AN H_∞ FILTERING APPROACH FOR ROBUST TRACKING OF OFDM DOUBLY-SELECTIVE CHANNELS

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ABSTRACT

In OFDM systems, fast time-varying channels can be effectively estimated and tracked by coupling a channel basis expansion model with a Kalman filtering approach. Although the Kalman filter is asymptotically MSE-optimal when the channel statistics are known, it may suffer some performance degradation in the presence of statistical mismatches. We propose to estimate OFDM doubly-selective channels by an H_∞ filtering approach that, although suboptimal in the MSE sense, is more robust than classical Kalman filtering to statistical mismatches. Simulation results and performance comparisons assess the effectiveness of the proposed solution.

1. INTRODUCTION

In mobile communication systems with high data-rate and fast mobility, the wireless fading channel is doubly-selective, i.e., both frequency-selective and time-selective. Orthogonal frequency-division multiplexing (OFDM) is a well known technique that is able to counteract frequency-selective fading channels [1] by means of parallel data transmission in the frequency domain. However, it is well known that OFDM systems are more sensitive than single-carrier systems to fast linear time-varying (LTV) channels, because the intercarrier interference (ICI) caused by Doppler effects destroys the orthogonality among the OFDM subcarriers [2] [3]. In order to counteract the ICI, many different methods have been proposed, based on linear equalization, decision-feedback equalization, and ICI cancellation [4] [5] [6] [7] [8]. All these techniques assume the knowledge of the channel variation over the whole OFDM block. Therefore, in the context of ICI mitigation, doubly-selective channel estimation plays an important role.

In order to estimate a doubly-selective channel, it is computationally convenient to assume a reduced-rank model of the time variation of the channel. This model, usually referred to as basis expansion model (BEM), allows for a parsimonious representation of the channel, thereby enabling low-complexity estimation of few parameters by means of least squares and linear minimum mean-squared error (LMMSE) techniques [9] [10]. Adaptive estimation and tracking of the BEM parameters by LMMSE prediction and Kalman filtering have been investigated in [11], [12], [13], and [14]. The Kalman channel estimator minimizes the error covariance and is therefore MSE-optimal among the unbiased estimators, under the assumption that the channel and noise covariances are known. However, in practical mobile applications, statistical information may be partial, or outdated, or absent. In some cases, the doubly-selective channel could even be non-stationary, due to frequent changes of speed or motion direction of a mobile terminal. In these scenarios, the optimality of the Kalman filter is no longer valid, and alternative approaches may be preferable.

In this paper, we investigate a minimax approach for robust estimation and tracking of the BEM parameters of OFDM doubly-selective channels. The key idea is to minimize the maximum possible estimation error in the presence of model uncertainties, such as the imperfect knowledge of the mobile speed, and of the Doppler power spectral density of the channel. This criterion leads to an H_∞ filter [15], which can be interpreted as a Kalman estimator in a Krein space [16]. Previous studies about H_∞ filtering for OFDM have been presented in [17], which deals with carrier frequency offset estimation, and in [18], which considers slowly time-varying channels. Specifically, [18] assumes a block-fading model, i.e., a BEM order equal to one. Differently, we deal with rapidly time-varying channels with BEM order larger than one. By taking into account the possible presence of unknown model uncertainties, such as the error covariance of the BEM coefficients, we design an H_∞ channel estimator that turns out to be more robust than the corresponding Kalman filter. Simulation results in different mismatched scenarios show the effectiveness of the proposed approach. Since the proposed H_∞ filter shares a similar structure with the Kalman filter, the computational complexity increase is negligible.

The remainder of this paper is structured as follows. Section 2 contains the OFDM system model. In Section 3, we present the H_∞ filtering approach for doubly-selective channel estimation. Section 4 compares the H_∞ estimator with the Kalman estimator, by means of simulation results. In Section 5, we conclude the paper.

2. SYSTEM MODEL

We consider a typical OFDM system, where a data block $\mathbf{s} = [s[0], \cdots, s[N-1]]^T$ is transmitted on N orthogonal subcarriers by the unitary IDFT matrix $\mathbf{F}^H$. A cyclic prefix (CP) of length L equal to the maximum delay spread of the discrete-time channel is appended to the IDFT output. The signal undergoes a doubly-selective fading channel $h[n,l]$ (n is the time index and l is the multipath index), which models the fast time-varying multipath propagation. After CP removal, the received vector is reshaped by a time-domain window $\mathbf{w} = [w_0, \cdots, w_{N-1}]^T$, which helps to reduce the Doppler effects [6]. Thus, the received vector $\mathbf{z}_w = [z_w[0], \cdots, z_w[N-1]]^T$ is expressed by

$$\mathbf{z}_w = \mathbf{D}_w \mathbf{H}_T \mathbf{F}^H \mathbf{s} + \mathbf{D}_w \mathbf{n}_T, \qquad (1)$$

where $\mathbf{n}_T = [n[0], \cdots, n[N-1]]^T$ is an additive white Gaussian noise (AWGN), $\mathbf{D}_w = \mathit{diag}(\mathbf{w})$ is a diagonal windowing matrix, and $\mathbf{H}_w = \mathbf{D}_w \mathbf{H}_T$ is the $N \times N$ time-domain channel matrix that stores on the lth lower diagonal the time evolution of the lth windowed channel tap $h_w[n,l] = w[n]h[n,l]$, as expressed by $[\mathbf{H}_w]_{n,m} = h_w[n, (n-m)_{\mathrm{mod}N}]$. In order to simplify the channel matrix identification, it is common to represent the LTV channel by a BEM, which approximates the time variation of each channel tap $\mathbf{h}_{T,l} = [h_w[0,l], \cdots, h_w[N-1,l]]^T$ as a linear combination of a fixed set of basis functions, as expressed by [19]

$$\mathbf{h}_{T,l} = \mathbf{B}\mathbf{h}_{C,l}, \qquad (2)$$

where $\mathbf{h}_{C,l} = [h_{-Q,l}, \cdots, h_{Q,l}]^T$ contains the $2Q+1$ BEM coefficients, and $\mathbf{B} = [\boldsymbol{\lambda}_{-Q}, \cdots, \boldsymbol{\lambda}_Q]$ is the $N \times (2Q+1)$ matrix that contains the $2Q+1$ BEM basis functions $\{\boldsymbol{\lambda}_q\}$. Thus, to identify the LTV channel, we only need to estimate $U = (L+1)(2Q+1)$ BEM coefficients $\{h_{q,l}\}$ for each OFDM block. In practical mobile scenarios, it is possible to accurately approximate the real channel by few basis functions e.g., $2Q+1 \in \{3,5,7\}$, using heuristic rules such as $Q \geq \lceil \nu_D \rceil$, with $\nu_D = f_D/\Delta f$, where f_D is the maximum Doppler spread and Δf is the OFDM subcarrier separation.

There are several possibilities for the choice of the BEM functions, such as polynomials, complex exponentials, and discrete prolate spheroidal functions [20]. Among them, we focus on the generalized complex exponential (GCE) BEM [21], summarized by

$$\mathbf{B} = \mathbf{D}_w \mathbf{B}_{GCE} \mathbf{T}, \tag{3}$$

where $\mathbf{B}_{GCE}$ is defined in [21], and $\mathbf{T}$ is a square matrix that makes the columns of $\mathbf{B}$ orthonormal. For instance, $\mathbf{T}$ can be chosen to be upper triangular using $\mathbf{T} = \tilde{\mathbf{R}}^{-1}$, where $\tilde{\mathbf{Q}}\tilde{\mathbf{R}} = \mathbf{D}_w \mathbf{B}_{GCE}$ represents the economy-size QR decomposition. In this case, $\mathbf{B} = \tilde{\mathbf{Q}}$. Plugging (2) into (1), after some standard matrix manipulations, the received vector is expressed by

$$\mathbf{z}_w = \sum_{q=-Q}^{Q} \boldsymbol{\Lambda}_q \mathbf{H}_q \mathbf{F}^H \mathbf{s} + \mathbf{D}_w \mathbf{n}_T, \tag{4}$$

where $\mathbf{H}_q$ is a circulant matrix containing the N dimensional vector $[h_{q,0}, \cdots, h_{q,L}, 0, \cdots, 0]^T$ and $\boldsymbol{\Lambda}_q = diag(\boldsymbol{\lambda}_q)$.

After FFT processing, the receiver observation equation is

$$\mathbf{y} = \mathbf{F}\mathbf{z}_w = \mathbf{H}_F \mathbf{s} + \mathbf{n}_F = \sum_{q=-Q}^{Q} \mathbf{C}_q \boldsymbol{\Delta}_q \mathbf{s} + \mathbf{n}_F, \tag{5}$$

where $\mathbf{n}_F = \mathbf{F}\mathbf{D}_w \mathbf{n}_T$ is the colored Gaussian noise, $\mathbf{C}_q = \mathbf{F}\boldsymbol{\Lambda}_q \mathbf{F}^H$ is a circulant matrix that represents the frequency-domain effect of qth basis $\boldsymbol{\lambda}_q$, with entries expressed by

$$[\mathbf{C}_q]_{k,m} = \frac{1}{N} \sum_{n=0}^{N-1} \lambda_q[n] \exp(-j2\pi(k-m)n/N), \tag{6}$$

and $\boldsymbol{\Delta}_q = \mathbf{F}\mathbf{H}_q \mathbf{F}^H = diag(\mathbf{F}_L \tilde{\mathbf{h}}_q)$ is a diagonal matrix that quantifies the frequency selectivity of the qth basis, where $\tilde{\mathbf{h}}_q = [h_{q,0}, \cdots, h_{q,L}]^T$, and $\mathbf{F}_L$ is the matrix containing the first $L+1$ columns of the FFT matrix $\sqrt{N}\mathbf{F}$. For linear time-invariant channels, $Q = 0$, $\mathbf{C}_q$ is just a scaled identity matrix, and $\boldsymbol{\Delta}_q$ is a diagonal matrix containing the channel frequency response. On the contrary, for LTV channels, the channel matrix $\mathbf{H}_F$ is not diagonal, and ICI is introduced by the discrete-Doppler spread contained in its super and subdiagonals. Actually, in realistic LTV channels, the discrete-Doppler support is practically limited, which means that the frequency-Doppler channel matrix $\mathbf{H}_F$ is almost banded, and that the out-of-band values can be neglected. Assuming a one-sided Doppler bandwidth B, each subcarrier receives ICI from $2B$ adjacent subcarriers.

In order to estimate the LTV channel, we employ a pilot-symbol-assisted modulation approach [9]. The known training pilots are grouped in P subblocks of length $L_P = 4B+1$, and are interleaved with the information data, as summarized by

$$\mathbf{s} = [\mathbf{d}^{(1)T}, \mathbf{p}^{(1)T}, \cdots, \mathbf{d}^{(P)T}, \mathbf{p}^{(P)T}, \mathbf{d}^{(P+1)T}]^T, \tag{7}$$

where $\mathbf{d}^{(i)}$ is the ith data subblock and $\mathbf{p}^{(i)}$ is the ith pilot subblock. We split (7) into $\mathbf{s} = \mathbf{d} + \mathbf{p}$, where the data vector $\mathbf{d}$ is zero in the pilot positions, and the pilot vector $\mathbf{p}$ is zero in the data positions. In addition, we define the pilot-selection matrix $\mathbf{S}$ as the $P(2B+1) \times N$ matrix with ones (zeros) in those columns corresponding to the the central positions of the pilot (data) vectors, thereby obtaining from (5) the observation vector

$$\tilde{\mathbf{y}} = \mathbf{S}\mathbf{y} = \mathbf{P}\mathbf{h} + \mathbf{S}\mathbf{H}_F \mathbf{d} + \tilde{\mathbf{n}}. \tag{8}$$

In (8), which is obtained by plugging (7) in (5), $\mathbf{P}$ is a known matrix that depends on the BEM basis functions and on the pilot vectors, defined as $\mathbf{P} = \mathbf{S}\boldsymbol{\Omega}(\mathbf{I}_U \otimes \mathbf{p})$, where $\boldsymbol{\Omega} = [\boldsymbol{\Omega}_{-Q,0}, \cdots, \boldsymbol{\Omega}_{-Q,L}, \cdots, \boldsymbol{\Omega}_{Q,0}, \cdots, \boldsymbol{\Omega}_{Q,L}]$, $\boldsymbol{\Omega}_{q,l} = \mathbf{C}_q \mathbf{F}\mathbf{Z}_l \mathbf{F}^H$, with $\mathbf{Z}_l$ defined as the N-size square cyclic-shift matrix with ones on the lth lower diagonal and zeros elsewhere, and $\tilde{\mathbf{n}} = \mathbf{S}\mathbf{n}_F$. In the same equation (8),

$$\mathbf{h} = [\tilde{\mathbf{h}}_{-Q}^T, \cdots, \tilde{\mathbf{h}}_Q^T]^T, \tag{9}$$

is the U-size vector containing the BEM unknowns such that

$$\mathbf{P}\mathbf{h} = \mathbf{S}\boldsymbol{\Omega}(\mathbf{I}_U \otimes \mathbf{p})\mathbf{h} = \mathbf{S}\boldsymbol{\Omega}(\mathbf{h} \otimes \mathbf{I}_N)\mathbf{p} = \mathbf{S}\mathbf{H}_F \mathbf{p}. \tag{10}$$

According to the frequency-domain Kronecker delta (FDKD) pilot design in [9] and [22], if each training block $\mathbf{p}^{(i)}$ contains a single nonzero pilot symbol in the middle position, surrounded by $4B$ trailing zeros, then the data-induced ICI $\mathbf{i} = \mathbf{S}\mathbf{H}_F \mathbf{d}$ on the pilot positions has a very low power. Therefore, in (8), the data-induced ICI can be neglected or incorporated into the noise term $\tilde{\mathbf{n}}$. In the following, we estimate the (windowed) LTV channel vector

$$\mathbf{h}^{(w)} = [h_w[0,0], \cdots, h_w[0,L], \cdots, h_w[N-1,0], \cdots, h_w[N-1,L]]^T. \tag{11}$$

3. H_∞ FILTERING

In order to estimate the LTV channel, we derive an H_∞ filter that models the evolution of the BEM coefficients in (9) as an autoregressive (AR) model of the first order, as expressed by

$$\mathbf{h}_k = \mathbf{A}_k \mathbf{h}_{k-1} + \mathbf{v}_k, \tag{12}$$

where k is the time index of the OFDM block, $\mathbf{A}_k$ takes into account the correlation between the BEM coefficients of consecutive blocks, and $\mathbf{v}_k$ represents the innovation of the BEM coefficients introduced in the kth block, with $E(\mathbf{v}_k) = \mathbf{0}_{U\times 1}$, $E(\mathbf{v}_k \mathbf{v}_{k-m}^H) = \mathbf{Q}_k \delta[m]$, where $\delta[m]$ is the Kronecker delta, and $E(\mathbf{h}_k \mathbf{v}_{k-m}^H) = \mathbf{0}_{U\times 1}$.

By neglecting the data-induced ICI in (8), the observation equation can be expressed by

$$\tilde{\mathbf{y}}_k = \mathbf{P}\mathbf{h}_k + \tilde{\mathbf{n}}_k, \tag{13}$$

where $\tilde{\mathbf{n}}_k$ is a colored Gaussian noise characterized by $E(\tilde{\mathbf{n}}_k) = \mathbf{0}_{P(2B+1)\times 1}$, $E(\tilde{\mathbf{n}}_k \tilde{\mathbf{n}}_{k-m}^H) = \mathbf{R}_{\tilde{n}} \delta[m]$, $E(\mathbf{v}_k \tilde{\mathbf{n}}_{k-m}^H) = \mathbf{0}_{U\times P(2B+1)}$, and $E(\mathbf{h}_k \tilde{\mathbf{n}}_{k-m}^H) = \mathbf{0}_{U\times P(2B+1)}$.

The vector to be estimated $\mathbf{h}_k^{(w)}$, which contains the $L+1$ taps of the LTV windowed channel in the kth block, is expressed by

$$\mathbf{h}_k^{(w)} = \mathbf{L}\mathbf{h}_k, \tag{14}$$

where $\mathbf{L} = \mathbf{B} \otimes \mathbf{I}_{L+1}$.

The H_∞ filter is designed starting from the cost function

$$J_1 = \frac{\sum_{k=1}^{M} ||\mathbf{h}_k^{(w)} - \hat{\mathbf{h}}_k^{(w)}||^2_{\boldsymbol{\Sigma}_k^{-1}}}{||\mathbf{h}_0 - \hat{\mathbf{h}}_0^+||^2_{\mathbf{M}_0^+} + \sum_{k=1}^{M} ||\mathbf{v}_k||^2_{\mathbf{Q}_k} + \sum_{k=1}^{M} ||\tilde{\mathbf{n}}_k||^2_{\mathbf{R}_{\tilde{n}}}}, \tag{15}$$

where $\hat{\mathbf{z}}_k$ is the H_∞ estimate of the LTV channel, $\mathbf{\Sigma}_k$ is an $(L+1)N$-size square matrix that weights the output error, $\hat{\mathbf{h}}_0^+$ is the initial BEM vector estimate, $\mathbf{M}_0^+$ is the U-size square matrix that represents the reliability of the initial estimate, M is the number of OFDM blocks, and $||\mathbf{x}||_{\mathbf{G}} = ||\mathbf{G}^{-1}\mathbf{x}||_2$ is the weighted two-norm of $\mathbf{x}$. The minimization of J_1 in (15) is intractable [15], so the H_∞ filter is designed to satisfy an user-defined bound, as expressed by

$$J_1 < \frac{1}{\theta}. \quad (16)$$

By expressing J_1 in (15) as $J_1 = \tilde{N}/\tilde{D}$, where $\tilde{N}$ is the numerator and $\tilde{D}$ is the denominator, (16) is equivalent to

$$J = \tilde{N} - \frac{\tilde{D}}{\theta} < 0, \quad (17)$$

whose minimax solution is expressed by

$$\hat{\mathbf{h}}_k^{(w)} = \arg\min_{\mathbf{h}_k^{(w)}}(\max_{\mathbf{h}_0,\mathbf{v}_k,\tilde{\mathbf{n}}_k} J). \quad (18)$$

This way, the solution $\hat{\mathbf{h}}_k^{(w)}$ is robust to the worst possible error caused by a bad initial state $\mathbf{h}_0$, a statistical mismatch in the innovation $\mathbf{v}_k$, and an erroneous assumption on the covariance of $\tilde{\mathbf{n}}_k$. Actually, the H_∞ filter does not attain the minimization of J, but it guarantees that the bound (17) is satisfied. The H_∞ solution can be obtained using the following equations:

$$\mathbf{M}_k^- = \mathbf{A}_k \mathbf{M}_{k-1}^+ \mathbf{A}_k^H + \mathbf{Q}_k, \quad (19)$$

which is the covariance of the state prediction error, where

$$\mathbf{M}_k^+ = \mathbf{M}_k^-(\mathbf{I}_U + (\mathbf{\Gamma} - \theta\tilde{\mathbf{\Sigma}}_k)\mathbf{M}_k^-)^{-1} \quad (20)$$

is the covariance of the state estimation error, with $\mathbf{\Gamma} = \mathbf{P}^H \mathbf{R}_{\tilde{n}}^{-1}\mathbf{P}$ and $\tilde{\mathbf{\Sigma}}_k = \mathbf{L}^H \mathbf{\Sigma}_k \mathbf{L}$,

$$\mathbf{K}_k^{(\infty)} = \mathbf{M}_k^+ \mathbf{P}^H \mathbf{R}_{\tilde{n}}^{-1} \quad (21)$$

is the H_∞ gain that, together with the predicted state

$$\hat{\mathbf{h}}_k^- = \mathbf{A}_k \hat{\mathbf{h}}_{k-1}^+, \quad (22)$$

lets to compute the estimated state $\hat{\mathbf{h}}_k^+$ by

$$\hat{\mathbf{h}}_k^+ = \hat{\mathbf{h}}_k^- + \mathbf{K}_k^{(\infty)}(\tilde{\mathbf{y}}_k - \mathbf{P}\hat{\mathbf{h}}_k^-), \quad (23)$$

which finally leads to the estimate $\hat{\mathbf{h}}_k^{(w)}$ of the windowed LTV channel in the kth OFDM block, as expressed by

$$\hat{\mathbf{h}}_k^{(w)} = \mathbf{L}\hat{\mathbf{h}}_k^+. \quad (24)$$

The mathematical derivation of (19)-(24), which is herein omitted, can be obtained by Lagrange multipliers method, similarly to [15].

From (16), it is clear that high values of θ reduce the upper bound of the cost function. However, if θ is too large, the covariance of the state estimation error $\mathbf{M}_k^+$ in (20) may lose its nonnegative definiteness. Therefore, θ should be chosen such that $\mathbf{M}_k^+$ remains positive definite for any k. This is guaranteed by choosing [18]

$$\theta < \frac{1}{e_{\max,k}}, \quad (25)$$

where $e_{\max,k}$ is the maximum eigenvalue of $\tilde{\mathbf{\Sigma}}_k[(\mathbf{M}_k^-)^{-1} + \mathbf{\Gamma}]^{-1}$.

It is worth noting that the H_∞ filter has the same structure, and hence the same complexity, of the well-known Kalman filter. Specifically, the choice $\theta = 0$ makes the H_∞ filter equivalent to the Kalman filter [15]. Indeed, when $\theta = 0$, the covariance update in (20) becomes identical to that of the Kalman filter, and therefore the H_∞ gain in (21) is identical to the Kalman gain. However, when θ tends to zero, the bound in (16) becomes looser, and hence the robustness to statistical mismatches decreases. Consequently, choosing $\theta = 0$ guarantees the optimal MSE performance only when the model evolution is perfectly known, i.e., when the estimator has a perfect knowledge of $\mathbf{A}_k$ and $\mathbf{Q}_k$. On the other hand, choosing a nonzero value for θ introduces some robustness when $\mathbf{A}_k$ and $\mathbf{Q}_k$ are unknown or badly estimated, sacrificing the MSE performance when $\mathbf{A}_k$ and $\mathbf{Q}_k$ are perfectly known. Clearly, since $\mathbf{A}_k$ and $\mathbf{Q}_k$ model the evolution of the BEM coefficients, both depend on the channel statistics, as expressed by

$$\mathbf{A}_k = \mathbf{R}_{h_{cross},k}\mathbf{R}_{h,k}^{-1}, \quad (26)$$

$$\mathbf{Q}_k = \mathbf{R}_{h,k} - \mathbf{R}_{h_{cross},k}\mathbf{A}_k^H, \quad (27)$$

$$\mathbf{R}_{h,k} = E(\mathbf{h}_k\mathbf{h}_k^H) = \mathbf{L}^H(\mathbf{R}_{k,\text{Doppler}}^{(t)} \otimes \mathbf{R}_{k,\text{Multipath}})\mathbf{L}, \quad (28)$$

$$\mathbf{R}_{h_{cross},k} = E(\mathbf{h}_k\mathbf{h}_{k-1}^H), \quad (29)$$

where $\mathbf{R}_{k,\text{Doppler}}^{(t)}$ is the N-size square Toeplitz matrix whose columns contains the (windowed) time autocorrelation function of a channel tap in the kth OFDM block, while $\mathbf{R}_{k,\text{Multipath}}$ is the $(L+1)$-size square matrix whose diagonal contains the power-delay profile of the channel in the kth OFDM block. Therefore, both $\mathbf{A}_k$ and $\mathbf{Q}_k$ strongly depend on the Doppler power spectral density, and consequently on the maximum Doppler spread f_D of the channel.

In case of imperfect knowledge of the channel statistics $\mathbf{R}_{h_{cross},k}$, $\mathbf{R}_{h,k}$, $\mathbf{R}_{k,\text{Multipath}}$, $\mathbf{R}_{k,\text{Doppler}}^{(t)}$, or in non-stationary environments where the statistics are time-varying, the matrices $\hat{\mathbf{A}}_k$ and $\hat{\mathbf{Q}}_k$ used by the channel estimator are different from the actual ones. Therefore, we have to distinguish whether the statistics are perfectly known at the initialization step ($\hat{\mathbf{A}}_1 = \mathbf{A}_1$ and $\hat{\mathbf{Q}}_1 = \mathbf{Q}_1$) or not ($\hat{\mathbf{A}}_1 \neq \mathbf{A}_1$ and $\hat{\mathbf{Q}}_1 \neq \mathbf{Q}_1$). In addition, during the tracking phase, we have to distinguish whether the estimator uses the same matrices employed for initialization ($\hat{\mathbf{A}}_k = \hat{\mathbf{A}}_1$ and $\hat{\mathbf{Q}}_k = \hat{\mathbf{Q}}_1$, for $k > 1$) or the estimator tracks the matrices after estimation, using the following update equations [13]:

$$\hat{\mathbf{R}}_{h_{cross},k} = \lambda_k\hat{\mathbf{R}}_{h_{cross},k-1} + (1-\lambda_k)\hat{\mathbf{h}}_k^+\hat{\mathbf{h}}_{k-1}^{+H}, \quad (30)$$

$$\hat{\mathbf{\Phi}}_k = \lambda_k^{-1}\hat{\mathbf{\Phi}}_{k-1} - \lambda_k^{-2}\frac{\hat{\mathbf{\Phi}}_{k-1}\hat{\mathbf{h}}_k^+\hat{\mathbf{h}}_k^{+H}\hat{\mathbf{\Phi}}_{k-1}}{(1-\lambda_k)^{-1} + \lambda_k^{-1}\hat{\mathbf{h}}_k^{+H}\hat{\mathbf{\Phi}}_{k-1}\hat{\mathbf{h}}_k^+}, \quad (31)$$

$$\hat{\mathbf{A}}_{k+1} = \hat{\mathbf{R}}_{h_{cross},k}\hat{\mathbf{\Phi}}_k, \quad (32)$$

$$\hat{\mathbf{v}}_k = \hat{\mathbf{h}}_k^+ - \hat{\mathbf{A}}_k\hat{\mathbf{h}}_{k-1}^+, \quad (33)$$

$$\hat{\mathbf{Q}}_{k+1} = \lambda_k\hat{\mathbf{Q}}_k + (1-\lambda_k)\hat{\mathbf{v}}_k\hat{\mathbf{v}}_k^H, \quad (34)$$

where λ_k is a (possibly time-varying) forgetting factor. Indeed, different choices for initialization and tracking of $\hat{\mathbf{A}}_k$ and $\hat{\mathbf{Q}}_k$ lead to different performances.

As a remark, we point out that, differently from the Kalman filter, the performance of the H_∞ filter also depend on the matrix $\mathbf{L}$ that links the LTV channel estimate $\hat{\mathbf{h}}_k^{(w)}$ to the estimated BEM coefficients $\hat{\mathbf{h}}_k^+$ (see (24)).

4. SIMULATION RESULTS

We consider an OFDM system with $N_A = 244$ active carriers, $N_V = N - N_A = 12$ zero subcarriers that act as guard frequency bands, CP length $L = 4$, QPSK data constellations, and FDKD pilots [9] with $P = 5$ subblocks of length $4B + 1 = 9$. The receiver window $\mathbf{w}$ has been designed according to the minimum band-approximation error [8] as the sum of 5 complex exponentials. The channel estimator employs a GCE-BEM with $2Q + 1 = 5$ basis functions. The LTV channel, which has been generated as in [23], has a constant power-delay profile of length $L + 1 = 5$, as expressed by $\mathbf{R}_{k,\text{Multipath}} = \mathbf{I}_{L+1}$, and a Jakes' power spectral density of the Doppler dispersion, as expressed by $[\mathbf{R}^{(t)}_{k,\text{Doppler}}]_{m,n} = J_0(2\pi(m-n)\nu_D/N)[\mathbf{w}]_m[\mathbf{w}]_n$. The signal-to-noise ratio, defined as $\text{SNR} = (L+1)E(||\mathbf{s}||^2)/E(||\mathbf{n}_F||^2)$, is equal to 20 dB. The estimator performance is assessed by means of the normalized MSE (NMSE) of the windowed channel, defined as

$$\text{NMSE} = \frac{E(||\mathbf{h}^{(w)}_k - \hat{\mathbf{h}}^{(w)}_k||^2)}{E(||\mathbf{h}^{(w)}_k||^2)}. \tag{35}$$

Each set of simulations considers the H_∞ channel tracking during the transmission of $M = 200$ consecutive OFDM blocks, stochastically iterated for 40 channel realizations. We compare the H_∞ filter performance versus its Kalman filter counterpart (e.g., $\theta = 0$). Both filters use a fixed forgetting factor $\lambda_k = 0.995$ to update the statistical knowledge of the AR model according to (30)-(34). The initial state is assumed as $\hat{\mathbf{h}}^+_{k-1} = \mathbf{0}_{U\times 1}$. The weight matrix used in the numerator of (15) is $\mathbf{\Sigma}_k = \mathbf{D}_w\mathbf{D}^H_w \otimes \mathbf{I}_{L+1}$. The noise covariance matrix $\mathbf{R}_{\tilde{n}}$ and the power-delay profile matrix $\mathbf{R}_{k,\text{Multipath}}$ are assumed to be perfectly known.

In Fig. 1, the LTV channels is simulated using a maximum Doppler spread $\nu_D = 0.5$. However, the H_∞ and Kalman filters operate in a mismatched mode, assuming a maximum Doppler spread $\hat{\nu}_D = \nu_D/2$ at the first OFDM block, and estimating the channel statistics using (30)-(34) for the successive OFDM blocks. The H_∞ filter uses $\theta = 0.05$. Fig. 1 shows that the H_∞ filter is more robust than the Kalman filter to the Doppler mismatch, since it exhibits both a lower NMSE degradation and a faster update of its channel statistical knowledge.

Fig. 2 illustrates the effect of the H_∞ filter parameter θ, in the same simulation scenario of Fig. 1. The average NMSE is obtained by averaging the last 100 OFDM blocks (i.e., from $k = 101$ to $k = 200$). It is evident that there exists a single minimum, which is $\theta = 0.07$ for this case. However, there is a significant range where the H_∞ filter outperforms the Kalman filter (characterized by $\theta = 0$). It should be noted that, for some channel realizations, using $\theta = 0.2$ does not satisfy the bound (25).

In the second set of simulations, after 50 OFDM blocks, the maximum Doppler spread is suddenly changed from $\nu_D = 0.5$ to $\nu_D = 0.75$. The H_∞ filter uses $\theta = 0.04$. Both the H_∞ and Kalman filters operate in matched mode at the first OFDM block, assuming a maximum Doppler spread $\hat{\nu}_D = 0.5$, and then operate in tracking mode for the successive blocks. The NMSE performance in Fig. 3 confirms the behavior of Fig. 1. Anyway, the faster channel variation grants a faster statistics update by (30)-(34). Consequently, after convergence, the Kalman filter reaches a lower NMSE, since it is the LMMSE estimator by design. Noteworthy, the H_∞ filter, which is designed to constrain the maximum error and not the MSE, in the presence of significant mismatch it is capable to outperform the Kalman filter also in terms of MSE. For the same simulations set of Fig. 3, Fig. 4 plots the maximum NMSE and the average NMSE

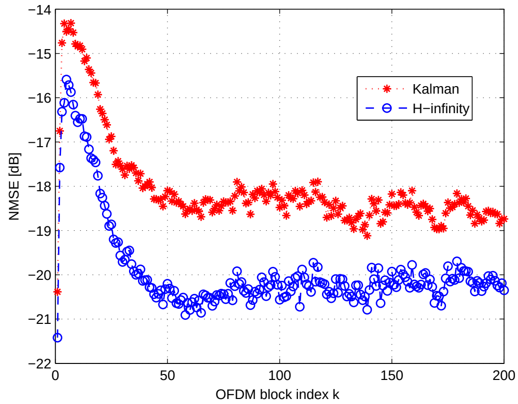

Fig. 1. Mismatched maximum Doppler spread.

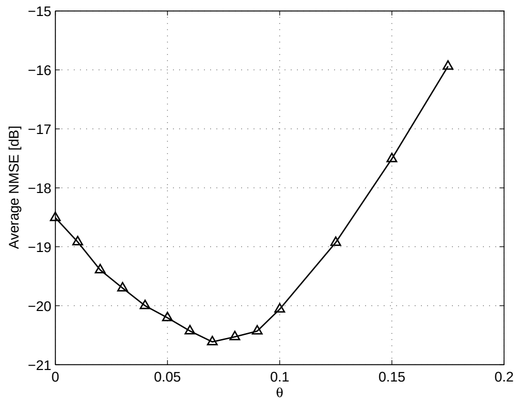

Fig. 2. Effect of θ on the NMSE.

for the 40 channel realizations. Fig. 4 clearly shows that the H_∞ channel estimator trades off average MSE in matched scenarios for a significant robustness in maximum MSE (and possibly also average MSE) in mismatched scenarios. This is a desirable characteristic because, while a certain average MSE penalty in channel-matched scenarios may not significantly influence the OFDM bit-error rate after channel equalization, a significant reduction of the worst-case MSE may let the OFDM system to not lose the channel tracking and to not completely disrupt the service in critical mismatched scenarios.

5. CONCLUSIONS

We have proposed a robust estimator of OFDM doubly-selective channels based on H_∞ filtering. The proposed estimator minimizes an upper bound on the maximum amount of channel estimation error that can be committed. Therefore, the proposed approach is suitable in the presence of mismatches about the statistical knowledge of the channel. Simulation comparisons have shown that, in case of imperfect statistical knowledge, our H_∞ estimator produces a superior tracking performance with respect to a Kalman estimator, with comparable computational complexity.

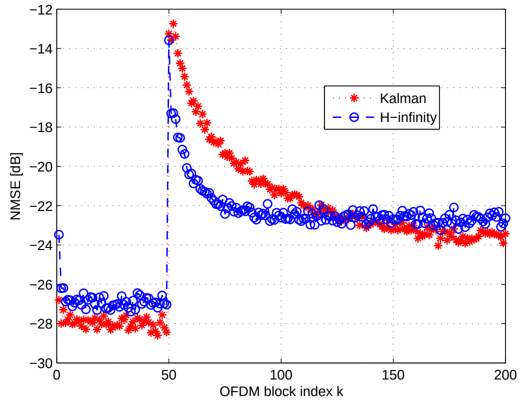

Fig. 3. Tracking a jump of the maximum Doppler spread.

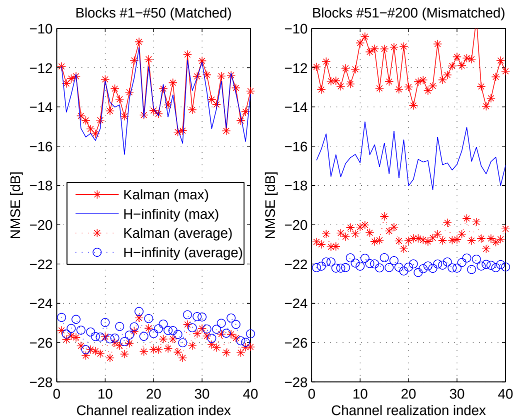

Fig. 4. Comparison of the worst MSE with the average MSE.

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