Theoretical Analysis and Performance of the Decorrelating Detector for DS-CDMA Signals in Nonlinear Channels

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***Abstract*—We analyze the effects of the nonlinear distortions, introduced by the transmitting high-power amplifier (HPA), on the performance of the linear decorrelating multiuser detector in direct-sequence code-division multiple-access (DS-CDMA) downlink systems. By assuming and motivating the Gaussian distribution of the nonlinear distortion noise, the symbol-error rate (SER) and the total degradation (TD) has been derived theoretically in additive white Gaussian noise (AWGN) and frequency-flat Rayleigh fading channels. Simulation results show that the analytical model is quite accurate in many scenarios.**

***Index Terms*—Code-division multiple access (CDMA), multiuser detection, nonlinear distortions.**

I. Introduction

DIRECT-SEQUENCE code-division multiple access (DS-CDMA) is a widely employed technique for wireless communications in both satellite and cellular mobile systems. The almost constant envelope exhibited by the single user signal in the uplink scenario is clearly an advantage with respect to other techniques, e.g., multicarrier CDMA, because of the reduced nonlinear distortions mainly introduced by the high-power amplifier (HPA) at the transmitter side. Anyway, in the downlink scenario, the transmitted signal is the sum of many independent signals, each one belonging to a different user. Since each signal may add constructively or not to the others, the overall signal amplitude is characterized by a high variability, and consequently it is exposed to nonlinear distortions that may be caused by the HPA. Moreover, when multiple codes are assigned to a single user, such nonlinear distortions are significant also in the uplink scenario [1], because of the high peak-to-average power ratio (PAR) of the signal transmitted by the mobile user.

Generally, a predistortion technique [2] is employed at the transmitter side to counteract the nonlinear characteristics of the HPA. On the other hand, even if the HPA is perfectly linearized, a residual clipping is not avoidable because of the maximum HPA output power. Therefore, it is of interest to quantify the nonlinear distortion effects in terms of SER performance and total degradation (TD) [3], which is a parameter that allows to optimize the mean output power for a given HPA and a target SER.

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In the downlink scenario, the effect of the HPA has been analyzed in [3] for the matched filter detection of DS-CDMA signals in AWGN channels. However, nonorthogonal DS-CDMA systems greatly benefit of multiuser detection techniques [4] to improve the SER performance. Indeed, multiuser detection techniques become attractive when the number of users increases, that is when the nonlinear distortions also are significant.

The main contribution of this paper is to consider how the nonlinear amplifier affects the SER performance of the decorrelating detector [4]–[7] in the downlink scenario. The paper is organized as follows. In Section II, the downlink DS-CDMA system model for frequency-flat nonlinear channels is presented. The statistical characterization of the nonlinear distortions of Section III allows to derive in Section IV the SER performance in AWGN and in frequency-flat Rayleigh fading channels, and, in Section V, the TD performance analysis. In Section VI, simulation results are provided in order to verify the analytical findings, and finally some concluding remarks are drawn.

II. System Model

In the downlink of a DS-CDMA system, the signal that carries the information to the kth user is expressed by

$$x_k(t) = A_k \sum_{i=-\infty}^{+\infty} b_k[i] s_k(t - iT) \tag{1}$$

where A_k, $s_k(t)$ and $b_k[i]$ are the amplitude, the spreading waveform, and the ith symbol, respectively, and T is the symbol duration. We assume that the symbols $\{b_k[i]\}$ belong to a set of zero-mean i.i.d. random variables with $E\{|b_k[i]|^2\} = 1$. The spreading waveform $s_k(t)$ of the kth user can be expressed as $s_k(t) = (1/\sqrt{N}) \sum_{j=0}^{N-1} c_k[j] p(t - jT_c)$, where N is the processing gain, $T_c = T/N$ is the chip duration, $p(t)$ is the impulse response of the chip pulse shaper, and $c_k[j]$ is the jth value of the kth user's spreading code, with $|c_k[j]| = 1$. Assuming that K users are active, the signal transmitted by the base station can be expressed by

$$w(t) = G\left(|x(t)|\right) e^{j\Phi(|x(t)|) + j\arg[x(t)]} \tag{2}$$

where $x(t) = \sum_{k=1}^{K} x_k(t)$, and $G(\cdot)$ and $\Phi(\cdot)$ model the AM/AM and AM/PM distortion, respectively, introduced by the HPA [2]. Alternatively, $w(t)$ in (2) can be expressed as

$$w(t) = \alpha_0 x(t) + n_d(t) \tag{3}$$

where the complex coefficient α_0 takes the linear amplification gain into account, and $n_d(t)$ is the nonlinear distortion noise. Although there exist many ways to choose the parameters α_0 and $n_d(t)$ that satisfy (3), in the next section we will resort to the

Bussgang Theorem [8] in order to identify the quantities used in this context.

Two different type of channels have been considered.

- AWGN: the channel impulse response is given by $g(\tau,t) = \beta(t)e^{j\theta(t)}\delta(\tau)$, with $\beta(t) = 1$ and $\theta(t) = \theta_0$;
- flat Rayleigh fading: the channel is supposed to be slowly time-varying with impulse response given by $g(\tau,t) = \beta(t)e^{j\theta(t)}\delta(\tau)$, in such a way that the amplitude $\beta(t)$ and the phase-shift $\theta(t)$ can be considered constant during the symbol interval T. The gain $\beta(t)$ is modeled as a stationary random process, with probability density function (pdf) expressed by $f(\beta) = 2\beta e^{-\beta^2}$, where $\beta \geq 0$.

At the kth user receiver, the signal affected by the frequency-flat channel is perturbed by a complex zero-mean AWGN $n(t)$, as expressed by $r(t) = \int_{-\infty}^{+\infty} g(t,\tau)w(t-\tau)d\tau + n(t) = r_{\mathrm{SIG}}(t) + r_{\mathrm{NL}}(t) + n(t)$, where $r_{\mathrm{SIG}}(t) = \beta(t)\alpha_0 e^{j\theta(t)}x(t)$, and $r_{\mathrm{NL}}(t) = \beta(t)e^{j\theta(t)}n_d(t)$. We assume a receiver with coherent demodulation, perfect time and frequency synchronization, and phase-shift recovery. The signal $r(t)$ is filtered by a chip-matched filter and successively sampled at the chip rate $1/T_c$, thus obtaining $r_n[l] = \int_{-\infty}^{+\infty} r(t)p^*(t - lT - nT_c)dt = r_{n,\mathrm{SIG}}[l] + r_{n,\mathrm{NL}}[l] + r_{n,\mathrm{AWGN}}[l]$. Defining, $\beta[l] = \beta(lT)$, $\varphi[l] = \theta(lT) + \arg(\alpha_0)$, $\mathbf{c}_k = N^{-1/2}[c_k[0]\cdots c_k[N-1]]^T$, $\mathbf{r}[1] = [r_0[l]\cdots r_{N-1}[l]]^T$, $\mathbf{A} = \mathrm{diag}(A_1,\cdots,A_K)$, $\mathbf{b}[l] = [b_1[l]\cdots b_K[l]]^T$, and $\mathbf{C} = [\mathbf{c}_1\cdots\mathbf{c}_K]$, it is easy to derive

$$\begin{aligned}\mathbf{r}[l] &= \mathbf{r}_{\mathrm{SIG}}[l] + \mathbf{r}_{\mathrm{NL}}[l] + \mathbf{r}_{\mathrm{AWGN}}[l]\\ &= \beta[l]|\alpha_0|e^{j\varphi[l]}\mathbf{CAb}[l] + \mathbf{r}_{\mathrm{NL}}[l] + \mathbf{r}_{\mathrm{AWGN}}[l].\end{aligned} \tag{4}$$

The linear decorrelating detector (LDD), or zero-forcing (ZF) detector, of the kth user can be expressed by

$$\mathbf{d}_k = (\mathbf{D})_k, \quad \mathbf{D} = \mathbf{R}^{-1}\mathbf{C}^H \tag{5}$$

where the subscript k stands for the kth row of a matrix, and $\mathbf{R} = \mathbf{C}^H\mathbf{C}$. The LDD can be obtained either by using the knowledge of the spreading codes in $\mathbf{C}$ [6], or blindly [7]. The decision variable is then obtained by compensating for all the attenuations and rotations introduced by the HPA and by the channel, as expressed by

$$\tilde{b}_k(l) = (\beta[l]|\alpha_0|)^{-1} e^{-j\varphi[l]} A_k^{-1}\mathbf{d}_k\mathbf{r}[l]. \tag{6}$$

III. Characterization of the Nonlinear Distortions

If all the amplitudes A_k are nearly equal with one another, and the number of users K is sufficiently high (e.g., $K > 20$), then the HPA input signal $x(t)$ can be approximated by a Gaussian random process [3], because of the central limit theorem (CLT) [8]. The hypothesis of a DS-CDMA system with a high number of users is not unrealistic when a multiuser detector counteracts the consequent multiuser interference.

Since $x(t)$ has been modeled as a zero-mean Gaussian process, we can invoke the complex extension of the Bussgang Theorem [3], [8], [9] in order to express the HPA output as the sum of two uncorrelated components: the useful one $\alpha(t)x(t)$, where $\alpha(t) = E\{w(t)x^*(t)\}/E\{|x(t)|^2\}$, and the nonlinear distortion noise $n_d(t)$. However, in many cases, e.g., when the pulse shaping waveform is a raised cosine function with a small roll-off factor, the time-variability of $\alpha(t)$ is small and hence it can be neglected [3], thereby motivating (3). Therefore, the autocorrelation function of the HPA output can be written as $R_{ww}(\tau) = |\alpha_0|^2 R_{xx}(\tau) + R_{n_dn_d}(\tau)$, where $|\alpha_0|^2 = \gamma_0$ and

$$R_{n_dn_d}(\tau) = \sum_{i=1}^{+\infty} \gamma_i R_{xx}(\tau)^{2i+1} \tag{7}$$

are obtained using the coefficients γ_i, $i \geq 0$, which only depend on the AM/AM and AM/PM characteristics, and on the input power $E\{|x(t)|^2\}$. These coefficients can be calculated by numerical integration or closed form expressions [10], [11]. Consequently, we have a complete characterization of both the effects (linear gain and nonlinear distortion noise) produced by the HPA.

IV. SER Performance of the Decorrelating Detector in Nonlinear Channels

For the sake of simplicity, we suppose that the data are QPSK-mapped, but the extension to other modulation schemes (e.g., M-PSK or M-QAM) is straightforward.

A. AWGN Channel

Since the nonlinear noise is generated at the transmitter, the *apparent* signal-to-noise ratio (SNR) $\mathrm{SNR}_{\mathrm{app}}$ measured at the decorrelator input is

$$\mathrm{SNR}_{\mathrm{app}} = \frac{\sigma^2_{\mathrm{SIG}} + \sigma^2_{\mathrm{NL}}}{\sigma^2_{\mathrm{AWGN}}} \tag{8}$$

where $\sigma^2_{\mathrm{SIG}} = E\{|r_{n,\mathrm{SIG}}[l]|^2\}$, $\sigma^2_{\mathrm{NL}} = E\{|r_{n,\mathrm{NL}}[l]|^2\}$ and $\sigma^2_{\mathrm{AWGN}} = E\{|r_{n,\mathrm{AWGN}}[l]|^2\}$. However, the SER performance are not directly imposed by the $\mathrm{SNR}_{\mathrm{app}}$, because an increase of σ^2_{NL} deteriorates the SER performance while increasing the $\mathrm{SNR}_{\mathrm{app}}$. As a consequence, we should define the *effective* SNR $\mathrm{SNR}_{\mathrm{eff}}$ as

$$\mathrm{SNR}_{\mathrm{eff}} = \frac{\sigma^2_{\mathrm{SIG}}}{\sigma^2_{\mathrm{NL}} + \sigma^2_{\mathrm{AWGN}}} \tag{9}$$

which is related to $\mathrm{SNR}_{\mathrm{app}}$ by

$$(\mathrm{SNR}_{\mathrm{eff}})^{-1} = (\mathrm{SNR}_{\mathrm{app}})^{-1} + \left[1 + (\mathrm{SNR}_{\mathrm{app}})^{-1}\right](\mathrm{SNR}_{nl})^{-1} \tag{10}$$

where $\mathrm{SNR}_{nl} = \sigma^2_{\mathrm{SIG}}/\sigma^2_{\mathrm{NL}}$ is imposed by the input back-off (IBO), defined as $\mathrm{IBO} = P_{x,\mathrm{sat}}/E\{|x(t)|^2\}$, being $P_{x,\mathrm{sat}}$ the input saturating power of the HPA. In order to obtain the SER performance, we want to characterize (in terms of first-order statistics) both the nonlinear and the thermal noise quantities at the decision variable, and to find a link with the SNRs, defined above, expressed at the input of the multiuser decorrelating detector.

In AWGN channels, the decorrelator output vector becomes $\mathbf{Dr}[l] = |\alpha_0|e^{j\varphi}\mathbf{Ab}[l] + \mathbf{Dr}_{\mathrm{NL}}[l] + \mathbf{Dr}_{\mathrm{AWGN}}[l]$, and, taking into account the QPSK mapping, only the phase-shift φ has to be compensated for, thus obtaining

$$\mathbf{v}[l] = e^{-j\varphi}\mathbf{Dr}[l] = \mathbf{s}[l] + \mathbf{n}[l] + \mathbf{a}[l] \tag{11}$$

where $\mathbf{s}[l] = |\alpha_0|\mathbf{Ab}[l]$, $\mathbf{n}[l] = e^{-j\varphi}\mathbf{Dr}_{\mathrm{NL}}[l]$, and $\mathbf{a}[l] = e^{-j\varphi}\mathbf{Dr}_{\mathrm{AWGN}}[l]$. Focusing on the user k, the kth element of the useful signal vector $\mathbf{s}[l]$ in (11) is equal to $|\alpha_0|A_k b_k[l]$, and therefore its power is expressed by

$$\sigma^2_{\mathrm{S},k} = E\left\{|\alpha_0 A_k b_k[l]|^2\right\} = |\alpha_0|^2 A_k^2. \tag{12}$$

As far as the thermal noise vector $\mathbf{a}[l]$ in (11) is concerned, its elements are jointly complex Gaussian random variables, because the detector $\mathbf{D}$ is linear. The power of the kth element of $\mathbf{a}[l]$ in (11) can be expressed by [4]

$$\sigma^2_{\mathrm{A},k} = \varepsilon_k \sigma^2_{\mathrm{AWGN}} \tag{13}$$

where $\varepsilon_k = (\mathbf{R}^{-1})_{k,k} \geq 1$ is the noise enhancement factor introduced by the decorrelating operation.

As far as the nonlinear distortion noise is concerned, it is noteworthy that each element of the vector $\mathbf{n}[l]$ in (11) is the linear combination of the N elements of $\mathbf{r}_{\mathrm{NL}}[l]$, by the N elements of $e^{-j\varphi}\mathbf{d}_k$. Therefore, if N is high enough, the nonlinear distortion noise can be approximated as a zero-mean Gaussian random variable because of the CLT. The accuracy of this approximation depends on the processing gain N, and on the used spreading codes $\mathbf{C}$ that affect the values of the weighting vector $e^{-j\varphi}\mathbf{d}_k$. Indeed, if few elements of $\mathbf{d}_k$ in (5) have amplitudes higher than the others, only these dominant elements will contribute to the nonlinear distortion noise at the decision variable, and, hence, the Gaussian approximation tends to fail. On the contrary, if many elements of $\mathbf{d}_k$ have a high modulus, the corresponding elements of $\mathbf{r}_{\mathrm{NL}}[l]$ are weighted with coefficients having almost-equal values, and consequently the approximation is very good. Moreover, also the HPA working point has an impact on the Gaussian assumption of the nonlinear distortion noise. Indeed, when the IBO is very high, most of the elements of $\mathbf{r}_{\mathrm{NL}}[l]$ are close to zero (at least for class A amplifiers). Consequently, for high values of the IBO, $\mathbf{n}[l]$ is practically obtained by the linear combination of few significant elements, thus violating the CLT hypothesis.

As a consequence of the Gaussian approximation, the statistical properties of $\mathbf{n}[l]$ are completely characterized by the covariance matrix $\mathbf{\Phi}_{\mathrm{NL}} = E\{\mathbf{n}[l]\mathbf{n}[l]^H\} = \mathbf{D\Psi D}^H$, with the matrix $\mathbf{\Psi} = E\{\mathbf{r}_{\mathrm{NL}}[l]\mathbf{r}_{\mathrm{NL}}[l]^H\}$ obtained as

$$(\mathbf{\Psi})_{m,n} = [R_{n_d n_d}(\tau) * R_{pp}(\tau)]_{\tau=(n-m)T_c} \tag{14}$$

where $R_{pp}(\tau)$ is the autocorrelation function of the pulse shaping waveform $p(t)$, $R_{n_d n_d}(\tau)$ is expressed by (7), and $*$ denotes the convolution operator. Consequently, the power $\sigma^2_{N,k}$ of the nonlinear noise component can be expressed by $\sigma^2_{\mathrm{N},k} = (\mathbf{D\Psi D}^H)_{k,k}$, which is the kth element of the main diagonal of $\mathbf{\Phi}_{\mathrm{NL}}$.

Moreover, if the spreading sequences are characterized by good autocorrelation properties (as the Gold codes used in this paper), the elements $(\mathbf{\Psi})_{m,n}$ with $m \neq n$ are very small with respect to $(\mathbf{\Psi})_{m,m}$. Therefore, the matrix $\mathbf{\Psi}$ is nearly diagonal and it can be well approximated by $\sigma^2_{\mathrm{NL}}\mathbf{I}_{\mathrm{N}}$, where $\sigma^2_{\mathrm{NL}} = (\mathbf{\Psi})_{m,m} = [R_{n_d n_d}(\tau) * R_{pp}(\tau)]_{\tau=0}$. In this case, the nonlinear noise power on the decision variable reduces to

$$\sigma^2_{\mathrm{N},k} = \varepsilon_k \sigma^2_{\mathrm{NL}}. \tag{15}$$

Finally, since $\mathbf{a}[l]$ and $\mathbf{n}[l]$ in (11) are mutually independent, the pdf of the nonlinear-plus-thermal noise is also Gaussian. Therefore, for QPSK modulation, the symbol-error probability for the user k can be expressed by [12]

$$\begin{aligned} P_{e,k,\mathrm{AWGN}} &= 2\mathrm{Q}(\sqrt{\mathrm{SNR}_{dec,k}}) - \mathrm{Q}^2(\sqrt{\mathrm{SNR}_{dec,k}}), \\ \mathrm{SNR}_{dec,k} &= \frac{\sigma^2_{\mathrm{S},k}}{\sigma^2_{\mathrm{N},k} + \sigma^2_{\mathrm{A},k}} \\ &= \frac{|\alpha_0|^2 A_k^2}{\varepsilon_k\left(\sigma^2_{\mathrm{NL}} + \sigma^2_{\mathrm{AWGN}}\right)}, \end{aligned} \tag{16}$$

where $\mathrm{SNR}_{dec,k}$ is the effective SNR at the decision variable. It can be related to the input $\mathrm{SNR}_{\mathrm{eff}}$ defined in (9) by

$$\mathrm{SNR}_{dec,k} = \frac{N A_k^2}{\varepsilon_k \sum_{i=1}^{K} A_i^2}\mathrm{SNR}_{\mathrm{eff}} \tag{17}$$

where N is the gain introduced by the despreading operation, ε_k is the noise amplification factor introduced by the decorrelation, and $A_k^2/\sum_{i=1}^{K} A_i^2$ is the fraction of the received signal power relative to the user k.

It is noteworthy that the analysis carried out for AWGN channels can be extended to frequency-selective fading channels by enlarging the receiver observation window [5]. In this case, the LDD is obtained by replacing the matrix $\mathbf{C}$ of the spreading codes in (5) with a matrix that contains the multipath-affected spreading codes. Given a channel realization, the conditional SER is obtained by applying the Gaussian approximation. A semi-analytical SER expression is then obtained by averaging over the multipath statistics [13].

B. Flat Rayleigh Fading Channel

In a slow flat-fading channel, the useful signal and the nonlinear distortion noise pass through the same channel, thus experiencing the same random complex gain. Supposing perfect phase compensation at the receiver side, the symbol-error probability conditioned to the knowledge of β can be approximated as

$$\begin{aligned} P_{e,k}(\beta) &\approx 2\mathrm{Q}\left(\sqrt{\mathrm{SNR}_{dec,k}(\beta)}\right), \\ \mathrm{SNR}_{dec,k}(\beta) &= \frac{\beta^2\sigma^2_{\mathrm{S},k}}{\beta^2\sigma^2_{\mathrm{N},k} + \sigma^2_{\mathrm{A},k}} \\ &= \frac{\beta^2|\alpha_0|^2 A_k^2}{\beta^2\varepsilon_k\sigma^2_{\mathrm{NL}} + \varepsilon_k\sigma^2_{\mathrm{AWGN}}} \end{aligned} \tag{18}$$

for practical symbol-error probabilities (i.e., when $P_{e,k}(\beta) \leq 10^{-2}$). The average SER, obtained by averaging $P_{e,k}(\beta)$ over the pdf of the channel gain β, is expressed by

$$\begin{aligned} P_{e,k,\mathrm{FLAT}} &\approx \int_0^{+\infty} 4\beta e^{-\beta^2}\mathrm{Q}\left(\sqrt{\frac{\beta^2\mu_k^2}{\beta^2\lambda^2+1}}\right)d\beta, \\ \mu_k^2 &= \frac{|\alpha_0|^2 A_k^2}{\varepsilon_k\sigma^2_{\mathrm{AWGN}}}, \\ \lambda^2 &= \frac{\sigma^2_{\mathrm{NL}}}{\sigma^2_{\mathrm{AWGN}}}. \end{aligned} \tag{19}$$

The analytical computation of the integral in (19) has been derived in [14]. The final result can be expressed by a series of generalized hypergeometric functions ${}_pF_q(\cdot;\cdot;\cdot)$ [15], as expressed by

$$P_{e,k,\mathrm{FLAT}} \approx 1 - \frac{\sqrt{2}}{2}\mu_k \exp\left(-\frac{\mu_k^2}{2\lambda^2}\right) \sum_{m=0}^{+\infty} \frac{1}{m!}\left(\frac{\mu_k^2}{2\lambda^2}\right)^m \times {}_2F_0\left(m+\frac{3}{2}, \frac{1}{2};;-\lambda^2\right). \quad (20)$$

V. Total Degradation

The TD to obtain a target SER can be defined as [3]

$$[\mathrm{TD}]_{\mathrm{dB}} = ([\mathrm{SNR}_{\mathrm{app}}]_{\mathrm{dB}} - [\mathrm{SNR}_{\mathrm{eff}}]_{\mathrm{dB}}) + [\mathrm{OBO}]_{\mathrm{dB}} \quad (21)$$

where the output back-off (OBO) is defined as $\mathrm{OBO} = P_{w,\mathrm{sat}}/E\{|w(t)|^2\}$, being $P_{w,\mathrm{sat}}$ the maximum HPA output power, $\mathrm{SNR}_{\mathrm{app}}$, defined in (8), is the apparent SNR required at the LDD input in order to obtain the target SER, and $\mathrm{SNR}_{\mathrm{eff}}$, defined in (9), is the input SNR required by the LDD to obtain the same SER in the linear scenario.

The term into the round brackets in (21) represents the power penalty with respect to the linear scenario, while the OBO represents the power penalty with respect to the maximum amplifier output power. Clearly, there exists a trade-off between these two power penalties, because, if the OBO is decreased, the HPA introduces a higher distortion, and consequently the required $\mathrm{SNR}_{\mathrm{app}}$ for the target SER must be greater. As a consequence, the minimization of the TD can be considered a fair criterion for the selection of the optimum OBO value, provided that other effects, like the adjacent channel interference (ACI), can be neglected or eliminated by linear filtering. Obviously, the TD can be analytically evaluated by substituting in (21) the quantities obtained in Section IV.

VI. Simulation Results

We consider two different amplifier models: the Saleh model [16], characterized by the AM/AM and AM/PM curves $G(|x|) = 2|x|/(1+|x|^2)$ and $\Phi(|x|) = (\pi/3)|x|^2/(1+|x|^2)$, respectively, and the ideally predistorted amplifier (IPA) model, characterized by $G(|x|) = |x|$ for $|x| \le A_{\mathrm{sat}}$, $G(|x|) = A_{\mathrm{sat}}$ when $|x| > A_{\mathrm{sat}}$, and $\Phi(|x|) = 0$. We assume the pulse shaping waveform $p(t)$ as a square-root raised cosine function with roll-off factor $\rho = 0.22$.

First, we consider a base station that transmits data with equal amplitudes to all the $K = 40$ users. Gold sequences of length $N = 63$ have been chosen for the short spreading codes. Fig. 1 shows the SER of the LDD as function of $\mathrm{SNR}_{\mathrm{app}}$ in AWGN channels when an IPA is used. It is noteworthy the good agreement between the theoretical curves and the simulated points for all the OBO values. On the contrary, in [17], a SER floor mismatch is present at low OBO. Such a difference is not evident herein, because a nonrectangular chip pulse shaper is assumed, and consequently, for a fixed OBO, the power of the nonlinear distortion noise at the output the chip-matched receiving filter is smaller than in [17]. Therefore, in this case, the nonlinear distortion noise is not enough to manifest the saturating behavior of the SER curves, unless extremely low OBO values are considered.

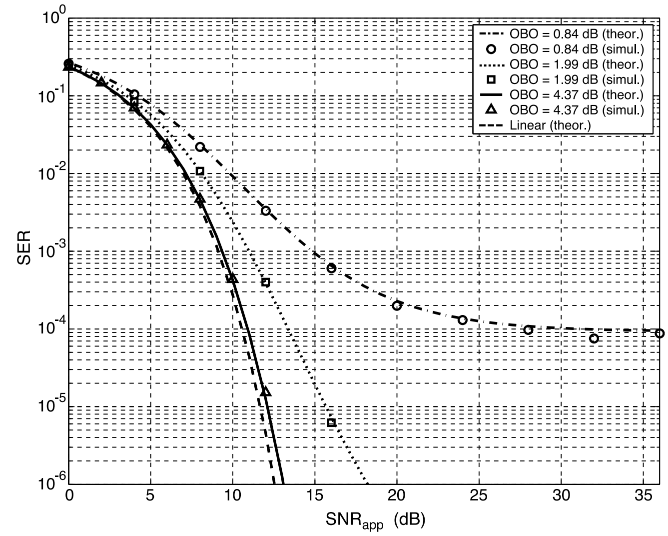

Fig. 1. SER performance in AWGN channels (IPA, 40 users, equal users' amplitudes).

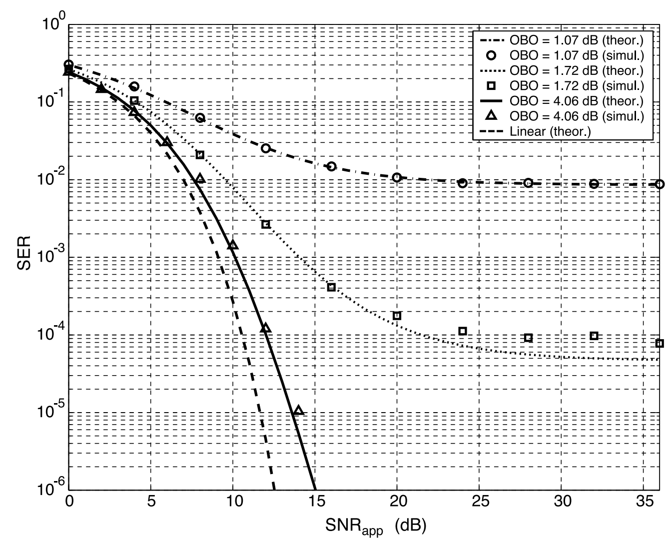

Fig. 2. SER performance in AWGN channels (Saleh amplifier, 40 users, equal users' amplitudes).

Moreover, when the Saleh HPA model replaces the IPA (Fig. 2), the SER behavior is similar to [17]. Indeed, there is a good agreement at high back-off ($\mathrm{OBO} = 4$ dB), while at low OBO values ($\mathrm{OBO} = 1.72$ dB), the simulated performance does not perfectly match with the theoretical one because of the non perfect Gaussianity of the nonlinear distortion noise. As in [17], the simulated SER slightly diverges from the theoretical one when the nonlinear distortion noise is dominant with respect to the AWGN, i.e., at high $\mathrm{SNR}_{\mathrm{app}}$. The good agreement between simulated and theoretical performance at low $\mathrm{SNR}_{\mathrm{app}}$ for any OBO proves that the signal loss induced by the HPA is correctly modeled by the coefficient $|\alpha_0|$. For very low OBOs (i.e., $\mathrm{OBO} = 1$ dB), the Gaussian approximation is accurate, despite of the increased nonlinear distortion noise. Furthermore, in flat Rayleigh fading channels (Fig. 3), considerations quite similar to the AWGN scenario hold true.

Fig. 4 shows the TD when $\mathrm{SER} = 10^{-3}$. The TD is obtained using the approximated analytical model, since little mismatch

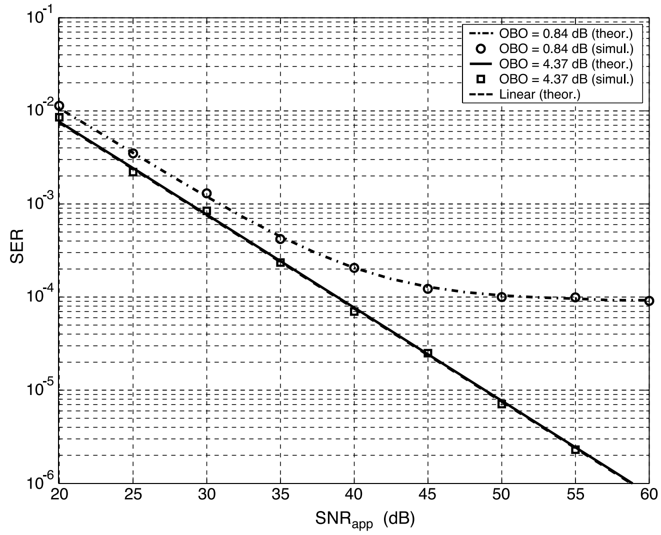

Fig. 3. SER performance in flat Rayleigh fading channels (IPA, 40 users, equal users' amplitudes).

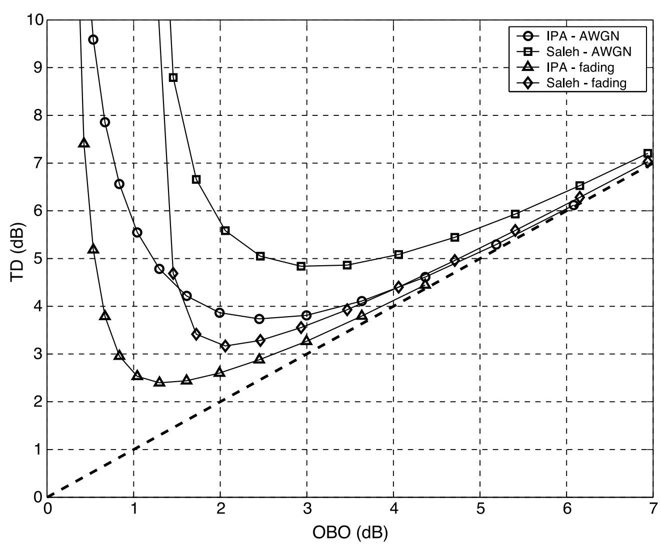

Fig. 4. TD performance (40 users, equal users' amplitudes).

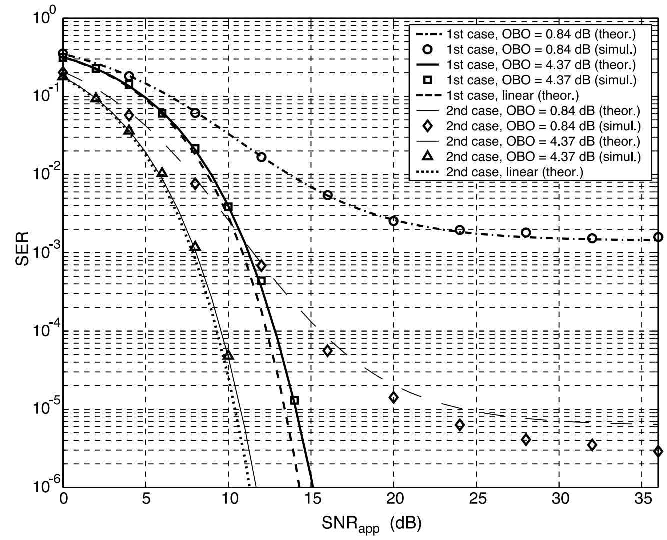

Fig. 5. SER performance in AWGN channels (IPA, 40 users, unequal users' amplitudes).

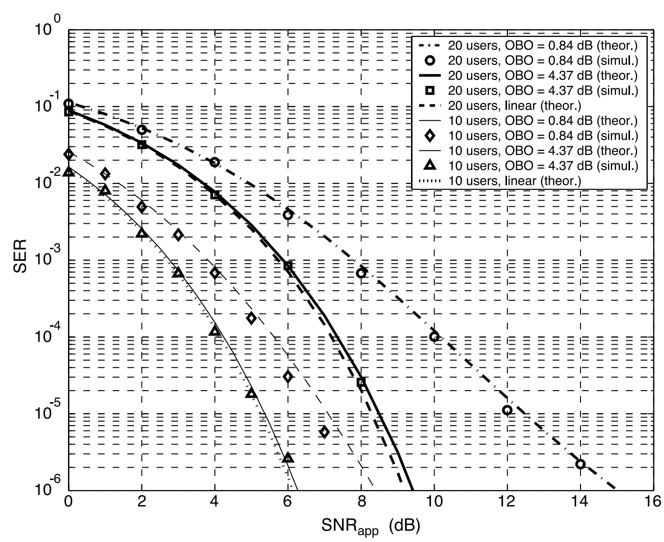

Fig. 6. SER performance in AWGN channels (IPA, 10 or 20 users, equal users' amplitudes).

with the simulations is introduced at this target SER. It is clear that for any OBO value the TD in flat Rayleigh fading scenarios is smaller than in AWGN channels. This can be intuitively explained by the following consideration. In a flat Rayleigh fading scenario, the nonlinear distortion noise is affected by the fading channel, and therefore its power is small (high) when the useful signal power also is small (high). Since most of the errors are committed when the useful signal power is small, the nonlinear distortion noise is partially masked by the thermal noise, whose power is independent of the fading gain.

It is interesting to check the differences between analytical and simulated performance when the operating conditions are not exactly those assumed in the theoretical model. Specifically, we check the model accuracy when the powers $\{A_k^2\}$ of the users' signals are not exactly the same, and when the number K of active users is not high enough to assume a Gaussian pdf for the HPA input. Particularly, we focus on two extreme scenarios that are for high and very low OBO values. In Fig. 5, we consider the situation with AWGN and IPA, even though analogous considerations hold true for other channels or amplifiers. In the *first case*, we assume that the power of the $K = 40$ users' signals varies in a linear manner, in such a way that the last user has double power than the first user, which is the user of interest. In this case the simulated performance exactly matches the analytical model when the OBO is high or very low. In a *second case* we assume that 20 out of 40 users, including that of interest, have double power than the others. In this case, Fig. 5 shows a little mismatch in the saturating point of the SER curve at low OBO, as in Fig. 4. Moreover, Fig. 6 shows that the analysis is still accurate when the number of active (equal power) users is reduced to $K = 20$. However, when the base station attends a smaller number of users, e.g., $K = 10$ users, the HPA input is no more Gaussian. Therefore, when $\mathrm{SER} < 10^{-4}$, the analytical results are slightly pessimistic if compared with the simulated ones.

VII. Conclusion

An analytical framework to evaluate the performance of decorrelating detectors for DS-CDMA systems subject to amplifier nonlinear distortions has been introduced. We have derived closed form SER expressions for AWGN and flat Rayleigh fading channels. Results for QPSK mapping with square-root raised cosine pulse shaping waveforms have been presented, and the OBO that minimizes the system TD has been evaluated. Simulation results have shown that the analytical model is quite appropriate in a large number of scenarios. Such performance analysis can be extended to other linear multiuser detectors (e.g., MMSE detectors) and to multipath channels [13].

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