THREE-STAGE CENTRALIZED SPECTRUM SENSING OF OFDM SIGNALS

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ABSTRACT

We propose a three-stage centralized spectrum sensing for cognitive radios in OFDM systems, where local decisions of cooperative sensors are combined at the fusion center. The fusion center, if necessary, can invoke a third stage for finer sensing. The first two stages employ distributed energy detectors and a centralized voting rule, while the third stage exploits the OFDM signal correlation. The overall probability of detection is optimized by jointly designing the optimal thresholds at the three stages, for a given probability of false alarm. The detection performance and the average sensing time are analyzed theoretically and compared by simulations with spectrum sensing that employs only the two-stage centralized detector or the correlation-based detector.

Index Terms**—**Autocorrelation, cognitive radio, cooperative detection, energy detector, OFDM, spectrum sensing, voting rule

1. INTRODUCTION

Cognitive radio is a promising paradigm to overcome the underutilization of the frequency spectrum [1]. In a cognitive radio network, unlicensed (secondary) users are allowed to transmit using the spectrum portion that is temporarily unused by licensed (primary) users. In this view, a critical issue is spectrum sensing [2], which permits to recognize whether a physical resource is occupied or available for a possible secondary-user transmission.

Spectrum sensing can be done in many ways. Each secondary user may perform independent sensing, or operate in a cooperative fashion [2]. Herein we consider cooperative centralized sensing, where the final sensing decision is taken at a fusion center (FC) after collection of the local decisions of the secondary users: among other benefits, cooperative sensing offers an improved reliability with respect to independent sensing. The possible presence of a primary signal may be inferred by using an energy detector (ED) [3], or by looking for some specific structure into the received signal [2]. For instance, when the primary signals are based on orthogonal frequency-division multiplexing (OFDM), as it happens for many standards (such as, IEEE 802.11a/g WLAN, IEEE 802.16 WiMAX, IEEE 802.20 MBWA, LTE-Advanced, etc.), the sensing device may look for some repeated patterns introduced by the OFDM cyclic prefix (CP), using either an autocorrelation (AC) detector [4], or a cyclostationary (CS) detector [5], or other detectors that exploit the CP presence [6]. Additionally, (multistage) sensing schemes may employ two or more serially concatenated detectors, in order to increase the probability of detection for a given probability of false alarm.

In this paper, we propose a three-stage cooperative spectrum sensing scheme for OFDM signals, whose philosophy is related to the two-stage local sensing scheme proposed in [7]. With respect to [7], our sensing scheme presents the following differences: first, we focus on a cooperative multiple-sensor scenario, while [7] deals with a single sensor; second, we apply the double-step philosophy at the FC rather than at the local sensor as proposed in [7]; third, in the final stage, we use an AC-based detector, while [7] resorts to a CS detector. Noteworthy, our additional stage represented by the combination of local decisions at the FC, grants to our approach an additional degree of freedom to optimize the detection performance. In summary, the main contributions of our paper are:

- The joint optimization of all the three stages to maximize the probability of detection for a given probability of false alarm;
- An approximated analytical characterization of the AC-stage performance, which is also instrumental to the overall optimization.

We remark that the three-stage philosophy may also be applied with other detectors. For instance, another ED could replace the AC detector at the FC. In any case, multistage sensing schemes seem appropriate especially when the first stage must be a time-limited or a complexity-constrained detection scheme.

2. THREE-STAGE CENTRALIZED SENSING

We consider a three-stage centralized spectrum sensing scheme with K secondary users and an FC. Each secondary user is equipped with a simple ED. In the first stage, each ED performs a coarse sensing and then sends its binary decision to the FC. We assume a decision $D_i = 1$ when the i th ED senses a primary signal as present, and $D_i = 0$ when a primary signal is supposed absent, for $i = 1,...,K$. In the second stage, the FC collects the K preliminary decisions, and possibly takes its own decision according to the following rule:

$$D = \sum_{i=1}^{K} D_i \underset{3^{\text{rd}}\text{ stage}}{\overset{H_1}{\gtrless}} t , \quad (1)$$

where t is an integer threshold, and H_1 is the "channel occupied" (signal present) hypothesis. Note that if $D < t$, rather than deciding for the hypotheses H_0 (signal absent), the FC considers the ED coarse decisions as unreliable, and enables a third stage. In the third stage, the FC, which is characterized by additional sensing capabilities, performs a fine spectrum sensing using a more powerful detector. We assume that the FC employs an AC-based detector, but the same approach can be extended to other detectors with minor modifications. The whole system is summarized in Fig. 1.

In order to design the spectrum sensing system, we have to specify how to select the integer threshold t , as well as the thresholds of the EDs of the secondary users and the threshold of the AC detector at the FC. To this purpose, we use the Neyman-Pearson approach, aiming at maximizing the probability of detection P_d subject to a constant false alarm rate (CFAR) $\bar{P}_{fa}$. By conditional probability rules, the probability of detection P_d after the three sensing stages can be expressed as

$$P_d = P_d^{(\mathrm{CE})} + P_d^{(\mathrm{AC})}(1 - P_d^{(\mathrm{CE})}), \quad (2)$$

where $P_d^{(\mathrm{CE})}$ is the probability of detection after the second stage, i.e., after the centralized collection of the ED results, and $P_d^{(\mathrm{AC})}$ is the probability of detection of the AC detector in the third stage. Analogously, the probability of false alarm P_{fa} after the three sensing stages can be expressed as

$$P_{fa} = P_{fa}^{(\mathrm{CE})} + P_{fa}^{(\mathrm{AC})}(1 - P_{fa}^{(\mathrm{CE})}) = \bar{P}_{fa}, \quad (3)$$

where $P_{fa}^{(\mathrm{CE})}$ is the probability of false alarm after the second stage, and $P_{fa}^{(\mathrm{AC})}$ is the probability of false alarm of the AC detector, and $\bar{P}_{fa}$ is the desired CFAR. In the following, we express $P_d^{(\mathrm{CE})}$ and $P_{fa}^{(\mathrm{CE})}$ in (2) after the second stage as functions of the parameters of the ED in the first stage. Successively, we will consider the third stage, and we will express $P_d^{(\mathrm{AC})}$ and $P_{fa}^{(\mathrm{AC})}$ in (3) as functions of the parameters of the AC detector.

3. FIRST STAGE: ENERGY DETECTORS

We assume that the primary users employ OFDM, with N subcarriers and CP of length L. We define the OFDM block length as $M = N + L$. These parameters are known to both the secondary users and the FC. In the first stage, each secondary user tries to distinguish between the hypotheses H_0 and H_1:

$$\begin{aligned} H_0:\ & \mathbf{x}_i = \mathbf{w}_i, \\ H_1:\ & \mathbf{x}_i = \mathbf{s}_i + \mathbf{w}_i, \end{aligned} \quad (4)$$

where $\mathbf{x}_i$ is the vector received by the i th ED, $i = 1,...,K$, of size $N^{(\mathrm{ED})}$ equal to the number of observations of the ED, $\mathbf{w}_i$ is an additive white Gaussian noise (AWGN) vector, with zero mean and covariance $\mathbf{C}_{\mathbf{w}_i} = \sigma_{\mathbf{w}_i}^2 \mathbf{I}_{N^{(\mathrm{ED})}}$, and $\mathbf{s}_i$ is the signal vector, of size $N^{(\mathrm{ED})}$, received by the i th ED. The number of observations $N^{(\mathrm{ED})}$ is assumed to be $N^{(\mathrm{ED})} = N_{block}^{(\mathrm{ED})} M$, where $N_{block}^{(\mathrm{ED})}$ is a positive integer equal for the K sensors. We assume that the AWGN vectors $\{\mathbf{w}_i,\ i = 1,...,K\}$ are independent, and that the EDs have the same noise power $\sigma_{\mathbf{w}_i}^2 = \sigma_{\mathbf{w}}^2$. We suppose that the number of subcarriers N is sufficiently large to approximate $\mathbf{s}_i$ as a jointly Gaussian vector with zero mean. Since L is usually much shorter than N, i.e., $L \le N/4$, the signal covariance $\mathbf{C}_{\mathbf{s}_i}$ can be well approximated as $\mathbf{C}_{\mathbf{s}_i} \approx \sigma_{\mathbf{s}_i}^2 \mathbf{I}_{N^{(\mathrm{ED})}}$, even if $N_{block}^{(\mathrm{ED})} N \le \mathrm{rank}(\mathbf{C}_{\mathbf{s}_i}) \le N_{block}^{(\mathrm{ED})} N + L$ [6]. In addition, we assume that the primary user is sufficiently far away from both the secondary users and the FC [8]. Under this condition, the signal powers received by the K EDs are roughly equal, i.e., $\sigma_{\mathbf{s}_i}^2 = \sigma_{\mathbf{s}}^2$ [8]. Therefore, the received signal-to-noise ratio (SNR) $\gamma^{(\mathrm{ED})} = \sigma_{\mathbf{s}}^2 / \sigma_{\mathbf{w}}^2$ is the same for all the K secondary users.

The ED of the i th local sensor first estimates the energy Λ_i of the received signal $\mathbf{x}_i$, which is used as the test statistic to take its local decision D_i, according to [3]

$$\Lambda_i = \|\mathbf{x}_i\|^2 = \sum_{j=1}^{N^{(\mathrm{ED})}} |[\mathbf{x}_i]_j|^2 \underset{D_i=0}{\overset{D_i=1}{\gtrless}} \eta^{(\mathrm{ED})}, \quad (5)$$

where $[\mathbf{x}_i]_j$ denotes the j th element of $\mathbf{x}_i$, and the threshold $\eta^{(\mathrm{ED})}$ is a design parameter assumed equal for the all the K EDs. The probability of false alarm of the i th ED is expressed as [3]

$$P_{fa}^{(\mathrm{ED})} = \Pr\{\Lambda_i \ge \eta^{(\mathrm{ED})} \mid H_0\} = 1 - F_{\chi^2_{2N^{(\mathrm{ED})}}}\left(\frac{2\eta^{(\mathrm{ED})}}{\sigma_{\mathbf{w}}^2}\right), \quad (6)$$

where $F_{\chi^2_{2N^{(\mathrm{ED})}}}(\cdot)$ denotes the cumulative distribution function (cdf) of a chi-square random variable with $2N^{(\mathrm{ED})}$ degrees of freedom. The probability of detection of the i th ED can be expressed as [3]

$$P_d^{(\mathrm{ED})} = \Pr\{\Lambda_i \ge \eta^{(\mathrm{ED})} \mid H_1\} \approx 1 - F_{\chi^2_{2N^{(\mathrm{ED})}}}\left(\frac{2\eta^{(\mathrm{ED})}}{\sigma_{\mathbf{s}}^2 + \sigma_{\mathbf{w}}^2}\right). \quad (7)$$

Note that the probabilities in (6) and (7) do not depend on the ED index i, since the threshold $\eta^{(\mathrm{ED})}$, the SNR $\gamma^{(\mathrm{ED})}$, and the number of observations $N^{(\mathrm{ED})}$ are the same for all the K secondary users.

4. SECOND STAGE: VOTING RULE

After the coarse sensing, the EDs send their one-bit decisions $\{D_i\}$ to the FC, which collects the aggregate parameter D expressed by (1). Now, the FC uses a *t*-out-of-*K* voting-rule (VR) detector to decide between two hypotheses: (a) H_1, i.e., the signal of a primary user is present, for $D \ge t$; or (b) undecided yet, for $D < t$. In this last case, a third sensing stage is employed in order to decide between H_0 and H_1. Therefore, the probability of detecting a primary user signal after the second stage is expressed by

$$P_d^{(\mathrm{CE})} = \sum_{i=t}^{K} \binom{K}{i} (P_d^{(\mathrm{ED})})^i (1 - P_d^{(\mathrm{ED})})^{K-i}, \quad (8)$$

where $P_d^{(\mathrm{ED})}$ is expressed by (7). The probability of centralized detection $P_d^{(\mathrm{CE})}$ in (8) is then used for the maximization of the probability of detection in (2). On the other hand, the probability of centralized false alarm is given by

$$P_{fa}^{(\mathrm{CE})} = \sum_{i=t}^{K} \binom{K}{i} (P_{fa}^{(\mathrm{ED})})^i (1 - P_{fa}^{(\mathrm{ED})})^{K-i}. \quad (9)$$

Due to the CFAR constraint (3), Equation (9) links the probability of false alarm $P_{fa}^{(\mathrm{AC})}$ of the third stage with the threshold t of the centralized VR detector and with the probability of false alarm $P_{fa}^{(\mathrm{ED})}$ of the EDs. Equivalently, due to (6), Equation (9) links $P_{fa}^{(\mathrm{AC})}$ with t and $\eta^{(\mathrm{ED})}$: therefore, the maximization of (2) subject to (3) can be performed by an opportune choice of three parameters. However, only two of them can be optimized, since the third one is univocally determined from the other two by the CFAR constraint, as we will detail in Section 6.

5. THIRD STAGE: AUTOCORRELATION

In the third stage, the FC tries to distinguish between the hypotheses H_0 and H_1:

$$\begin{aligned} H_0:\ & \mathbf{x}_0 = \mathbf{w}_0, \\ H_1:\ & \mathbf{x}_0 = \mathbf{s}_0 + \mathbf{w}_0, \end{aligned} \quad (10)$$

where $\mathbf{x}_0$ is the vector received by the FC, of size $N^{(\mathrm{AC})}$ equal to the number of observations, $\mathbf{w}_0$ is an AWGN vector, with zero mean and covariance $\mathbf{C}_{\mathbf{w}_0} = \sigma_{\mathbf{w}_0}^2 \mathbf{I}_{N^{(\mathrm{ED})}}$, and $\mathbf{s}_0$ is the signal vector, of size $N^{(\mathrm{AC})}$, received by the FC. $N^{(\mathrm{AC})}$ is assumed to be $N^{(\mathrm{AC})} = N_{block}^{(\mathrm{AC})} M$, with $N_{block}^{(\mathrm{AC})}$ integer. Since N is supposed large, $\mathbf{s}_0$ can be well approximated by a jointly Gaussian vector with zero mean and covariance $\mathbf{C}_{\mathbf{s}_0} = \sigma_{\mathbf{s}_0}^2 (\mathbf{I}_{N^{(\mathrm{AC})}} + \mathbf{J}_{N^{(\mathrm{AC})}})$, where $\mathbf{J}_{N^{(\mathrm{AC})}}$ is a matrix whose elements $[\mathbf{J}_{N^{(\mathrm{AC})}}]_{l,l+N} = [\mathbf{J}_{N^{(\mathrm{AC})}}]_{l+N,l} = 1$ when the index l belongs to the CP, and zero otherwise. Under the assumption that the primary user is sufficiently far away from both the secondary users and the FC [8], $\sigma_{\mathbf{s}_0}^2 \approx \sigma_{\mathbf{s}}^2$, and the SNR is expressed by $\gamma^{(\mathrm{AC})} = \sigma_{\mathbf{s}}^2 / \sigma_{\mathbf{w}_0}^2$. When the front-end of the detector employed at the FC has the same noise bandwidth of the EDs, then $\sigma_{\mathbf{w}_0}^2 = \sigma_{\mathbf{w}}^2$, and hence $\gamma^{(\mathrm{AC})} \approx \gamma^{(\mathrm{ED})} = \gamma$.

Since the FC can afford more complexity than the local sensors, the FC can employ a detector different from the ED. The optimal Neyman-Pearson detector for OFDM has been analyzed in [6], while AC-based detectors have been proposed in [9] and [4]. In this third stage, we assume an AC-based detector, similar to that in [9], which exploits the periodicity of the OFDM signal $\mathbf{s}_0$ introduced by the CP. The considered AC detector estimates the sliding-window correlation between two L-size received signals, separated by N, as expressed by:

$$R_{cp}[m] = \frac{1}{(N_{block}^{(\mathrm{AC})}-1)L} \sum_{j=0}^{N_{block}^{(\mathrm{AC})}-2} \sum_{l=0}^{L-1} \mathrm{Re}\{[\mathbf{x}_0]_{m+l+jM+N} [\mathbf{x}_0]^*_{m+l+jM}\} , \quad (11)$$

for $m = 0,...,M-1$. Differently from [9], (11) takes the real part rather than the absolute value. The test statistic of our AC-based detector is expressed by

$$\rho = \max_{0\le m\le M-1} \{R_{cp}[m]\} . \quad (12)$$

For the AC-based detector of (11)-(12), we derive the probability of detection $P_d^{(\mathrm{AC})}$ and the probability of false alarm $P_{fa}^{(\mathrm{AC})}$.

To simplify the following explanation, without loss of generality, we assume that $m = 0$ corresponds to an AC detector that is perfectly synchronous with the CP, i.e., that the first sample of $\mathbf{s}_0$ in (10) is the first sample of the CP. If the FC knows the timing information $m = 0$, the test statistic of the AC detector could depend on $R_{cp}[0]$ only. However, we remark that our FC does not know the timing information and uses the test statistic (12).

Finding the cdf $F_\rho(\cdot)$ of ρ is not an easy task, since the random variables $\{R_{cp}[m]\}$ are correlated over the sliding-window support L. If the random variables $\{R_{cp}[m]\}$ were independent, we would have $F_\rho(x) = \prod_{m=0}^{M-1} F_{R_{cp}[m]}(x)$ [10]. To take into account the correlation among the random variables $\{R_{cp}[m]\}$, we assume that the cdf $F_\rho(x)$ can be approximated as the product of $\tilde{M} < M$ cdfs $\{F_{R_{cp}[m]}(x)\}$. Basically, we want to approximate the maximum of M correlated random variables with the maximum of $\tilde{M} < M$ independent random variables. Therefore, we obtain

$$F_\rho(x) = \prod_{m\in\Im} F_{R_{cp}[m]}(x) , \quad (13)$$

where $\Im \subset \{0,...,M-1\}$ is a subset of indices with cardinality $\tilde{M}$, whose choice will be detailed later on. Each random variable $R_{cp}[m]$ is obtained by the sum of $2(N_{block}^{(\mathrm{AC})}-1)L$ products of real-valued zero-mean Gaussian random variables: despite in general each product is not Gaussian distributed, when $2(N_{block}^{(\mathrm{AC})}-1)L$ is large $R_{cp}[m]$ can be approximated as Gaussian. It is easy to show that its mean value $\mu_{R_{cp}[m]} = E\{R_{cp}[m]\}$ can be expressed as

$$\mu_{R_{cp}[m]|H_0} = 0 , \quad (14)$$

$$\mu_{R_{cp}[m]|H_1} = \begin{cases} \frac{L-m}{L}\sigma_\mathbf{s}^2, & 0\le m\le L-1, \\ 0, & L\le m\le N, \\ \frac{m-N}{L}\sigma_\mathbf{s}^2, & N+1\le m\le M-1. \end{cases} \quad (15)$$

After some computations, omitted due to the lack of space, the variance $\sigma^2_{R_{cp}[m]} = E\{(R_{cp}[m]-\mu_{R_{cp}[m]})^2\}$ can be expressed as

$$\sigma^2_{R_{cp}[m]|H_0} = \frac{\sigma_\mathbf{w}^4}{2(N_{block}^{(\mathrm{AC})}-1)L} , \quad (16)$$

$$\sigma^2_{R_{cp}[m]|H_1} = \frac{(\sigma_\mathbf{s}^2+\sigma_\mathbf{w}^2)^2 + \mu_{R_{cp}[m]|H_1}\sigma_\mathbf{s}^2}{2(N_{block}^{(\mathrm{AC})}-1)L} . \quad (17)$$

Under the H_0 hypothesis, the means (14) and the variances (16) are equal. Therefore, using (13), the probability of false alarm for the AC detector can be expressed as

$$P_{fa}^{(\mathrm{AC})} = \Pr\{\rho\ge\eta^{(\mathrm{AC})} | H_0\} \approx 1 - \left[1 - Q\left(\frac{\sqrt{2(N_{block}^{(\mathrm{AC})}-1)L}\,\eta^{(\mathrm{AC})}}{\sigma_\mathbf{w}^2}\right)\right]^{\tilde{M}_0} , (18)$$

where $\eta^{(\mathrm{AC})}$ is the threshold of the AC detector, and $\tilde{M} = \tilde{M}_0$ is the cardinality of $\Im = \Im_0$ under H_0. After extensive simulation results, we have verified that $\tilde{M}_0 = 6M/L$ is a good approximation. Besides, the probability of detection can be expressed as

$$P_d^{(\mathrm{AC})} = \Pr\{\rho\ge\eta^{(\mathrm{AC})} | H_1\} \approx 1 - p(\eta^{(\mathrm{AC})}) , \quad (19)$$

$$p(\eta) = \prod_{m\in\Im_1} \left[1 - Q\left(\frac{\eta - \mu_{R_{cp}[m]|H_1}}{\sqrt{\sigma^2_{R_{cp}[m]|H_1}}}\right)\right] , \quad (20)$$

where $\mu_{R_{cp}[m]|H_1}$ and $\sigma^2_{R_{cp}[m]|H_1}$ are expressed by (15) and (17), and $\Im = \Im_1$ is the set of indices under H_1. Again, we have verified that $\Im_1 = \{iL/2 + L/4\}$, $i = 0,...,2M/L-1$, is a good approximation.

6. THRESHOLD OPTIMIZATION

We express the CFAR constraint (3) as

$$P_{fa}^{(\mathrm{AC})} = \frac{\bar{P}_{fa} - P_{fa}^{(\mathrm{CE})}}{1 - P_{fa}^{(\mathrm{CE})}} , \quad (21)$$

which clearly explains that $P_{fa}^{(\mathrm{AC})} \le \bar{P}_{fa}$ and $P_{fa}^{(\mathrm{CE})} \le \bar{P}_{fa}$ should be both satisfied. Equation (21) can be employed to link the three thresholds $\eta^{(\mathrm{ED})}$, t, and $\eta^{(\mathrm{AC})}$. Indeed, by (6), Equation (9) becomes

$$P_{fa}^{(\mathrm{CE})} = b(\eta^{(\mathrm{ED})}, t) , \quad (22)$$

$$b(\eta,t) = \sum_{i=t}^{K} \binom{K}{i} \left[1 - F_{\chi^2_{2N^{(\mathrm{ED})}}}\left(\frac{2\eta}{\sigma_\mathbf{w}^2}\right)\right]^i \left[F_{\chi^2_{2N^{(\mathrm{ED})}}}\left(\frac{2\eta}{\sigma_\mathbf{w}^2}\right)\right]^{K-i} , \quad (23)$$

and, by using (18), (21), and (22), we obtain

$$\eta^{(\mathrm{AC})} = q(\eta^{(\mathrm{ED})}, t) = \frac{\sigma_\mathbf{w}^2}{\sqrt{2(N_{block}^{(\mathrm{AC})}-1)L}} Q^{-1}\left(1 - \sqrt[\tilde{M}_0]{\frac{1-\bar{P}_{fa}}{1-b(\eta^{(\mathrm{ED})},t)}}\right), \quad (24)$$

where $b(\eta^{(\mathrm{ED})}, t)$ is expressed by (23). From (24), it is clear that the threshold $\eta^{(\mathrm{AC})}$ of the AC detector can be expressed as a function $q(\eta^{(\mathrm{ED})}, t)$ of the other two thresholds. Therefore, only the two thresholds $\eta^{(\mathrm{ED})}$ and t need to be optimized.

We now express the revenue function (2) as a function of the two thresholds $\eta^{(\mathrm{ED})}$ and t. By inserting (7) into (8), we obtain

$$P_d^{(\mathrm{CE})} = b\left(\frac{\eta^{(\mathrm{ED})}}{\gamma+1}, t\right) , \quad (25)$$

and, by (25) and (19), the probability of detection (2) becomes

$$P_d = 1 - p(\eta^{(\mathrm{AC})})\left[1 - b\left(\frac{\eta^{(\mathrm{ED})}}{\gamma+1}, t\right)\right]. \quad (26)$$

Using (24), the probability of detection can be expressed by

$$P_d = g(\eta^{(\mathrm{ED})}, t) = 1 - p\left(q(\eta^{(\mathrm{ED})}, t)\right)\left[1 - b\left(\frac{\eta^{(\mathrm{ED})}}{\gamma+1}, t\right)\right], \quad (27)$$

which is the revenue function we want to maximize for $\eta^{(\mathrm{ED})} \in \mathbb{R}^+$ and $t \in \{1,...,K\}$. Therefore, the constrained maximization of (2) has been transformed into the unconstrained maximization of (27). The steps of the optimization algorithm follow.

Threshold Optimization Algorithm

1. Parameter selection: choose the number of observations $N^{(\mathrm{ED})} = N^{(\mathrm{ED})}_{block} M$ for the ED and $N^{(\mathrm{AC})} = N^{(\mathrm{AC})}_{block} M$ for the AC detector; select the target probability of false alarm $\overline{P}_{fa}$.
2. Initialization: set the initial threshold $t = \overline{t} = 1$ for the VR; set the initial best solution $(P_{d,opt}, \eta^{(\mathrm{ED})}_{opt}, t_{opt}) = (0, +\infty, 0)$.
3. Optimization: find the maximum of $P_d = g(\eta^{(\mathrm{ED})}, \overline{t})$ over $\eta^{(\mathrm{ED})} \in \mathbb{R}^+$, using a numerical algorithm (e.g., trust-region methods [11]); store the possible solution as $(\overline{P}_d, \overline{\eta}^{(\mathrm{ED})}, \overline{t})$.
4. Solution update: if $\overline{P}_d > P_{d,opt}$, update the current best solution as $(P_{d,opt}, \eta^{(\mathrm{ED})}_{opt}, t_{opt}) \leftarrow (\overline{P}_d, \overline{\eta}^{(\mathrm{ED})}, \overline{t})$.
5. Cycle control: If $\overline{t} = K$, then go to Step 6; otherwise, let $\overline{t} \leftarrow \overline{t} + 1$ and go to Step 3.
6. End of algorithm: by (24), calculate $\eta^{(\mathrm{AC})}_{opt} = q(\eta^{(\mathrm{ED})}_{opt}, t_{opt})$, and produce the final solution $(P_{d,opt}, \eta^{(\mathrm{ED})}_{opt}, t_{opt}, \eta^{(\mathrm{AC})}_{opt})$.

7. AVERAGE SENSING TIME

Similarly to the two-stage scheme [7], the average sensing time T of the proposed three-stage scheme can be computed as

$$T = T^{(\mathrm{ED})} + P_3 T^{(\mathrm{AC})}, \tag{28}$$

where $T^{(\mathrm{ED})}$ is the sensing time of ED, $T^{(\mathrm{AC})}$ is the sensing time of the AC detector, and P_3 is the probability of enabling the third stage. Since the K EDs sense in parallel, we have $T^{(\mathrm{ED})} = N^{(\mathrm{ED})} T_s$, where $T_s = 1/W$ is the sampling period and W is the bandwidth of the OFDM signal. Similarly, we have $T^{(\mathrm{AC})} = N^{(\mathrm{AC})} T_s$. The probability of enabling the third stage can be expressed as

$$\begin{aligned} P_3 &= (1 - P^{(\mathrm{CE})}_{fa}) \Pr\{H_0\} + (1 - P^{(\mathrm{CE})}_{d}) \Pr\{H_1\} \\ &= 1 - b(\eta^{(\mathrm{ED})}_{opt}, t_{opt}) \Pr\{H_0\} - b\left(\frac{\eta^{(\mathrm{ED})}_{opt}}{\gamma + 1}, t_{opt} \right) \Pr\{H_1\}, \end{aligned} \tag{29}$$

where $b(\eta, t)$ is expressed by (23), $\Pr\{H_0\}$ is the probability of absence of a primary signal, and $\Pr\{H_1\} = 1 - \Pr\{H_0\}$ is the probability of presence of a primary signal.

8. SIMULATION RESULTS

We compare the proposed three-stage centralized sensing scheme with one-stage-only schemes, such as ED and AC detection, and with a two-stage centralized scheme that uses K ED at the first stage and a t_{opt}-out-of-K VR at the second stage. We assume an OFDM signal with bandwidth $W = 10$ MHz, $N = 128$ subcarriers, and CP length $L = 32$. We select $N^{(\mathrm{ED})}_{block} = 3$ and $N^{(\mathrm{AC})}_{block} = 62$, which lead to $N^{(\mathrm{ED})} \approx 500$ and $N^{(\mathrm{AC})} \approx 10^4$. Unless otherwise stated, we assume $\overline{P}_{fa} = 0.1$, $K = 4$, and $\Pr\{H_0\} = 0.5$.

Fig. 2 illustrates the probability of detection of various detection schemes. The proposed three-stage detector exploits the good performance of the two-stage collaborative ED-VR detector at low SNR and of the AC-based detector at high SNR, and hence outperforms both of them. In particular, the three-stage detector is useful for the SNR range where the AC-based detector outperforms the two-stage ED-VR detector.

Fig. 3 shows the optimum threshold t_{opt} for the same simulation scenario of Fig. 2. It is worth noting that the optimum threshold t_{opt} is different from the majority logic $t = K/2 = 2$ (typically optimal for the two-stage VR alone) both at low SNR, where $t_{opt} = 3$, and at high SNR, where $t_{opt} = 1$ corresponds to the "OR" rule.

Fig. 4 compares the average sensing time of the proposed three-stage scheme with ED and AC detection, for a moderately loaded scenario ($\Pr\{H_0\} = 0.5$). Clearly, the average sensing time of the proposed scheme is greatly reduced with respect to the AC detector, especially when there are many local sensors K and at moderately high SNR, where the third stage is seldom enabled.

Fig. 5 displays the probability of detection of the proposed three-stage detector as a function of the number K of local sensors. Undoubtedly, an increase of K produces an improved performance, as well as a reduced probability P_3 of enabling the third stage, as testified by the average sensing time reduction of Fig. 4.

Finally, the good performance the proposed three-stage detector is confirmed by Fig. 6, which exhibits the receiver operating characteristic (ROC) of different detectors, when the SNR is $\gamma = -15$ dB and $\gamma = -12$ dB.

9. CONCLUSIONS

We have proposed a three-stage centralized spectrum sensing scheme for OFDM cognitive radio systems. The first stage employs K local EDs, while the second stage gathers the local decisions using a t-out-of-K VR. The optional third stage is based on an AC-based detector. The proposed design, based on the Neyman-Pearson approach, allows for a simple selection of the thresholds of the constituent detectors, through a numerical maximization of the overall probability of detection subject to a CFAR constraint. Simulation results have shown that the proposed three-stage detector outperforms conventional detectors, such as the AC detector and the collaborative ED with VR. The proposed detector also presents a reduced average sensing time. The proposed design philosophy can be applied not only to the ED and the AC detector herein considered, but also with other constituent detectors.

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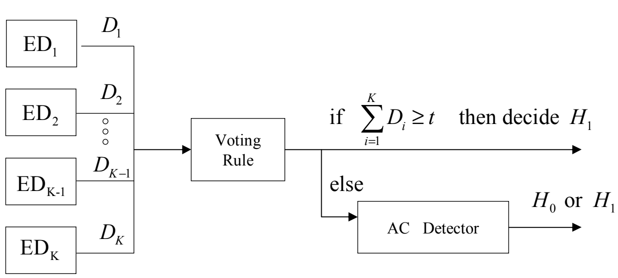

Fig. 1. Three-stage centralized spectrum sensing scheme.

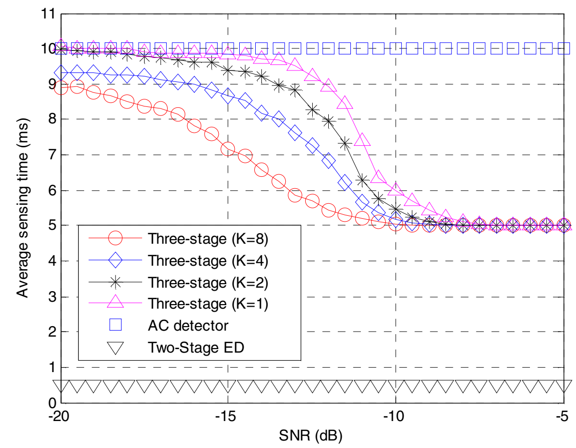

Fig. 4. Average sensing time of different detectors.

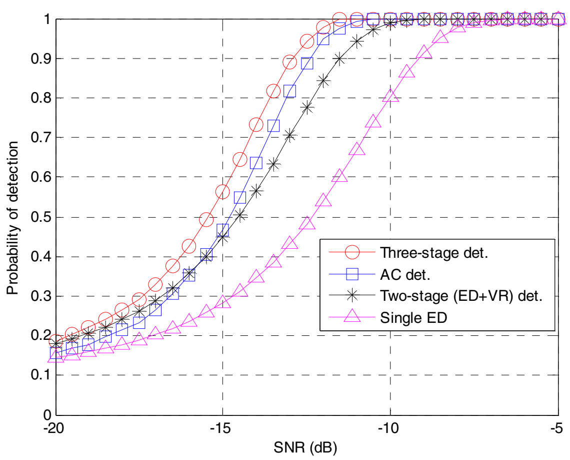

Fig. 2. Performance comparison among various detectors.

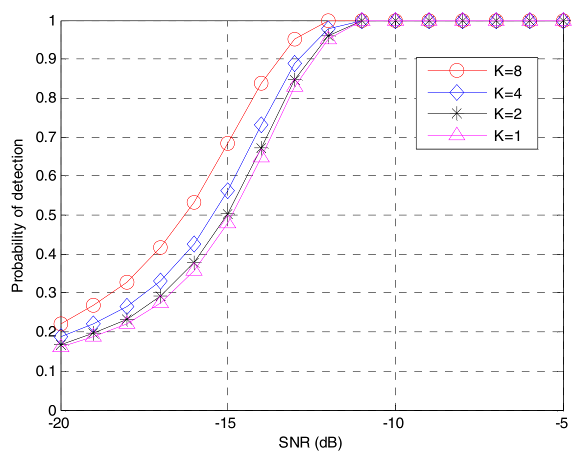

Fig. 5. Three-stage detector performance for different number of sensors.

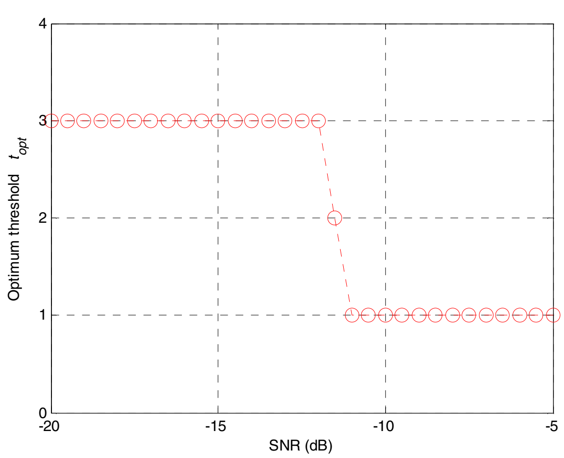

Fig. 3. Optimum threshold t_{opt} for the VR stage of the three-stage detector.

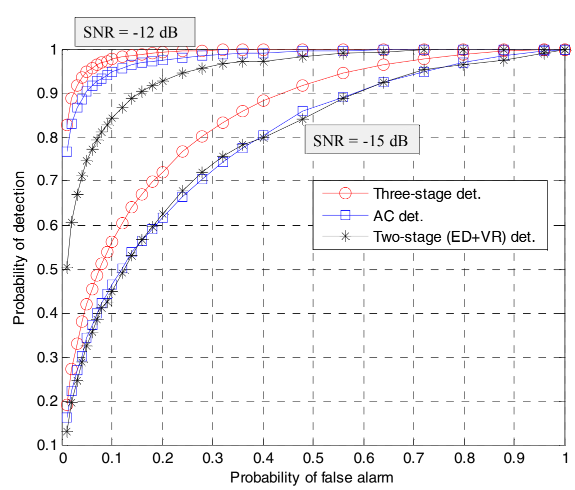

Fig. 6. ROC of different detectors.