An Analytical Upper Bound on MMSE Performance Using Approximated MMSE Multiuser Detector in Flat Rayleigh Fading Channels

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ABSTRACT
This paper focuses on the performance of the minimum mean-squared error (MMSE) multiuser detector (MUD) in the downlink of direct-sequence code-division multiple-access (DS-CDMA) systems. An analytical upper bound on the bit-error probability in frequency-nonselective Rayleigh fading channels is derived. This bound is obtained by evaluating the bit-error probability of a linear detector, which is characterised by a reduced complexity with respect to the MMSE. The closeness of this theoretical bound to the ideal MMSE bit-error rate (BER) performance is analysed by simulation in several scenarios, evidencing the performance gain at low signal-to-noise (SNR) with respect to the decorrelator detector. Thus, this receiver can be considered a possible compromise between MMSE and decorrelator detector as far as performance and complexity is concerned.

1. INTRODUCTION

Multiuser Detection (MUD) techniques for DS-CDMA systems give significant performance improvement with respect to the conventional matched-filter receiver. Since the optimum maximum likelihood receiver has a great computational complexity, in the last years much attention has been paid to many suboptimal receivers [1]. Among the linear ones, the minimum mean-squared error (MMSE) detector offers many advantages [2] and it can also be implemented by means of a digital adaptive filter. An important feature of the MMSE receiver is the capability to counteract both the multiple-access interference (MAI) and the thermal noise, thus maximising the signal to interference-plus-noise ratio at the detector output [1].

The bit-error rate (BER) performance of the MMSE detector in additive white Gaussian noise (AWGN) channels has been studied extensively in [3], which provides a good approximation for the bit-error probability. An empirically derived formula is presented in [4] to establish the MMSE detector BER performance in frequency-flat Rayleigh fading channels, while the multipath scenario is usually addressed by using semi-analytical approaches based on computer simulations (see e.g. [5] [6]).

This paper, differently from [4], establishes an upper bound on the MMSE BER performance for flat Rayleigh fading channels in downlink scenarios by using a fully theoretical approach. A reduced complexity receiver, called Approximated MMSE (AMMSE) detector, is introduced by approximating the analytical expression of the MMSE detector. The BER performance of the AMMSE detector will be studied analytically and by simulation, and it will be shown to be a tight upper bound to the MMSE performance when the signal-to-noise ratio (SNR) is not too high.

2. SYSTEM MODEL

The baseband signal transmitted by the base station to K users, in the downlink of a CDMA system, is expressed by

$$z(t)=\sum_{k=1}^{K}x_k(t)=\sum_{k=1}^{K}A_k\sum_{i=-\infty}^{+\infty}b_k[i]s_k(t-iT), \quad (1)$$

where T is the symbol duration, A_k and $s_k(t)$ are the amplitude and the spreading waveform, respectively, and $b_k[i]$ is the ith symbol. The spreading waveform can be expressed as

$$s_k(t)=\frac{1}{\sqrt{N}}\sum_{j=0}^{N-1}c_k[j]p(t-jT_c), \quad (2)$$

where N is the processing gain, $T_c=T/N$ is the chip duration, $p(t)$ is the impulse response of the chip pulse shaping filter and $c_k[j]$ is the jth value of the kth user binary spreading code. If BPSK modulation is used, then $b_k[i]$ belongs to a set of independent and equally probable $\{\pm 1\}$ random variables.

This paper assumes that the signal $z(t)$, transmitted by the base station, passes through a frequency-flat channel with impulse response

$$g(\tau,t)=\beta(t)\exp(j\theta(t))\cdot\delta(\tau), \quad (3)$$

where $\beta(t)$ and $\theta(t)$ are the gain and the phase shift of the channel respectively, and $\delta(\tau)$ is the Dirac delta function. The amplitude $\beta(t)$ and the phase shift $\theta(t)$ are supposed to be slowly time-varying, and consequently they can be considered constant during one symbol interval. For any given time t, the gain $\beta(t)$ is modeled as a Rayleigh random variable, of unit power (i.e. $E\{\beta^2\}=1$), with probability density function (PDF)

$$f(\beta)=2\beta\cdot\exp(-\beta^2)\cdot u_{-1}(\beta), \quad (4)$$

where $u_{-1}(\beta)$ is the unitary step function.

At the receiver side, the channel-affected signal is perturbed by a complex zero-mean additive white Gaussian noise (AWGN) $n(t)$, as expressed by (5),

$$r(t)=\int_{-\infty}^{+\infty}g(\tau,t)z(t-\tau)dt+n(t)=$$
$$=\beta(t)\exp(j\theta(t))\cdot z(t)+n(t). \quad (5)$$

The received signal $r(t)$ is firstly filtered by a chip matched filter and successively sampled at the chip rate $1/T_c$, thus obtaining (6), where $r_{n,\text{SIGNAL}}[l]$ is the useful part related to $z(t)$ and $r_{n,\text{AWGN}}[l]$ is the in-band component of the thermal noise

$$r_n[l] = \int_{-\infty}^{+\infty} r(t)p^*(t - lT - nT_c)dt = \\ = r_{n,\text{SIGNAL}}[l] + r_{n,\text{AWGN}}[l]. \quad (6)$$

By using the following definitions

$$\mathbf{r}[l] = [\, r_0[l] \cdots r_{N-1}[l]\,]^T, \quad (7)$$

$$\mathbf{C} = \frac{1}{\sqrt{N}} \begin{bmatrix} c_1[0] & \cdots & c_K[0] \\ \vdots & \cdots & \vdots \\ c_1[N-1] & \cdots & c_K[N-1] \end{bmatrix},$$

$$\mathbf{A} = \text{diag}(A_1, \cdots, A_K), \quad \mathbf{b}[l] = [\, b_1[l] \cdots b_K[l]\,]^T, \quad (8)$$

$$\beta[l] = \beta(lT), \quad \psi[l] = \exp(j\theta(lT)), \quad (9)$$

we obtain, in matrix form,

$$\mathbf{r}[l] = \mathbf{r}_{\text{SIGNAL}}[l] + \mathbf{r}_{\text{AWGN}}[l] = \\ = \beta[l]\psi[l] \cdot \mathbf{CAb}[l] + \mathbf{r}_{\text{AWGN}}[l], \quad (10)$$

where $\mathbf{r}_{\text{SIGNAL}}[l]$ and $\mathbf{r}_{\text{AWGN}}[l]$ are the vectors obtained collecting the $r_{n,\text{SIGNAL}}[l]$ and $r_{n,\text{AWGN}}[l]$ values, respectively, as in (7), and the entries of the vector $\mathbf{r}_{\text{AWGN}}[l]$ are zero mean i.i.d. complex Gaussian random variables with power $2\sigma^2$, as expressed by

$$E\{\mathbf{r}_{\text{AWGN}}[l]\mathbf{r}_{\text{AWGN}}[l]^H\} = 2\sigma^2 \mathbf{I}_N.$$

3. MMSE RECEIVER IN FLAT FADING CHANNELS

Since the transmitted data are BPSK mapped, the receiver decision rule can be expressed as

$$\hat{b}_k[l] = \text{sgn}\{\text{Re}[\mathbf{w}_k[l]^H \mathbf{r}[l]]\}, \quad (11)$$

where the $\mathbf{w}_k[l]$ column vector represents the detector for the user k at the discrete time l. The MMSE detector is obtained by minimising the mean-squared error $E\{|\, b_k[l] - \mathbf{w}_k[l]^H \mathbf{r}[l]\,|^2\}$, which leads to [7]

$$\mathbf{w}_k[l] = \psi[l](\mathbf{CH}[l]^2\mathbf{C}^T + \sigma^2\mathbf{I}_N)^{-1}\mathbf{C}\left(\mathbf{H}[l]\right)_{:,k} = \\ = \psi[l]\mathbf{CM}\left(\mathbf{H}[l]^{-1}\right)_{:,k}, \quad (12)$$

$$\mathbf{M} = (\mathbf{R} + \sigma^2\mathbf{H}[l]^{-2})^{-1} = (\mathbf{R} + \sigma^2\beta[l]^{-2}\mathbf{A}^{-2})^{-1}, \quad (13)$$

where $\mathbf{R} = \mathbf{C}^T\mathbf{C}$ is the matrix containing the cross-correlation coefficients of the users spreading codes, $\mathbf{H}[l] = \beta[l]\mathbf{A}$ is the channel-affected amplitude diagonal matrix, the subscript :,k indicates the extraction of the kth column of the relative matrix, and $\psi[l]$, defined in (9), represents the channel phase-shift.

Assuming perfect channel state information, and defining the real vector $\mathbf{m}_k[l]$ as

$$\mathbf{m}_k[l] = \psi^*[l]\mathbf{w}_k[l] = \mathbf{CM}\left(\mathbf{H}[l]^{-1}\right)_{:,k}, \quad (14)$$

from (10) and (14) it follows that the decision variable in (11) can be evaluated by exploiting the following expression

$$\text{Re}[\mathbf{w}_k[l]^H\mathbf{r}[l]\,] = \mathbf{m}_k[l]^T(\mathbf{CH}[l]\mathbf{b}[l] + \tilde{\mathbf{r}}_{\text{AWGN}}[l]), \quad (15)$$

where $\tilde{\mathbf{r}}_{\text{AWGN}}[l] = \text{Re}[\psi^*[l]\mathbf{r}_{\text{AWGN}}[l]]$ is a real vector with autocorrelation matrix

$$E\{\tilde{\mathbf{r}}_{\text{AWGN}}[l]\tilde{\mathbf{r}}_{\text{AWGN}}[l]^T\} = \sigma^2\mathbf{I}_N.$$

3.1 Probability of Error of the MMSE Detector

The decision variable expressed by (15) is similar to the one in AWGN channels [1] [8], with the only exception that the amplitude matrix $\mathbf{H}[l]$ and the receiver vector $\mathbf{m}_k[l]$ are time dependent. Therefore, in order to obtain the MMSE bit-error probability, we can use the same approach of [3] [1] by exploiting the Gaussian approximation of the residual multiple-access interference (MAI) at the detector output. Indeed, it has been shown in [3] that this approximation is very good in a wide range of situations. As a consequence, supposing that user 1 is the user of interest, and replacing the amplitudes A_k with the β-dependent one βA_k, the MMSE bit-error probability conditioned to the knowledge of β is [3]

$$P_{e1}(\beta) = Q\left(\frac{1}{\sqrt{\dfrac{\sigma^2(\mathbf{MRM})_{1,1}}{\left[\beta A_1(\mathbf{MR})_{1,1}\right]^2} + \sum_{k=2}^{K}\left[\dfrac{A_k(\mathbf{MR})_{1,k}}{A_1(\mathbf{MR})_{1,1}}\right]^2}}\right), \quad (16)$$

where $Q(x) = (2\pi)^{-1/2}\int_x^{+\infty}\exp(-v^2/2)dv$ and the subscript 1,k indicates the extraction of the element 1,k of the corresponding matrix. The average BER can be obtained integrating (16) over the PDF of the channel gain β, which for Rayleigh fading is expressed by (4). The average BER is consequently expressed by

$$P_{e1,\text{FAD}} = \int_{-\infty}^{+\infty} f(\beta)P_{e1}(\beta)d\beta = \\ = \int_0^{+\infty} 2\beta\exp(-\beta^2)Q\left(\sqrt{\frac{\beta^2A_1^2(\mathbf{MR})_{1,1}^2}{\sigma^2(\mathbf{MRM})_{1,1} + \beta^2\sum_{k=2}^{K}A_k^2(\mathbf{MR})_{1,k}^2}}\right)d\beta. \quad (17)$$

However, an analytical evaluation of the integral (17) is difficult, mainly because the integration variable β is contained inside a matrix that has to be inverted (see (13)). To the knowledge of the authors, the analytical solution of this integral is not known.

3.2 The Approximated MMSE (AMMSE) Detector

In this paragraph we introduce a reduced complexity MMSE-like detector. An approximation of the MMSE detector in fading channels can be obtained if the detector is made independent of the instantaneous channel gain, with the same fashion of the precombining LMMSE detector of [9] in multipath fading channels. Accordingly, we can replace the matrix $\mathbf{M}$ in (13), that depends on the instantaneous amplitudes βA_k, with a matrix $\mathbf{X}$ that depends only on the average amplitudes A_k (because $E\{\beta^2\} = 1$). As a consequence of this modification, the MMSE detector (12) becomes the AMMSE detector $\mathbf{a}_k[l]$, as expressed by (18)

$$\mathbf{a}_k[l] = \psi[l]\mathbf{CX}\left(\mathbf{A}^{-1}\right)_{:,k}, \quad (18)$$

$$\mathbf{X} = (\mathbf{R} + \sigma^2\mathbf{A}^{-2})^{-1}. \quad (19)$$

It is interesting to notice that when the fading gain is very high, i.e. $\beta^2A_k^2 \gg \sigma^2$, the MMSE detector is similar to the decorrelator detector, since (13) reduces to $\mathbf{M} \approx \mathbf{R}^{-1}$. Indeed, in this situation, the thermal noise power is very small and consequently only the MAI affects the BER performance. Hence the MMSE detector has to counteract more MAI than noise. On the other hand, when the fading gain is small, i.e. $\beta^2 A_k^2 << \sigma^2$, the MMSE detector is similar to the conventional detector (matched filter), because (13) reduces to $\mathbf{M} \approx \sigma^{-2}\beta^2\mathbf{A}^2$, and consequently $\mathbf{m}_k[l]$ in (14) tends to $(\mathbf{C})_{:,k}$ up to a scalar factor. Indeed, when the thermal noise power is high, the MMSE detector suppresses more noise than MAI. On the contrary, since the matrix $\mathbf{X}$ does not depend on the instantaneous channel gain β, the AMMSE detector (18) tries to suppress the same amount of MAI and noise for all the β values, disregarding that the MAI only is significant when $\beta^2 A_k^2 >> \sigma^2$ and, on the contrary, the noise only is meaningful when $\beta^2 A_k^2 << \sigma^2$. Therefore, the bit-error rate of the AMMSE detector will be higher than the one of the ideal MMSE receiver. As a consequence, the bit-error probability of the AMMSE receiver is an upper bound on the bit-error probability of the MMSE detector.

The performance-complexity trade-off is evident by comparing the MMSE detector and the AMMSE one. Indeed the AMMSE detector in (18) does not require any inverse matrix update, being matrix $\mathbf{X}$ in (19) constant in time. On the other hand, the MMSE detector in (12) has a higher computational complexity, because the matrix $\mathbf{M}$ in (13) must be updated when β changes.

4. UPPER BOUND EVALUATION

In the following, we derive the upper bound for the MMSE bit-error probability in Rayleigh fading channel. As stated in section 3, this bound is the probability of error of the AMMSE detector.

4.1 Probability of Error of the AMMSE Detector

In order to obtain the AMMSE bit-error probability, we use the Gaussian approximation of the residual MAI, which holds true for all the linear detectors [3]. As a consequence, the AMMSE bit-error probability conditioned to the knowledge of β has the same expression of (16), where the new matrix $\mathbf{X}$ replaces the β-dependent matrix $\mathbf{M}$ as expressed by

$$P_{e1}(\beta) = Q\left(\frac{1}{\sqrt{\dfrac{\sigma^2(\mathbf{XRX})_{1,1}}{\left[\beta A_1(\mathbf{XR})_{1,1}\right]^2} + \sum_{k=2}^{K}\left[\dfrac{A_k(\mathbf{XR})_{1,k}}{A_1(\mathbf{XR})_{1,1}}\right]^2}}\right). \quad (20)$$

Consequently, the average BER is

$$\begin{aligned} P_{e1,\mathrm{FAD}} &= \int_{-\infty}^{+\infty} f(\beta) P_{e1}(\beta) d\beta = \\ &= \int_0^{+\infty} 2\beta\exp(-\beta^2) Q\left(\sqrt{\frac{\beta^2 A_1^2(\mathbf{XR})_{1,1}^2}{\sigma^2(\mathbf{XRX})_{1,1} + \beta^2\sum_{k=2}^{K} A_k^2(\mathbf{XR})_{1,k}^2}}\right) d\beta = \\ &= \int_0^{+\infty} 2\beta\exp(-\beta^2) Q\left(\sqrt{\beta^2\mu^2/(1+\beta^2\lambda^2)}\right) d\beta, \end{aligned} \quad (21)$$

where μ^2 and λ^2, defined in (22), are the average signal-to-noise ratio and the average residual interference-to-noise ratio for the decision variable $b_1(l)$, respectively.

$$\mu^2 = \frac{A_1^2(\mathbf{XR})_{1,1}^2}{\sigma^2(\mathbf{XRX})_{1,1}}, \quad \lambda^2 = \frac{\sum_{k=2}^{K} A_k^2(\mathbf{XR})_{1,k}^2}{\sigma^2(\mathbf{XRX})_{1,1}}. \quad (22)$$

By inserting in (21) both $Q(x) = 0.5\cdot\mathrm{erfc}(x/\sqrt{2})$ and $y = \beta\mu/[(2+2\beta^2\lambda^2)^{1/2}]$, we obtain

$$P_{e1,\mathrm{FAD}} = \\ = \int_0^{\mu/(\sqrt{2}\lambda)} \exp\left(\frac{-2y^2}{\mu^2 - 2\lambda^2 y^2}\right) \mathrm{erfc}(y) \, \frac{2y\mu^2}{(\mu^2 - 2\lambda^2 y^2)^2} \, dy. \quad (23)$$

By integrating (23) by parts it follows that

$$P_{e1,\mathrm{FAD}} = \\ = \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^{\mu/(\sqrt{2}\lambda)} \exp\left(\frac{-2y^2}{\mu^2 - 2\lambda^2 y^2}\right) \exp\left(-y^2\right) dy, \quad (24)$$

and substituting $t = \mu^2 - 2\lambda^2 y^2$ in (24) it is easy to obtain

$$P_{e1,\mathrm{FAD}} = \frac{1}{2} - \frac{\sqrt{2}}{4\sqrt{\pi}\lambda} \exp\left(\frac{2-\mu^2}{2\lambda^2}\right) \cdot \\ \cdot \int_0^{\mu^2} \exp\left(-\frac{\mu^2}{\lambda^2}\frac{1}{t}\right) \exp\left(\frac{t}{2\lambda^2}\right) \left(\mu^2 - t\right)^{-1/2} dt. \quad (25)$$

By using the Taylor series expansion of the function $\exp(t/(2\lambda^2))$, (25) becomes

$$P_{e1,\mathrm{FAD}} = \frac{1}{2} - \frac{\sqrt{2}}{4\sqrt{\pi}\lambda} \exp\left(\frac{2-\mu^2}{2\lambda^2}\right) \sum_{k=0}^{+\infty} \frac{1}{k!}\left(\frac{1}{2\lambda^2}\right)^k \cdot \\ \cdot \int_0^{\mu^2} \exp\left(-\frac{\mu^2}{\lambda^2}\frac{1}{t}\right) t^k (\mu^2 - t)^{-1/2} dt. \quad (26)$$

The integral in (26) is in the same form of the integral 3.471.2 in [10], hence we obtain

$$P_{e1,\mathrm{FAD}} = \\ = \frac{1}{2} - \frac{\sqrt{2}}{4}\frac{\mu}{\lambda} \exp\left(\frac{1-\mu^2}{2\lambda^2}\right) \sum_{k=0}^{+\infty} \frac{1}{k!}\left(\frac{\mu^2}{2\lambda^3}\right)^k W_{-\frac{k+1}{2}, \frac{k+1}{2}}\left(\frac{1}{\lambda^2}\right), \quad (27)$$

where $W_{-(k+1)/2,\,(k+1)/2}(x)$ is the confluent hyper-geometric Whittaker W function [11] of order $-(k+1)/2$, $(k+1)/2$.

The solution (27) to the integral (21) can also be expressed in function of the confluent hypergeometric functions U and ${}_2F_0$ by means of the relations 13.1.33 and 13.1.10 in [11]

$$W_{-\frac{k+1}{2}, \frac{k+1}{2}}(x) = \exp(-x/2) x^{k/2+1} U\left(k + \frac{3}{2}, k+2, x\right), \quad (28)$$

$$U(a,b,z) = z^{-a} {}_2F_0(a, 1+a-b;;-1/z). \quad (29)$$

which lead to the expressions

$$P_{e1,\mathrm{FAD}} = \\ = \frac{1}{2} - \frac{\sqrt{2}}{4}\frac{\mu}{\lambda^3} \exp\left(\frac{-\mu^2}{2\lambda^2}\right) \sum_{k=0}^{+\infty} \frac{1}{k!}\left(\frac{\mu^2}{2\lambda^4}\right)^k U\left(k+\frac{3}{2}, k+2, \frac{1}{\lambda^2}\right), \quad (30)$$

$$= \frac{1}{2} - \frac{\sqrt{2}}{4}\mu \exp\left(\frac{-\mu^2}{2\lambda^2}\right) \sum_{k=0}^{+\infty} \frac{1}{k!}\left(\frac{\mu^2}{2\lambda^2}\right)^k {}_2F_0\left(k+\frac{3}{2}, \frac{1}{2};; -\lambda^2\right). \quad (31)$$

We have to point out that the solution (27), (30) or (31) of the integral (21) could be useful also for the performance evaluation of other communication systems (in Rayleigh fading channels) impaired by some transmitter generated noise, provided that this interference can be modeled as Gaussian at the decision variable. Indeed, when the interference is generated at the transmitter, the interference-to-noise ratio λ^2 is attenuated by the same β^2 factor of the signal-to-noise ratio μ^2. An example of this situation could be the nonlinear distortion noise generated by a High Power Amplifier (HPA) in Orthogonal Frequency Division Multiplexing (OFDM) systems that use a BPSK or a QPSK data mapping. As shown in [12], the nonlinear noise can be modeled, for practical input power back-off values, as a Gaussian noise, and consequently the equation (27) can be used in order to evaluate the OFDM performance in frequency-selective Rayleigh fading channels. Indeed, the OFDM technique converts a frequency-selective fading channel in a number of parallel frequency-flat fading channels, each one associated to a single sub-carrier.

5. SIMULATION RESULTS

In this section, some computer simulation results are presented and discussed. Different situations are considered, in order to understand a) how much the upper bound is close to the BER of the MMSE detector and b) how much this upper bound is apart from the decorrelator BER curve.

Fig. 1 shows the BER performance of the user of interest as function of the number of active users *K*. It is supposed that the base station transmits each user signal with the same power. Gold codes with processing gain $N = 31$ have been used. The signal-to-noise ratio (SNR), defined as

$$\mathrm{SNR} = A_1^2 \big/ (2\sigma_{\mathrm{AWGN}}^2)\,, \qquad (32)$$

is equal to 20 dB. The BER in fig. 1 shows that no appreciable differences exist among the performance of the three detectors under investigation (MMSE, AMMSE, and decorrelator), except when the loading factor K/N is close to 1. This behavior can be explained by the small cross-correlation of the Gold codes, which renders the users quasi-orthogonal. Indeed, when the users' codes are orthogonal, the matrix $\mathbf{R}$ is diagonal, and then all the three detectors are scaled versions of the same detector (conventional single-user detector).

Fig. 2 exhibits the BER performance as function of the SNR in a situation where all the users are active ($K = N = 31$). Random codes have been used in this case. At low SNR, the MMSE upper bound is close to the simulated MMSE detector BER, and the AMMSE detector significantly outperforms the decorrelator. However, at high SNR, the upper bound tends to the decorrelator detector BER, with a large power penalty with respect to the MMSE performance. Indeed, at high SNR, the matrix $\mathbf{X}$ in (19) is almost equal to the matrix $\mathbf{R}^{-1}$ that appears in the decorrelating detector. As a consequence, the AMMSE detector in (18) is nearly equal to the decorrelating one and, of course, the performance must be nearly the same. On the other hand, the MMSE detector outperforms the other ones because it takes into account the effect of the β channel gain, suppressing mainly the noise when β is low (high channel attenuation) and mainly the MAI when β is high (i.e. when the channel amplifies the users' signals), as observed in section 3.

In fig. 3 and fig. 4, random codes with $K = N = 31$ are used again, but the sets of codes are different from the previous one. Moreover, in fig. 4, the power A_k^2 of each interfering user is supposed to be doubled with respect to the power A_1^2 of the user of interest . Figs. 3-4 clarify that the upper bound is close to the MMSE performance for low SNR, and it tends to the decorrelator performance at high SNR, as shown in the previous situation (fig. 2).

Fig. 5 displays the BER performance as function of the multiple-access interference (MAI), defined as

$$\mathrm{MAI} = A_k^2 \big/ A_1^2\,, \qquad (33)$$

where it is supposed that the interfering users' signals have the same amplitude A_k. Since the SNR is not very high (SNR = 25 dB), the upper bound is close to the MMSE performance. The difference from the decorrelator BER tends to increase when the MAI decreases, because the decorrelating detector does not take into account the increased power of the user of interest.

6. SUMMARY AND CONCLUSIONS

A theoretical upper bound on the MMSE probability of error in flat Rayleigh fading downlink channels has been derived as the BER of a reduced complexity receiver (AMMSE receiver). It has been shown by simulation that this upper bound is very close to the ideal MMSE error probability at low SNR, while it approaches the decorrelator performance at higher SNR. The analytical expression of this upper bound could be useful in order to evaluate the BER performance of other system (such as OFDM systems) impaired by transmitter induced distortions that appear as Gaussian at the decision variable.

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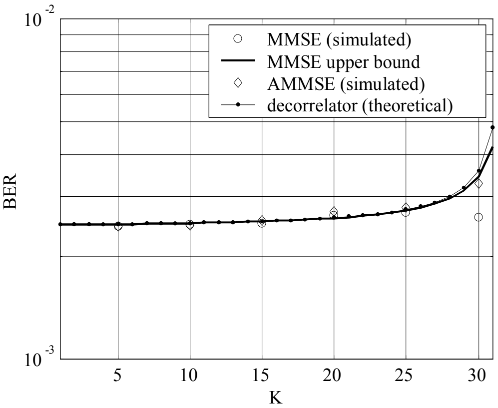

Figure 1: BER vs. number of active users. Gold codes, $N = 31$, SNR = 20 dB.

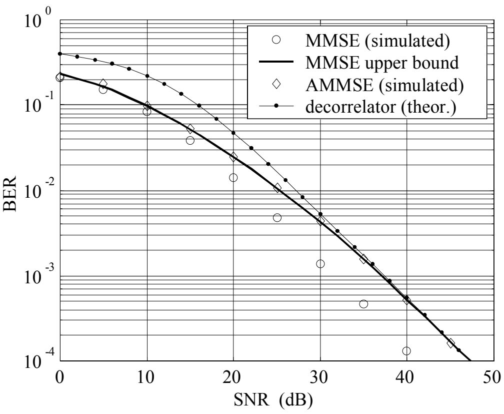

Figure 2: BER vs. SNR. Random codes, $K = N = 31$.

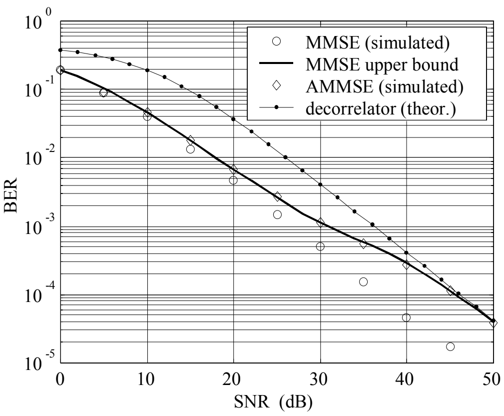

Figure 3: BER vs. SNR. Random codes, $K = N = 31$ (different codes set with respect to Fig.2).

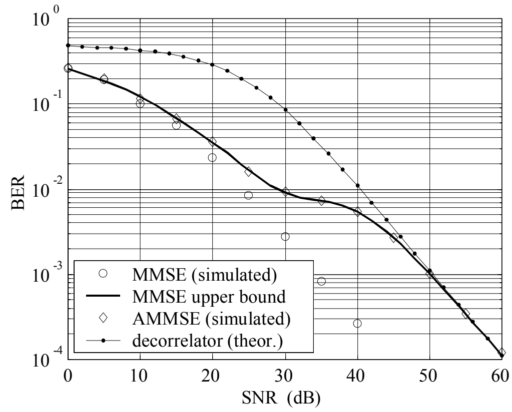

Figure 4: BER vs. SNR. Random codes, MAI = 3 dB, $K = N = 31$ (different codes set with respect to Figs 2-3).

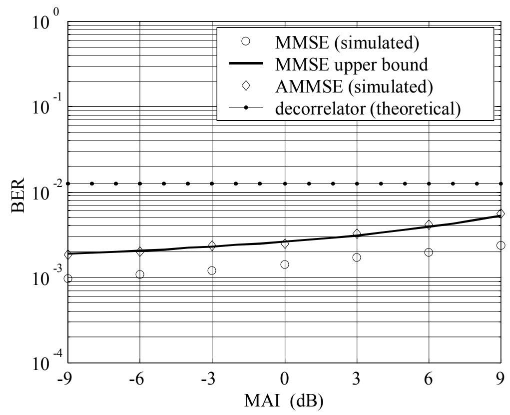

Figure 5: BER vs. MAI. Random codes, $K = N = 31$, SNR = 25 dB.