ of the user of interest.

We also note that the theoretical model of the nonlinear distortions could also be applied when the spreading codes are long. However, in this case, the multiuser detectors become time varying and, hence, more complex. In addition, the statistics of the nonlinear distortions also become time varying; therefore, the validity of the approximations made should be checked.

V. SER Performance in Nonlinear Frequency-Selective Channels

For frequency-selective channels, the detector $\underline{\mathbf{d}}_k$, expressed by

$$\underline{\mathbf{d}}_k = e^{-j\arg(\alpha_0)}(\underline{\mathbf{D}})_{K+k,:} \tag{28}$$

has to consider not only the MAI, but also has to combine the resolvable paths. The received vector expression in (10) suggests to treat the multipath situation, summarized by $\underline{\mathbf{G}}$, like the AWGN scenario in (21), with a modified code matrix expressed by

$$\underline{\mathbf{H}} = \underline{\mathbf{G}}\underline{\mathbf{C}}. \tag{29}$$

As a consequence, likewise (23), (25), and (26). For frequency-selective channels, we can define the RAKE receiver, LDD, and mmse detector, respectively, as

$$\underline{\mathbf{D}}_{\text{RAKE}} = \underline{\mathbf{H}}^H \tag{30}$$

$$\underline{\mathbf{D}}_{\text{LDD}} = \underline{\mathbf{H}}^\dagger \tag{31}$$

$$\underline{\mathbf{D}}_{\text{mmse}} = \underline{\mathbf{M}} \tag{32}$$

where

$$\begin{aligned}\underline{\mathbf{M}} &= \left(\mathbf{I}_{3K} + |\alpha_0|^2\underline{\mathbf{A}}\underline{\mathbf{H}}^H\underline{\mathbf{W}}^{-1}\underline{\mathbf{H}}\underline{\mathbf{A}}\right)^{-1}\underline{\mathbf{A}}\underline{\mathbf{H}}^H\underline{\mathbf{W}}^{-1} \\ &= \underline{\mathbf{A}}\underline{\mathbf{H}}^H\left(|\alpha_0|^2\underline{\mathbf{H}}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H + \underline{\mathbf{W}}\right)^{-1}\end{aligned} \tag{33}$$

and $\underline{\mathbf{W}} = \underline{\mathbf{G}}\underline{\mathbf{\Psi}}\underline{\mathbf{G}}^H + \sigma^2_{\text{AWGN}}\mathbf{I}_{2N}$, with $\underline{\mathbf{\Psi}}$ equivalent to the square matrix $\mathbf{\Psi}$ but with higher dimension $2N + L$.

By applying the detector $\underline{\mathbf{D}}$ to the received signal $\underline{\mathbf{r}}[l]$ in (10), we obtain

$$\underline{\mathbf{v}}[l] = e^{-j\arg(\alpha_0)}\underline{\mathbf{D}}\underline{\mathbf{r}}[l] = \underline{\mathbf{s}}_k[l] + \underline{\mathbf{m}}_k[l] + \underline{\mathbf{n}}[l] + \underline{\mathbf{a}}[l] \tag{34}$$

where $\underline{\mathbf{s}}_k[l] = |\alpha_0|\underline{\mathbf{D}}\underline{\mathbf{H}}_{\text{SIG},k}\underline{\mathbf{A}}\underline{\mathbf{b}}[l]$ is the useful signal of the user k, $\underline{\mathbf{m}}_k[l] = |\alpha_0|\underline{\mathbf{D}}\underline{\mathbf{H}}_{\text{MAI},k}\underline{\mathbf{A}}\underline{\mathbf{b}}[l]$ is the intersymbol interference (ISI) plus MAI term, $\underline{\mathbf{n}}[l] = e^{-j\arg(\alpha_0)}\underline{\mathbf{D}}\underline{\mathbf{r}}_{\text{NL}}[l]$ is the nonlinear distortion noise, $\underline{\mathbf{a}}[l] = e^{-j\arg(\alpha_0)}\underline{\mathbf{D}}\underline{\mathbf{r}}_{\text{AWGN}}[l]$ is the thermal noise part, and $\underline{\mathbf{H}}_{\text{SIG},k} = [\mathbf{0}_{2N\times K+k-1}(\underline{\mathbf{H}})_{:,K+k}\mathbf{0}_{2N\times 2K-k}]$ and $\underline{\mathbf{H}}_{\text{MAI},k} = [(\underline{\mathbf{H}})_{:,1:K+k-1}\mathbf{0}_{2N\times 1}(\underline{\mathbf{H}})_{:,K+k+1:3K}]$ are obtained by partitioning the channel-code matrix $\underline{\mathbf{H}}$ in (29) as $\underline{\mathbf{H}} = \underline{\mathbf{H}}_{\text{SIG},k} + \underline{\mathbf{H}}_{\text{MAI},k}$.

As in the AWGN scenario, we suppose that both the MAI and nonlinear distortion noise can be approximated as Gaussian. Since these approximations work reasonably well in AWGN channels, a similar behavior is expected in multipath channels, because the received signal can be thought as the superposition of many replicas of AWGN-like contributions and the sum of Gaussian-random variables is still Gaussian. By (28), the decision variable $\underline{\mathbf{d}}_k\underline{\mathbf{r}}[l]$ in (11) is equal to the $(K+k)$th element of the vector $\underline{\mathbf{v}}[l]$ in (34). Therefore, in order to evaluate the conditional SER given the channel realization $g(\tau)$, we need to calculate the power of the $(K+k)$th element of the vectors on the right-hand side of (34).

As far as the thermal noise is concerned, the elements of the vector $\underline{\mathbf{a}}[l]$ are jointly complex Gaussian-random variables, because they are obtained as a linear combination $\underline{\mathbf{D}}$ of the jointly complex Gaussian-random variables contained in $\underline{\mathbf{r}}_{\text{AWGN}}[l]$. The covariance matrix $\underline{\mathbf{\Phi}}_{\text{AWGN}} = E\{\underline{\mathbf{a}}[l]\underline{\mathbf{a}}[l]^H\}$ is expressed by $\underline{\mathbf{\Phi}}_{\text{AWGN}} = \sigma^2_{\text{AWGN}}\underline{\mathbf{D}}\underline{\mathbf{D}}^H$ and, hence, the $(K+k)$th element of the thermal noise vector $\underline{\mathbf{a}}[l]$ in (34) has power $\sigma^2_{\underline{A},k}$, expressed by

$$\sigma^2_{\underline{A},k} = (\underline{\mathbf{D}}\underline{\mathbf{D}}^H)_{K+k,K+k}\sigma^2_{\text{AWGN}}. \tag{35}$$

For the nonlinear distortion noise, we observe that the element $(\underline{\mathbf{n}}[l])_{K+k,1}$ of the vector $\underline{\mathbf{n}}[l]$ is the sum of the $2N$ elements of $\underline{\mathbf{r}}_{\text{NL}}[l]$, weighted by the $2N$ elements of $(\underline{\mathbf{D}})_{K+k,:}$ and, consequently, if the processing gain N is high enough, the nonlinear distortion noise $(\underline{\mathbf{n}}[l])_{K+k,1}$ can be well approximated as a Gaussian-random variable. The accuracy of this approximation depends not only on the processing gain N, but also on the channel realization, which affects the values of the weighting vector $(\underline{\mathbf{D}})_{K+k,:}$ by means of $\underline{\mathbf{H}}$ and on the input backoff (IBO) to the HPA, which is defined as the ratio between the saturation power $P_{x,\max}$ of the HPA input signal $x(t)$ and its mean power, as expressed by

$$\mathrm{IBO} = \frac{P_{x,\max}}{E\{|x(t)|^2\}}. \quad (36)$$

Indeed, for some channel realizations, few elements of $(\underline{\mathbf{D}})_{K+k,:}$ are characterized by higher amplitudes with respect to the others and, consequently, the Gaussian approximation will tend to fail. On the contrary, for other channel realizations, many elements of $(\underline{\mathbf{D}})_{K+k,:}$ have significant amplitude and, hence, the approximation accuracy is very good, because many elements of the vector $\underline{\mathbf{n}}[l]$ are weighted with coefficients having almost-equal values. Moreover, if the IBO is too high, most of the elements of $\underline{\mathbf{r}}_{\mathrm{NL}}[l]$ are close to zero (at least for class A or ideally predistorted amplifiers) and, consequently, $\underline{\mathbf{n}}[l]$ is practically obtained by the linear combination of few significant elements, thus violating the CLT hypothesis as well.

As a consequence of the Gaussian approximation, the covariance matrix $\underline{\mathbf{\Phi}}_{\mathrm{NL}}$ of the vector $\underline{\mathbf{n}}[l]$, expressed by $\underline{\mathbf{\Phi}}_{\mathrm{NL}} = \underline{\mathbf{D}}\underline{\mathbf{G}}\underline{\mathbf{\Psi}}\underline{\mathbf{G}}^H\underline{\mathbf{D}}^H$, completely characterizes the statistical properties of the nonlinear distortion noise and the power $\sigma^2_{\underline{N},k}$ of the $(K+k)$th element of the $\underline{\mathbf{n}}[l]$ in (34) is expressed by

$$\sigma^2_{\underline{N},k} = \left(\underline{\mathbf{D}}\underline{\mathbf{G}}\underline{\mathbf{\Psi}}\underline{\mathbf{G}}^H\underline{\mathbf{D}}^H\right)_{K+k,K+k}. \quad (37)$$

As far as the ISI-plus-MAI term $\underline{\mathbf{m}}_k[l]$ in (34) is concerned, the good accuracy of the Gaussian approximation has been already tested in [8] for the RAKE and mmse receivers in linear scenarios. Since the covariance matrix $\underline{\mathbf{\Phi}}_{\mathrm{MAI}}$ of the column vector $\underline{\mathbf{m}}_k[l]$ is expressed by $\underline{\mathbf{\Phi}}_{\mathrm{MAI}} = |\alpha_0|^2\underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{MAI},k}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H_{\mathrm{MAI},k}\underline{\mathbf{D}}^H$, the $(K+k)$th element of the vector $\underline{\mathbf{m}}_k[l]$ in (34) is characterized by a power $\sigma^2_{\underline{M},k}$, expressed by

$$\sigma^2_{\underline{M},k} = |\alpha_0|^2\left(\underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{MAI},k}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H_{\mathrm{MAI},k}\underline{\mathbf{D}}^H\right)_{K+k,K+k}. \quad (38)$$

Moreover, it is well known that the LDD is designed with the aim of eliminating both MAI and ISI, thereby achieving $\sigma^2_{\underline{M},k} = 0$. For example, the necessary condition $3K \leq 2N$ guarantees that the LDD has a sufficiently high dimensionality (i.e., $2N$) in order to distinguish up to $3K$ different symbols. By the aid of simulations, we have verified that, if $2K + \max\{K, L\} \leq 2N$, both MAI and ISI are eliminated (almost surely).

Finally, analogously to the MAI case, the power of the $(K+k)$th element of the signal vector $\underline{\boldsymbol{S}}_k[l]$ in (34) can be obtained as

$$\sigma^2_{\underline{S},k} = |\alpha_0|^2(\underline{\mathbf{D}}\underline{\mathbf{H}}_{\mathrm{SIG},k}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H_{\mathrm{SIG},k}\underline{\mathbf{D}}^H)_{K+k,K+k}. \quad (39)$$

Given the channel realization $g(\tau)$, the conditional symbol-error probability $P_k(g)$ is equal to

$$P_k(g) = 2Q\left(\sqrt{\frac{\sigma^2_{\underline{S},k}}{\sigma^2_{\underline{M},k} + \sigma^2_{\underline{N},k} + \sigma^2_{\underline{A},k}}}\right) - Q^2\left(\sqrt{\frac{\sigma^2_{\underline{S},k}}{\sigma^2_{\underline{M},k} + \sigma^2_{\underline{N},k} + \sigma^2_{\underline{A},k}}}\right) \quad (40)$$

where the quantities $\sigma^2_{\underline{S},k}$, $\sigma^2_{\underline{M},k}$, $\sigma^2_{\underline{N},k}$, and $\sigma^2_{\underline{A},k}$ are defined in (35)–(39). Therefore, the average SER can be obtained by averaging (40) over the joint probability density function (pdf) $p(\beta_1, \ldots, \beta_Q, \theta_1, \ldots, \theta_Q, \tau_1, \ldots, \tau_Q)$ of the channel parameters $\beta_1, \ldots, \beta_Q, \theta_1, \ldots, \theta_Q, \tau_1, \ldots, \tau_Q$ contained in $g(\tau)$, as expressed by

$$P_{\mathrm{SEL},k} = \int P_k(g)p(\beta_1, \ldots, \beta_Q, \theta_1, \ldots, \theta_Q, \tau_1, \ldots, \tau_Q) \cdot d\beta_1 \ldots d\beta_Q d\theta_1 \ldots d\theta_Q d\tau_1 \ldots d\tau_Q. \quad (41)$$

The solution of the multivariate integral (41) seems to be quite prohibitive, even for simple statistical channel characterizations (e.g., Rayleigh-fading channels). Consequently, $P_{\mathrm{SEL},k}$ can be estimated using a semianalytical approach by

$$\hat{P}_{\mathrm{SEL},k} = \frac{1}{N_{ch}}\sum_{i=1}^{N_{ch}} P_k(g_i) \quad (42)$$

where the N_{ch} channel realizations g_i are generated according to the joint pdf $p(\beta_1, \ldots, \beta_Q, \theta_1, \ldots, \theta_Q, \tau_1, \ldots, \tau_Q)$ and $P_k(g_i)$ is calculated as in (40).

It should be noted that the approach that we used to estimate (41) is quite similar to that of [8] for linear scenarios and it is valid for frequency-selective channels with any statistical characterization. The main difference with [8] is that, since we assume the spreading codes as fixed, the average over the spreading codes is not necessary.

Moreover, we also highlight that, in frequency-selective scenarios, the LDD and the mmse detector can be obtained without explicit knowledge of the spreading codes of the other users [23]. For example, the matrix $\underline{\mathbf{P}} = |\alpha_0|^2\underline{\mathbf{H}}\underline{\mathbf{A}}^2\underline{\mathbf{H}}^H + \underline{\mathbf{W}}$ inverted in (33) can be evaluated by estimating $\underline{\mathbf{P}} = E\{\underline{\mathbf{r}}[l]\underline{\mathbf{r}}[l]^H\}$ from the received signal $\underline{\mathbf{r}}[l]$, as in nonlinear AWGN channels. In any case, the proposed approach for the SER evaluation can also be applied to low-complexity linear detectors obtained by code-matched filtering after the channel equalization [24].

In addition, we note that the theoretical analysis of the mmse detector, which assumes the knowledge of $\underline{\mathbf{H}}$ at the receiver side, could be used to obtain a lower bound for the steady-state SER of adaptive mmse receivers [25]. Indeed, since adaptive receivers do not assume $\underline{\mathbf{H}}$ to be known, their performance will also be impaired by the estimation error on the detector vector.

VI. Simulation Results

In this section, we validate the analytical results by means of computer simulations. Our main purpose is to examine the impact of the approximations introduced in the theoretical analysis. In all the simulations, the signal $w(t)$ transmitted by the HPA has been generated by using the exact relation (3) instead of its simplified version (4), employed in the theoretical analysis.

The scenario with a base station that transmits data to K users is considered. Unless otherwise stated, it is assumed that $K = 40$ users are active. The transmitted amplitudes $\{A_k\}$ are assumed to be equal for all the users. Since we mainly focus on multipath channels, we employ Gold codes of length $N = 63$, which are able to reduce both ISI and MAI at the output of the

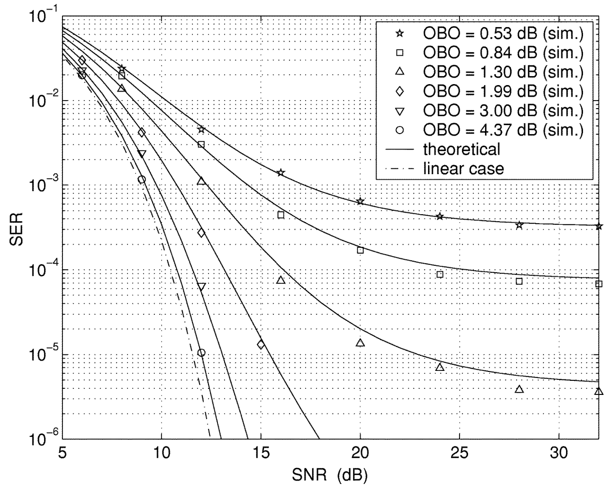

Fig. 1. SER of the mmse detector in AWGN channels (soft limiter $K = 40$).

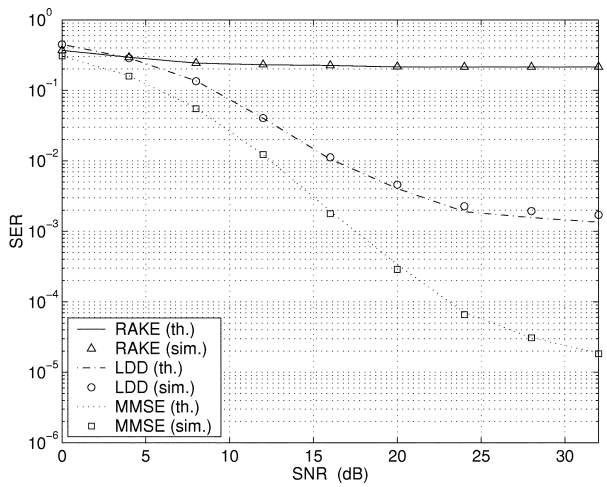

Fig. 2. SER comparison in frequency-selective channels (soft limiter, OBO = 1.30 dB, $K = 40$).

RAKE receiver. For the chip pulse-shaping waveform $p(t)$, a square-root raised cosine with a rolloff factor equal to $\rho = 0.22$ has been chosen. An oversampling factor $P = 10$ with respect to the chip rate $1/T_c$ has been used, leading to a simulation sampling frequency approximately eight times higher than the Nyquist rate. The nonlinearities considered for the HPA model are the soft limiter [14], which is the envelope input–output characteristic of an ideally predistorted HPA, with AM/AM and AM/PM curves expressed by

$$G(|x|) = \begin{cases} |x|, & |x| \le A_{\rm sat} \\ A_{\rm sat}, & |x| > A_{\rm sat} \end{cases} \quad \Phi(|x|) = 0 \qquad (43)$$

where $A_{\rm sat}$ is the maximum amplitude of the amplifier and the Saleh HPA model [26] with AM/AM and AM/PM is expressed by

$$G(|x|) = \frac{2|x|}{1+|x|^2} \quad \Phi(|x|) = \frac{\pi}{3}\frac{|x|^2}{1+|x|^2}. \qquad (44)$$

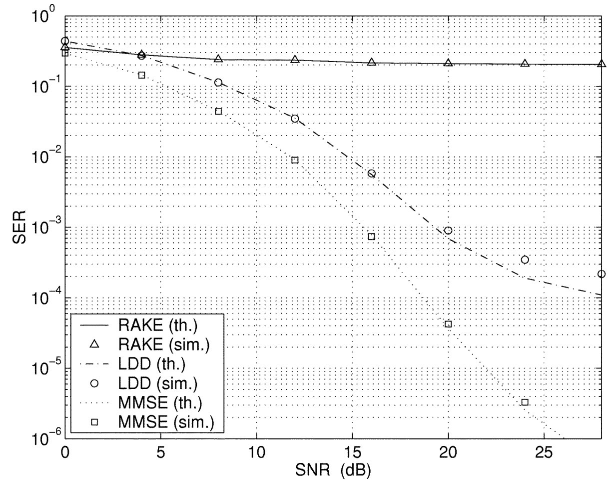

Fig. 3. SER comparison in frequency-selective channels (soft limiter, OBO = 1.99 dB, $K = 40$).

The signal-to-noise ratio (SNR) is defined at the input of the detector as

$$\text{SNR} = \frac{E\left\{|r_n[l] - r_{n,\text{AWGN}}[l]|^2\right\}}{E\left\{|r_{n,\text{AWGN}}[l]|^2\right\}} \qquad (45)$$

while the output backoff (OBO), similar to the IBO in (36), is defined as

$$\text{OBO} = \frac{P_{w,\max}}{E\{|w(t)|^2\}} \qquad (46)$$

where $P_{w,\max}$ represents the maximum HPA output power

Fig. 1 shows the SER performance of the mmse receiver in AWGN channels when the soft-limiter model (43) is used for the HPA. It is evident that there is a very good agreement between the analytical model and the simulation results for high OBO and for very low OBO values, while there is a little mismatch at high SNR for relatively low OBO values $(\text{OBO} = 1.30 \text{ dB})$. As in [13], this mismatch is mainly due to the Gaussian approximation of the nonlinear distortion noise and is marginally caused by the time variability of the linear gain of the HPA.

Figs. 2–4 illustrate the SER performance of the three linear receivers (RAKE, LDD, and mmse) in frequency-selective fading channels for different OBO values. The soft-limiter model (43) has been chosen for the HPA. The amplitudes $\{\beta_q \exp(j\theta_q)\}$ of the $Q = 15$ channel paths are modeled as independent zero-mean complex Gaussian-random variables with variance $E\left\{|\beta_q|^2\right\} = 1/Q$, while the path delays $\{\tau_q\}$ are fixed quantities, multiple of the chip duration T_c. The theoretical SER has been evaluated by averaging (40) over $N_{ch} = 400$ independent channel realizations. As expected, the mmse receiver, which takes into account the presence of the nonlinear distortion noise, outperforms the other two detectors. Moreover, for both the RAKE and the mmse detector, the comparison between theoretical and simulation results does not give evidence of significant differences, whereas a little mismatch exists for the LDD at high SNR. This mismatch is caused by the noise-amplification effect produced by the LDD, which also emphasizes the nonlinear distortion noise. On the other hand, the mmse detector tries to find a good tradeoff between the residual MAI and the noise, producing a smaller

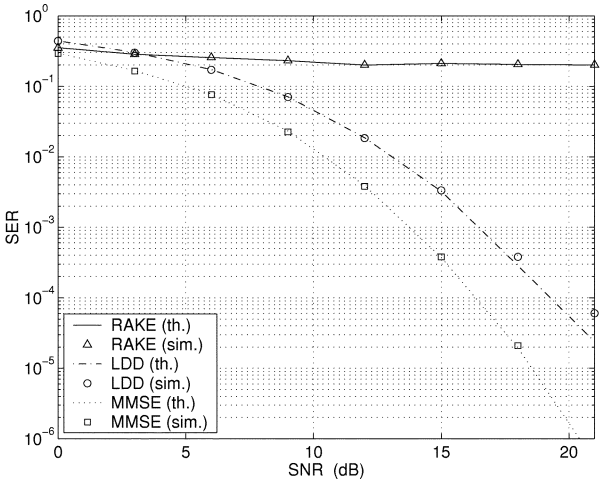

Fig. 4. SER comparison in frequency-selective channels (soft limiter, OBO = 3.00 dB, $K = 40$).

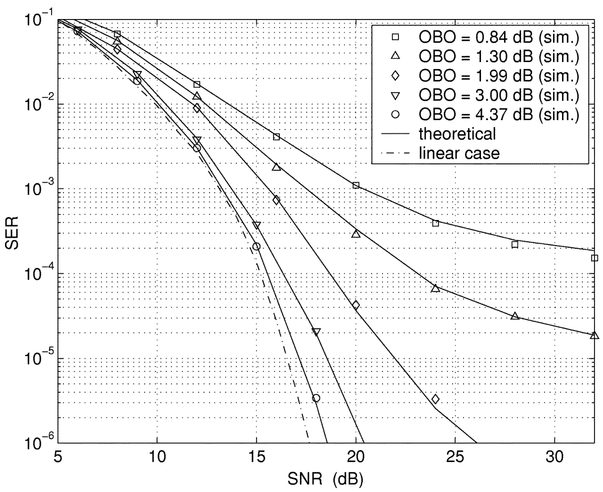

Fig. 5. SER of the mmse in frequency-selective channels (soft limiter, $K = 40$).

amplification of the nonlinear distortion noise. Therefore, for the LDD, the modeling errors of the nonlinear distortion noise are more evident than for the mmse detector.

Figs. 5 and 6 exhibit the SER performance of the mmse detector for different OBO values, using the frequency-selective channel model described above and the HPA models (43) and (44), respectively. As in AWGN channels, for practical uncoded SERs, there is a good agreement between analytical model and simulation results, especially if the OBO is very low or quite high. Moreover, a little mismatch is present when the OBO is roughly 2 dB (Fig. 6).

Finally, Figs. 7 and 8 show the SER performance when the number of active users is reduced to $K = 20$ and $K = 10$, respectively. We consider the mmse detector in frequency-selective channels, assuming the soft-limiter model (43) as HPA. In both the Figs. 7 and 8, it is evident that the SER analysis is less accurate with respect to the case with $K = 40$ active users (Fig. 5). However, the SER mismatch is quite small; therefore, the SER analysis could also be useful in these cases.

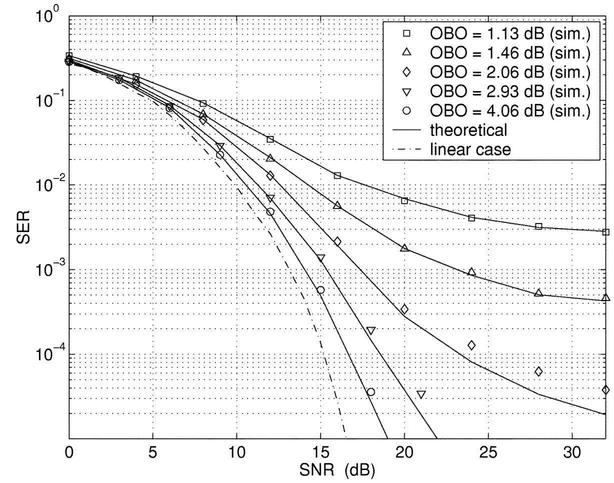

Fig. 6. SER of the mmse in frequency-selective channels (Saleh model, $K = 40$).

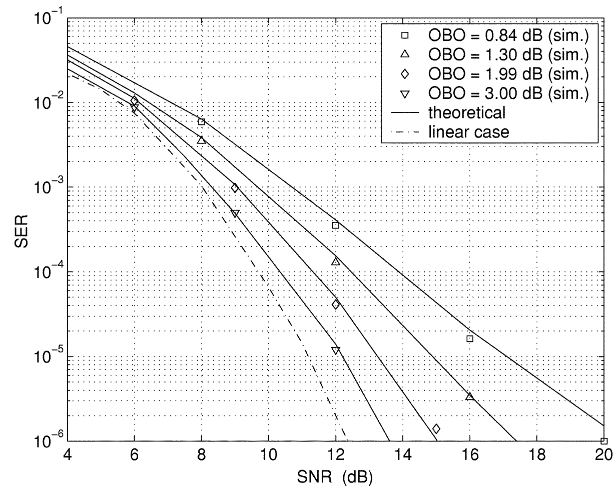

Fig. 7. SER of the mmse in frequency-selective channels (soft limiter, $K = 20$).

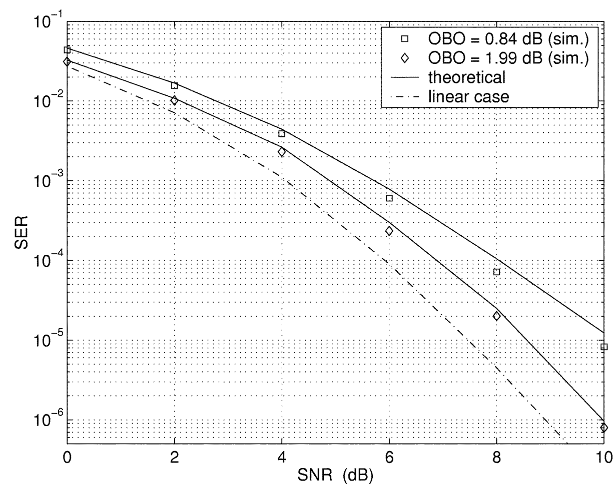

Fig. 8. SER of the mmse in frequency-selective channels (soft limiter $K = 10$).

VII. Conclusion

An analytical framework to evaluate the SER performance of linear multiuser detectors in the presence of a nonlinear amplifier in DS-CDMA downlink channels has been introduced. Results for QPSK mapping with square-root-raised cosine pulse shaping have been presented. Simulation results have shown that the analytical model is quite appropriate, especially under the hypothesis of high number of users. Therefore, the proposed approach represents a valuable tool to predict the SER performance, with dramatically reduced computation time with respect to simulations, which require signal interpolation in the presence of nonlinearities. The design of nonlinear multiuser detectors that take into account the presence of nonlinear distortions may be the subject of future work.

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